Occupational mobility and vocational training over the life cycle*

Abstract

This paper examines the inefficiency issues of occupation-specific human capital investments in a model with search frictions and occupational mobility over the life cycle. Firstly, we highlight how the existence of social externalities related to training, not taken into account by employers, cause an under-investment in occupation-specific vocational training compared with what it would be optimal to do. Secondly, we show that this lack of training leads to some occupational mobility inefficiencies. Finally, we calibrate the model on the French economy and show that implementing the optimal policy would account for significant increase in employment and productivity, through its impact on vocational training, occupational mobility and search frictions.

- **JEL Classification**: J24, J62, J64, J68
- **Keywords:** Human capital, Social externalities, Occupational mobility, Life cycle

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1 Introduction

Becker [1964] classified human capital investment as *general* if it raises the worker's productivity equally in all firms, and *specific* if it increases the worker's productivity only in the firm providing the training. According to standard human capital theory, workers in perfectly competitive labour markets will pay for general training whereas employers pay only for the firm-specific component of training that does not help the worker receive higher wages elsewhere. As training is mostly financed by firms, it has long been considered by most economists as specific.

However, perhaps somewhat surprisingly in light of the emphasis that labor economists have placed on specific training, recent empirical work in the human capital literature suggests that part of the skills acquired through training are transferable. Loewenstein and Spletzer [1999] use data from both the Employer Opportunity Pilot Project survey (EOPP) and the National Longitudinal Survey of Youth (NLSY) to show that the majority of employer-provided training is general. Blundell et al. [1996, 1999] use data from the National Child Development Survey (NCDS) to show that the return from on-the-job training undertaken with a previous employer is similar to the return from on-the-job training undertaken with the current employer. Parent [1999] also states that employers seem to reward skills acquired through training with previous employers as much as skills they provide themselves but notes that part of the skills acquired through training programs provided by the current employer seem to be fairly specific as they are shown to reduce mobility. From a theoretical point of view, Acemoglu and Pischke [1999a,b] proposed an alternative to the Becker's on-the-job training model by highlighting that imperfections in the labor market (asymmetric information, labor market frictions) can justify the fact that firms bear the cost of general training.

Nevertheless, more detailed studies suggest that vocational training is neither purely general, nor purely specific. Neal [1995] shows that displaced workers who find new jobs in their predisplacement industry earn significantly greater returns to both their predisplacement experience and tenure than observationally similar workers who switch industries following displacement. Parent [2000] hightlights that what matters most for the wage profile in terms of human capital is industry specificity, not firm-specificity. Over the last few years, Kambourov and Manovskii [2008, 2009a,b] have demonstrated that an important component of human capital is occupation-specific.

Considering that human capital acquired through firm-funded training is transferable to other firms, vocational training gives rise to some inefficiency issues. Firstly, Acemoglu [1997] and Acemoglu and Pischke [1999a] highlighted the impact of a poaching externality: as general human capital investments can benefit, with some probability, to some future employers, the current firm's private return of training investment is lower than its social return. Belan and Chéron [2014] also argued that due to a higher job finding rate of workers with a higher general human capital, the training of workers implies an additional unemployment externality: the social return of training indeed embodies the fact that unemployed worker with higher general human capital will switch faster from home production to market production. Chéron and Terriau [2016, 2017] extended this infinitely-lived agents approach by considering a finite horizon of workers in the labor market in order to determine to what extent search frictions and externalities related to training can interact each other over the life cycle. However, this model does not allow for on-the-job search and human capital inverstments are considered as purely general.

We propose a lifecycle model with search frictions, endogenous occupational mobility and endogenous occupational training decisions to analyse the inefficiency issues of employer-provided training. The remaining of the paper is organized as follows. We first present some stylized facts regarding training and mobility and their interactions with age. Next, we present the model, lay out the economic environment and characterize both equilibrium and optimal age-dynamics of training policy to identify related externalities over the life cycle. We lastly implement an empirical investigation with a calibration of the model on the French economy. A final section concludes.

2 Stylized facts

2.1 Effect of training on productivity

Theoretically, employers may fully or partially fund the training of workers in the hope of gaining a profitable return on this investment by productivity improvements. Many economists have attempted to demonstrate empirically the relationship between training and labor productivity using data on individual workers. Since information on productivity is very limited, these studies take an indirect approach, relying on the observed relationship between training and wages as evidence of a relationship between training and productivity (Lillard and Tan [1986], Brown [1989], Booth [1991], Lynch [1992], Bartel [1995], Blundell et al. [1996]). Blundell et al. [1996] state that individuals undertaking employer-provided or vocational training earn, on average, just above 5 per cent higher real earnings than individuals who have not undertaken such training, with some studies showing higher rates. In the French case, Chéron et al. [2010] conclude that training participation increases wages by 7%, the wage premium remaining flat along the wage distribution. Although these studies are informative, they only tell half the story if we consider that part of the gains from training are retained by firms, so that the impact of training on productivity is under-estimated. Some of the researchers who have been able to examine the linkage between training and productivity (Barron et al. [1987], Bishop [1994]) have used a subjective measure of productivity such as the answer to the survey question: "on a scale of 1-4 how has your productivity changed over the last year?". Bishop [1994] concludes that employer-provided training raises this subjective productivity measure by almost 16 percent. Using more objective measures of performance, other studies find that vocational training is associated with significantly higher productivity (Holzer et al. [1993], Bartel [1994], Black and Lynch [1996], Barrett and O'Connell [2001], Dearden et al. [2006], Konings and Vanormelingen [2015]). In particular, Konings and Vanormelingen [2015] estimate the productivity premium for a trained worker at 23%, while the wage premium of training is estimated at 12%. In France, Carriou and Jeger [1997], Delame and Kramarz [1997], Ballot et al. [2006] and Aubert et al. [2009] confirm positive effects of training on productivity and show that firms indeed obtain more than half of the returns to their human capital investments.

2.2 Effect of training on mobility

Recent studies show that firm-provided training reduce the probability of leaving an employer. Lynch [1991], using data taken from NLSY on workers in their first jobs after leaving school, finds that those who receive company training are likely to have longer employment durations than those who receive no company training, while workers receiving training from outside the firm are more likely to leave their jobs. Dearden et al. [1997] show with data from the Labour Force Survey (LFS) and NCDS that receiving training decreases the probability of moving jobs. Finally, Parent [1999]) using NLSY data from 1979-91 finds that workplace training reduce turnover, thus resulting in longer employment duration. In the french case, Chéron et al. [2010] confirm that the probability to switch firms is higher for untrained than for trained workers. While Belan and Chéron

[2014] and Chéron and Terriau [2016, 2017] integrate the impact of training on transitions from unemployment to employment in their theoretical models, the interactions between human capital investments and mobility has received far less attention.

2.3 The determinants of training

Numerous studies focused on the determinants of training (Lillard and Tan [1986], Blundell et al. [1996], Bassanini et al. [2005]; see Blasco et al. [2009] in the french case) and there is broad consensus that training decisions of the firms are motivated by individual characteristics. In particular, training decreases with age as a result of an effect of horizon and strongly increases with education, so that only skilled workers are trained. We propose to summarize training decisions by determining an endogenous ability level required to be trained, occupation-specific and age-dependent, in order to determine to what extent search frictions, occupational mobility and externalities related to training investments in occupation-specific human capital can interact each other over the life cycle.

3 Framework

Time is discrete. There are a finite number of occupations indexed by $k = 1, 2, 3...K$ which differ in terms of their productivity with p_k the productivity of the k^{th} occupation and $p_1 < p_2 < p_3 < ... < p_K$. Each occupation is represented by a firm. Workers are characterized by their ability level, denoted by a, distributed over the inteval $[\underline{a}, \bar{a}]$ according to p.d.f. $f(a)$, and by their age, denoted by t. Once matched, the worker and the firm produce according to an occupation-specific production technology. The production function combines worker skills a and the productivity of the occupation k to create value added:

- $(1+\Delta)ap_k$, if the worker has occupation-specific skills (as a result of a previous training in this occupation) or if the firm choose to finance occupation-specific human capital investments
- ap_k, if he has no occupation-specific skills and an ability level a too low to be trained

At the time of hiring, firms can choose to train workers without occupation-specific human capital (OSHC) in order to improve their skills from a to $(1+\Delta)a$[1]. This leads firms to bear an instantaneous training cost γ_f. Obviously, this intertemporal decision will depend on workers' ability, so that the training policy will consist in determining an ability threshold that is age-dependent and occupation-specific, denoted $\tilde{a}_{t,k}$.

Therefore, workers are heterogenous according to three dimensions: (i) ability a, (ii) age t, and (iii) OSHC. This implies in particular that we need to distinguish three types of agents at the time of hiring:

- Type-0, without OSHC and unable for training in occupation k, $(a < \tilde{a}_{t,k})$, with expected instantaneous productivity ap_k;
- Type-1, without OSHC and able for training in occupation k, $(a \geq \tilde{a}_{t,k})$, with expected instantaneous productivity $(1+\Delta)ap_k$ once the cost γ_f will have been paid;
- Type-2, with OSHC and expected instantaneous productivity $(1+\Delta)ap_k$ without any additional cost.

As the ability threshold $\tilde{a}_{t,k}$ required to be trained is specific to an occupation, note that some workers can have a level of ability high enough to be trained in occupation k but too low to be trained in an other occupation. Furthermore, it can be the case that the ability of a worker is high enough at age t to be trained in occupation k, but no longer at age $t+1$ as a result of an horizon effect.

3.1 Matching and bargaining

Unemployed workers meet firms with probability λ^{u_j} while employed workers receive outside offers from another occupation at rate λ^{e_j}. All workers, employed and unemployed, sample from the same exogenous job offer distribution $F(\omega)$. Wages are restricted to fixed wage contracts and can only be re-bargained when either party has a *credible threat.*

[1] This decomposition of human capital can be viewed as a particular case of Wasmer [2006] in which general skills are acquired at school, at the beginning of the worker's life, while occupation-specific skills can be obtained at work through employer-provided training.

Let $E_{j,t,a}(k, NB)$, $U_{j,t,a}(k)$, $J_{j,t,a}(k, NB)$ and $S_{j,t,a}(k)$ denote respectively the value of an employed worker, the value of an unemployed worker, the value of a job and the joint surplus between a worker of type j, age t and ability a and a firm (occupation) k. Using the terminology of Jarosch [2014], the worker's wage $w_{j,t,a}(k, NB)$, value $E_{j,t,a}(k, NB)$ and value $J_{j,t,a}(k, NB)$ are a function of her current employer k and the firm (or value of unemployment) he used as outside option in his last wage negotiation, the "Negociation Benchmark", denoted NB.

Then wages are pinned down in the tradition of the sequential auction framework pioneered by Postel-Vinay and Robin [2002a,b] and developped further in Cahuc et al. [2006]. Specifically, if an unemployed worker and a firm k choose to form a match, the wage implements a surplus split with worker share $\alpha \in [0, 1]$. The outside option of the worker is the unemployment, so $NB = U_{j,t,a}(k)$.

$$E_{j,t,a}(k, u) - U_{j,t,a}(k) = \alpha S_{j,t,a}(k) \tag{1}$$

I denote with $M_{t,a}^{U_j \to E_0}$ and $M_{t,a}^{U_j \to E_1}$ the sets of firms an unemployed worker is willing to work for, respectively if he is unable for training $S_{1,t,a}(\omega) < S_{0,t,a}(\omega)$ and if he is able for training $S_{1,t,a}(\omega) \geq S_{0,t,a}(\omega)$.

If a worker employed at firm k receive an offer from an outside firm ω, there are three cases:

First, if $S_{j,t,a}(\omega) < S_{j,t,a}(k)$, the worker stays with his current employer, but may use the outside offer to **renegotiate** his wage according to :

$$E_{j,t,a}(k, \omega) - U_{j,t,a} = S_{j,t,a}(\omega) + \alpha \left(S_{j,t,a}(k) - S_{j,t,a}(\omega)\right) \tag{2}$$

Therefore I denote the set of firms that belong to the first case with $M_{t,a}^{R_j}(k, NB)$ where $\omega \in M_{t,a}^{R_j}(k, NB)$ if $S_{j,t,a}(k) > S_{j,t,a}(\omega) > S_{j,t,a}(NB)$.

Second, if the worker has a higher joint surplus with firm ω, $S_{j,t,a}(\omega) > S_{j,t,a}(k)$, he transfers to firm ω. In that case, his old employer k becomes her new negotiation benchmark.

$$E_{j,t,a}(\omega, k) - U_{j,t,a} = S_{j,t,a}(k) + \alpha \left(S_{j,t,a}(\omega) - S_{j,t,a}(k)\right) \tag{3}$$

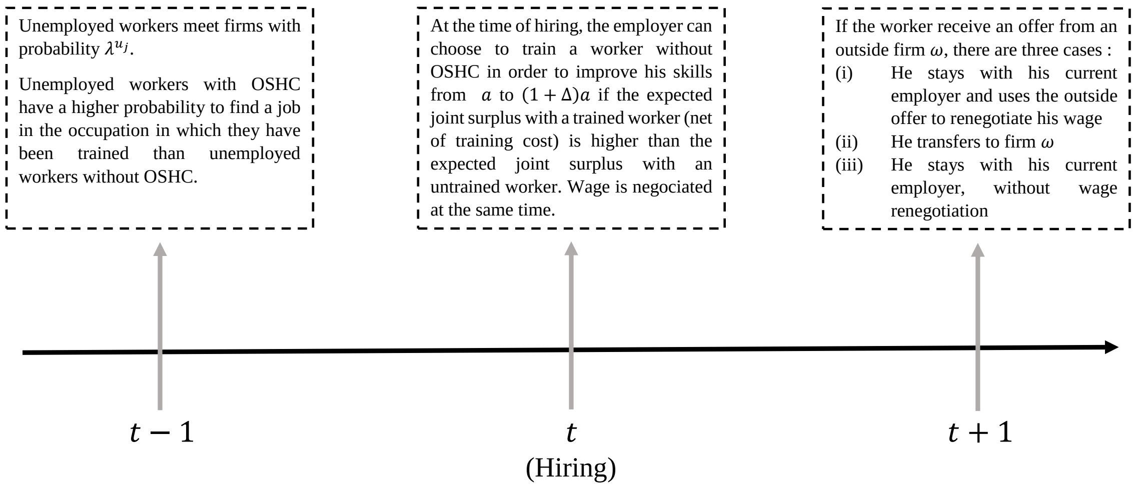
Thus, he receives the full surplus of his former job in occupation k plus a share α of the net gains from the move to firm ω. I denote the set of firms that correspond to this second case as $M_{t,a}^{E_j \to E_0}(k)$ for a worker unable to be trained in his new occupation (type 0) and as $M_{t,a}^{E_j \to E_1}(k)$ if it is profitable for the new firm to train the worker (type 1). That is on the one hand $\omega \in M_{t,a}^{E_j \to E_0}(k)$ if $S_{j,t,a}(\omega) > S_{j,t,a}(k)$ and $S_{1,t,a}(\omega) < S_{0,t,a}(\omega)$, and on the other hand $\omega \in M_{t,a}^{E_j \to E_1}(k)$ if $S_{j,t,a}(\omega) > S_{j,t,a}(k)$ and $S_{1,t,a}(\omega) \geq S_{0,t,a}(\omega)$. Note that a type-2 worker loses his OSHC if he decides to move to another firm (to change occupation) so that there is no direct transition between type-2 jobs.

In the third case, the outside offer is dominated by a previous outside offer and the worker just discards it and continues to work with k at his current wage.

$$E_{j,t,a}(k, NB) - U_{j,t,a} = S_{j,t,a}(NB) + \alpha \left(S_{j,t,a}(k) - S_{j,t,a}(NB)\right) \tag{4}$$

The determination of joint surplus and value functions is detailed in appendix A, B, C, D, E. The figure 1 displays the timing of the model.

Figure 1: Timing



Note that occupational mobility changes endogenously over the life cycle because the horizon effect reduces outside training opportunities or because the negociation benchmark rises with the worker's tenure. Therefore, our life cycle approach is particularly well suited to understanding the interactions between hiring decisions, training policy and occupational mobility.

3.2 Value functions

3.2.1 Value of employment

A worker employed at firm k who receive an offer from an outside firm ω, decide to:

(i) Stay with his current employer and use the outside offer to renegotiate his wage if $\omega \in M_{t,a}^{R_j}(k, NB)$

(ii) Transfer to firm ω, without training, if $\omega \in M_{t,a}^{E_j \to E_0}(k)$

(iii) Transfer to firm ω, with a training provided by his new employer, if $\omega \in M_{t,a}^{E_j \to E_1}(k)$

(iv) Stay with his current employer, without wage renegotiation, if the outside offer is dominated by a previous outside offer

It then follows that the expected value of employment for a worker with type j, age t and ability a matched with a firm of type k with negociation benchmark NB is given by:

Type 0:

$$E_{0,t,a}(k, NB) = w_{0,t,a}(k, NB) + \beta \Bigg[(1-\delta) \Bigg[\lambda^{e_0} \Bigg(\int_{\omega \in M_{t+1,a}^{R_0}(k,NB)} E_{0,t+1,a}(k,\omega)\, \mathrm{d}F(\omega) + \int_{\omega \in M_{t+1,a}^{E_0 \to E_0}(k)} E_{0,t+1,a}(\omega,k)\, \mathrm{d}F(\omega) + \int_{\omega \in M_{t+1,a}^{E_0 \to E_1}(k)} E_{1,t+1,a}(\omega,k)\, \mathrm{d}F(\omega) \Bigg) + \Bigg(1 - \lambda^{e_0} \int_{\omega \in M_{t+1,a}^{R_0}(k,NB) \cup M_{t+1,a}^{E_0 \to E_0}(k) \cup M_{t+1,a}^{E_0 \to E_1}(k)} \mathrm{d}F(\omega) \Bigg) E_{0,t+1,a}(k, NB) \Bigg] + \delta U_{0,t+1,a} \Bigg] \tag{5}$$

Type 1:

$$E_{1,t,a}(k, NB) = w_{1,t,a}(k, NB) + \beta \Bigg[(1-\delta) \Bigg[\lambda^{e} \Bigg(\int_{\omega \in M_{t+1,a}^{R_1}(k,NB)} E_{1,t+1,a}(k,\omega)\, \mathrm{d}F(\omega) + \int_{\omega \in M_{t+1,a}^{E_1 \to E_0}(k)} E_{0,t+1,a}(\omega,k)\, \mathrm{d}F(\omega) + \int_{\omega \in M_{t+1,a}^{E_1 \to E_1}(k)} E_{1,t+1,a}(\omega,k)\, \mathrm{d}F(\omega) \Bigg) + \Bigg(1 - \lambda^{e} \int_{\omega \in M_{t+1,a}^{R_1}(k,NB) \cup M_{t+1,a}^{E_1 \to E_0}(k) \cup M_{t+1,a}^{E_1 \to E_1}(k)} \mathrm{d}F(\omega) \Bigg) E_{1,t+1,a}(k, NB) \Bigg] + \delta U_{t+1,a}(k) \Bigg] \tag{6}$$

Type 2:

$$E_{2,t,a}(k,NB) = w_{2,t,a}(k,NB) + \beta\Bigg[(1-\delta)\Bigg[\lambda^e\Bigg(\int_{\omega\in M^{R_2}_{t+1,a}(k,NB)} E_{2,t+1,a}(k,\omega)\,\mathrm{d}F(\omega)$$

$$+\int_{\omega\in M^{E_2\to E_0}_{t+1,a}(k)} E_{0,t+1,a}(\omega,k)\,\mathrm{d}F(\omega) + \int_{\omega\in M^{E_2\to E_1}_{t+1,a}(k)} E_{1,t+1,a}(\omega,k)\,\mathrm{d}F(\omega)\Bigg) \tag{7}$$

$$+\Bigg(1-\lambda^e\int_{\omega\in M^{R_2}_{t+1,a}(k,NB)\cup M^{E_2\to E_0}_{t+1,a}(k)\cup M^{E_2\to E_1}_{t+1,a}(k)} \mathrm{d}F(\omega)\Bigg)E_{2,t+1,a}(k,NB)\Bigg] + \delta U_{t+1,a}(k)\Bigg]$$

3.2.2 Value of unemployment

All workers enter the labour market as unemployed without OSHC. We also assume that layoffs occur at the exogenous rate δ. In such cases, a laid off worker move into unemployment U_0 if he has no OSHC (that is he was previously employed in a type-0 job) or into unemployent U if he has OSHC (that is he was previously employed in a type-1 or type-2 job). Note that unemployed workers with OSHC have a higher probability to find a job in the occupation in which they have been trained than unemployed workers without OSHC (as a result of the unemployment externality highlighted by Belan and Chéron [2014]).

The expected value of unemployment for a worker with type j, age t and ability a is given by:

For a worker without OSHC:

$$U_{0,t,a} = b + \beta\Bigg[\lambda^{u_0}\Bigg(\int_{\omega\in M^{U_0\to E_0}_{t+1,a}} E_{0,t+1,a}(\omega,u)\,\mathrm{d}F(\omega) + \int_{\omega\in M^{U_0\to E_1}_{t+1,a}} E_{1,t+1,a}(\omega,u)\,\mathrm{d}F(\omega)\Bigg)$$

$$+\Bigg(1-\lambda^{u_0}\int_{\omega\in M^{U_0\to E_0}_{t+1,a}\cup M^{U_0\to E_1}_{t+1,a}} \mathrm{d}F(\omega)\Bigg)U_{0,t+1,a}\Bigg] \tag{8}$$

For a worker with OSHC:

$$U_{t,a}(k) = b + \beta\Bigg[\lambda^{u_0}\Bigg(\int_{\omega\in M^{U\to E_0}_{t+1,a}(k)} E_{0,t+1,a}(\omega,u)\,\mathrm{d}F(\omega) + \int_{\omega\in M^{U\to E_1}_{t+1,a}(k)} E_{1,t+1,a}(\omega,u)\,\mathrm{d}F(\omega)\Bigg)$$

$$+\lambda^u\Bigg(\int_{\omega\in k} E_{2,t+1,a}(\omega,u)\,\mathrm{d}F(\omega)\Bigg)$$

$$+\Bigg(1-\lambda^{u_0}\int_{\omega\in M^{U\to E_0}_{t+1,a}(k)\cup M^{U\to E_1}_{t+1,a}(k)} \mathrm{d}F(\omega) - \lambda^u\int_{\omega\in k}\mathrm{d}F(\omega)\Bigg)U_{t+1,a}(k)\Bigg] \tag{9}$$

Note that we can rewritte the values of unemployment as functions of joint surplus (see Appendix A, B):

$$U_{0,t,a} = b + \beta\left[\alpha\lambda^{u_0}\left(\int_{\omega\in M_{t+1,a}^{U_0\to E_0}} S_{0,t+1,a}(\omega)\,\mathrm{d}F(\omega) + \int_{\omega\in M_{t+1,a}^{U_0\to E_1}} S_{1,t+1,a}(\omega)\,\mathrm{d}F(\omega)\right) + U_{0,t+1,a}\right] \tag{10}$$

$$\begin{aligned} U_{t,a}(k) = {} & b + \beta\Bigg[\alpha\lambda^{u_0}\left(\int_{\omega\in M_{t+1,a}^{U\to E_0}(k)} S_{0,t+1,a}(\omega)\,\mathrm{d}F(\omega) + \int_{\omega\in M_{t+1,a}^{U\to E_1}(k)} S_{1,t+1,a}(\omega)\,\mathrm{d}F(\omega)\right) \\ & +\alpha\lambda^{u}\int_{\omega\in k} \mathrm{d}F(\omega) S_{2,t+1,a}(\omega) \\ & -\lambda^{u_0}\left(\int_{\omega\in M_{t+1,a}^{U\to E_0}(k)\cup M_{t+1,a}^{U\to E_1}(k)} \left(U_{t+1,a}(k) - U_{0,t+1,a}\right)\mathrm{d}F(\omega)\right) + U_{t+1,a}(k)\Bigg] \end{aligned} \tag{11}$$

3.2.3 Value of a filled job

The expected value of a filled job by a worker of type j, age t and ability a matched with a firm of type k with negociation benchmark NB is given by:

Type 0:

$$\begin{aligned} J_{0,t,a}(k, NB) = {} & ap_k - w_{0,t,a}(k, NB) + \beta(1-\delta)\Bigg[\lambda^{e_0}\int_{\omega\in M_{t+1,a}^{R_0}(k,NB)} J_{0,t+1,a}(k,\omega)\,\mathrm{d}F(\omega) \\ & +\left(1 - \lambda^{e_0}\int_{\omega\in M_{t+1,a}^{R_0}(k,NB)\cup M_{t+1,a}^{E_0\to E_0}(k)\cup M_{t+1,a}^{E_0\to E_1}(k)} \mathrm{d}F(\omega)\right) J_{0,t+1,a}(k, NB)\Bigg] \end{aligned} \tag{12}$$

Type 1:

$$\begin{aligned} J_{1,t,a}(k, NB) = {} & (1+\Delta)ap_k - w_{1,t,a}(k, NB) + \beta(1-\delta)\Bigg[\lambda^{e}\int_{\omega\in M_{t+1,a}^{R_1}(k,NB)} J_{1,t+1,a}(k,\omega)\,\mathrm{d}F(\omega) \\ & +\left(1 - \lambda^{e}\int_{\omega\in M_{t+1,a}^{R_1}(k,NB)\cup M_{t+1,a}^{E_1\to E_0}(k)\cup M_{t+1,a}^{E_1\to E_1}(k)} \mathrm{d}F(\omega)\right) J_{1,t+1,a}(k, NB)\Bigg] \end{aligned} \tag{13}$$

Type 2:

$$\begin{aligned} J_{2,t,a}(k, NB) = {} & (1+\Delta)ap_k - w_{2,t,a}(k, NB) + \beta(1-\delta)\Bigg[\lambda^{e}\int_{\omega\in M_{t+1,a}^{R_2}(k,NB)} J_{2,t+1,a}(k,\omega)\,\mathrm{d}F(\omega) \\ & +\left(1 - \lambda^{e}\int_{\omega\in M_{t+1,a}^{R_2}(k,NB)\cup M_{t+1,a}^{E_2\to E_0}(k)\cup M_{t+1,a}^{E_2\to E_1}(k)} \mathrm{d}F(\omega)\right) J_{2,t+1,a}(k, NB)\Bigg] \end{aligned} \tag{14}$$

3.3 Equilibrium training policy

The firm's training policy consists in determining, for a given age t and a given occupation k, an ability threshold $\tilde{a}_{t,k}$ above which any worker without occupation-specific human capital is trained at the time of hiring. Hence, $\tilde{a}_{t,k}$ satisfies the following condition :

$$S_{1,t,\tilde{a}_{t,k}}(k) = S_{0,t,\tilde{a}_{t,k}}(k) \tag{15}$$

With the following joint surplus (see Appendix C, D, E) and $a = \tilde{a}_{t,k}$:

$$\begin{aligned} S_{0,t,a}(k) \;=\;& ap_k - b + \beta\Bigg[(1-\delta)\Bigg[S_{0,t+1,a}(k) \\ &+\alpha\lambda^{e_0}\Bigg(\int_{\omega\in M^{E_0\to E_0}_{t+1,a}(k)}\Big(S_{0,t+1,a}(\omega) - S_{0,t+1,a}(k)\Big)\,\mathrm{d}F(\omega) \\ &+\int_{\omega\in M^{E_0\to E_1}_{t+1,a}(k)}\Big(S_{1,t+1,a}(\omega) - S_{0,t+1,a}(k)\Big)\,\mathrm{d}F(\omega)\Bigg)\Bigg] \\ &-\alpha\lambda^{u_0}\Bigg(\int_{\omega\in M^{U_0\to E_0}_{t+1,a}} S_{0,t+1,a}(\omega)\,\mathrm{d}F(\omega) + \int_{\omega\in M^{U_0\to E_1}_{t+1,a}} S_{1,t+1,a}(\omega)\,\mathrm{d}F(\omega)\Bigg)\Bigg] \end{aligned} \tag{16}$$

$$\begin{aligned} S_{1,t,a}(k) \;=\;& (1+\Delta)ap_k - b - \gamma_f + \beta\Bigg[\delta\Big[U_{t+1,a}(k) - U_{0,t+1,a}\Big] + (1-\delta)\Bigg[S_{1,t+1,a}(k) + \gamma_f \\ &+\alpha\lambda^{e}\Bigg(\int_{\omega\in M^{E_1\to E_0}_{t+1,a}(k)}\Big(S_{0,t+1,a}(\omega) - \big(S_{1,t+1,a}(k)+\gamma_f\big)\Big)\,\mathrm{d}F(\omega) \\ &+\int_{\omega\in M^{E_1\to E_1}_{t+1,a}(k)}\Big(S_{1,t+1,a}(\omega) - \big(S_{1,t+1,a}(k)+\gamma_f\big)\Big)\,\mathrm{d}F(\omega)\Bigg)\Bigg] \\ &-\alpha\lambda^{u_0}\Bigg(\int_{\omega\in M^{U_0\to E_0}_{t+1,a}} S_{0,t+1,a}(\omega)\,\mathrm{d}F(\omega) + \int_{\omega\in M^{U_0\to E_1}_{t+1,a}} S_{1,t+1,a}(\omega)\,\mathrm{d}F(\omega)\Bigg)\Bigg] \end{aligned} \tag{17}$$

As $\Big[U_{t+1,a}(k) - U_{0,t+1,a}\Big]$ can be rewritten as a function of joint surplus (see equations 10 & 11), the problem can be solved recursively starting from terminal condition at $t = T - 1$ using only the joint surplus functions.

3.4 Efficient training policy

The efficient training policy is now derived by determining, for a given age t and a given occupation k, an ability threshold $a^\star_{t,k}$ above which any worker without occupation-specific human capital is trained by the social planner at the time of hiring. Hence, $a^\star_{t,k}$ satisfies the following condition :

$$\tilde{Y}_{t,a}(k) - \gamma_f = \hat{Y}_{t,a}(k) \tag{18}$$

With the following social values (see Appendix G) and $a = a^\star_{t,k}$:

$$\begin{aligned}
\hat{Y}_{t,a}(k) \quad = \quad & ap_k + \beta\Bigg[(1-\delta)\Bigg[\lambda^e\Bigg(\int_{\omega \in M^{\hat{Y}\to\hat{Y}}_{t+1,a}(k)} \hat{Y}_{t+1,a}(\omega)\,\mathrm{d}F(\omega) \\
& + \int_{\omega \in M^{\hat{Y}\to\tilde{Y}}_{t+1,a}(k)} (\tilde{Y}_{t+1,a}(\omega) - \gamma_f)\,\mathrm{d}F(\omega)\Bigg) \\
& + \Bigg(1 - \lambda^e \int_{\omega \in M^{\hat{Y}\to\hat{Y}}_{t+1,a}(k) \cup M^{\hat{Y}\to\tilde{Y}}_{t+1,a}(k)} \mathrm{d}F(\omega)\Bigg)\hat{Y}_{t+1,a}(k)\Bigg] + \delta Y^{u_0}_{t+1,a}\Bigg]
\end{aligned} \tag{19}$$

$$\begin{aligned}
\tilde{Y}_{t,a}(k) \quad = \quad & (1+\Delta)ap_k + \beta\Bigg[(1-\delta)\Bigg[\lambda^e\Bigg(\int_{\omega \in M^{\tilde{Y}\to\hat{Y}}_{t+1,a}(k)} \hat{Y}_{t+1,a}(\omega)\,\mathrm{d}F(\omega) \\
& + \int_{\omega \in M^{\tilde{Y}\to\tilde{Y}}_{t+1,a}(k)} (\tilde{Y}_{t+1,a}(\omega) - \gamma_f)\,\mathrm{d}F(\omega)\Bigg) \\
& + \Bigg(1 - \lambda^e \int_{\omega \in M^{\tilde{Y}\to\hat{Y}}_{t+1,a}(k) \cup M^{\tilde{Y}\to\tilde{Y}}_{t+1,a}(k)} \mathrm{d}F(\omega)\Bigg)\tilde{Y}_{t+1,a}(k)\Bigg] + \delta Y^{u}_{t+1,a}(k)\Bigg]
\end{aligned} \tag{20}$$

Comparing the firms' training policy with the social planner's policy and using equations 22, 23, 24, 29, 30 and 31, we can note that if employees have the full bargaining power, $\alpha = 1$, equilibrium training policy and mobility are efficient (see Appendix L).

4 On life cycle effects of training externalities: An illustration with a two-occupation model

For a worker with age t, occupational mobility depends on his ability level a and the productivity of the occupation k. We first present a simplified version of the model with two occupations ($k = 1, 2$ with $p_1 < p_2$) in order to highlight the social externalities related to occupation-specific human capital investments and occupational mobility. Figure 2 displays the ability thresholds required to be trained over the life cycle for each occupation.

- If a worker is employed in the most productive occupation ($k = 2$), there is no incentive to move to another occupation. In other words, the employee refuses any outside offer and prefers to stay in his current occupation. As a consequence, there is no occupational mobility externalities for this type of worker.

- If a worker is employed in a less productive occupation ($k = 1$), there are some incentives to move to another occupation, according to the level of ability.

  – At point **A** (worker with age t and ability $a > \tilde{a}_{t,1}$), the worker is able for training in each occupation, both from the firm and the social planer point of view. Present training decision and occupational mobility are not affected by taking into account the social externalities related to training. However, there are some differences concerning future outside opportunities. We denote the age limit up to which a worker, with age t and ability a, is trained in occupation k respectively by $t_k^{\sim}(a)$ at equilibrium and $t_k^*(a)$ at optimum. A worker employed in occupation $k = 1$ accepts unconditionally an offer from occupation $k = 2$ until the age of $t_2^{\sim}(A)$ at equilibrum, compared to $t_2^*(A)$ at optimum. After this age, the outside offer can be refused if the additional productivity related to training received in occupation $k = 1$ is higher than the productivity gain by moving from $k = 1$ to $k = 2$ without training in the new occupation. As $t_2^*(A) > t_2^{\sim}(A)$, the value of the outside offer at optimum is higher at optimum between these two age limits, so that a worker in occupation $k = 1$ with age $t \in]t_2^{\sim}(A), t_2^*(A)]$ can move to occupation $k = 2$ at optimum but not at equilibrium. From this point of view, there is an **under-mobility** of workers with ability $a > \tilde{a}_{t,1}$ in occupation $k = 1$ with age $t \in]t_2^{\sim}(A), t_2^*(A)]$, comparing to what optimal policy suggests to do.

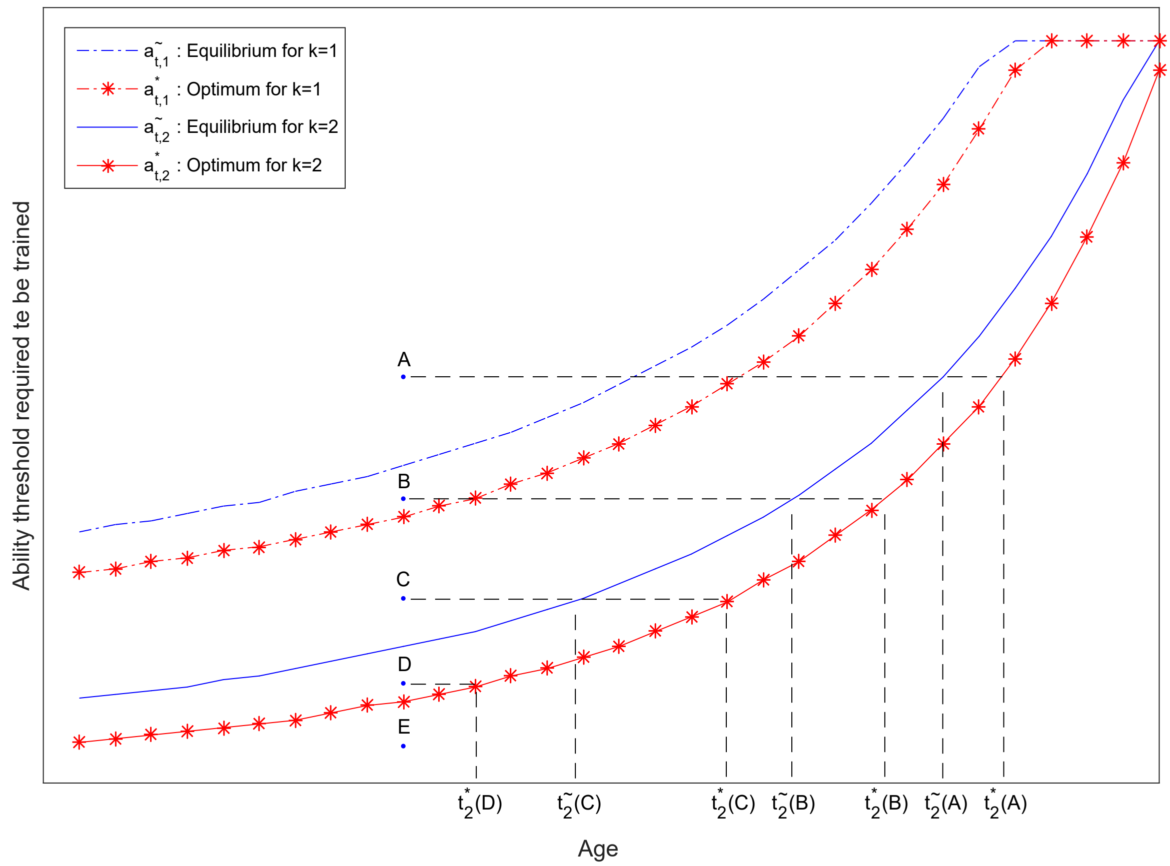
– At point **B** (worker with age t and ability $a \in [a^\star_{t,1}, \tilde{a}_{t,1}[$), the worker is able for training in occupation $k = 2$, both from the firm and the social planer point of view, and has a sufficient level to be trained by the social planer in $k = 1$ but too low to be trained by a firm in this occupation. As a consequence, training lead to a retention effect at optimum, but not at equilibrium. More precisely, at equilibrum, a worker employed in occupation $k = 1$ accepts any offer to move to $k = 2$, whatever his age. In contrast, at optimum, this offer can be refused from the age of $t^*_2(B)$ if the additional productivity due to the training provided by the social planer in occupation $k = 1$ is higher than the productivity gain by moving from $k = 1$ to $k = 2$ without training in the new occupation. From this point of view, there is an **over-mobility** of workers with ability $a \in [a^\star_{t,1}, \tilde{a}_{t,1}[$ in occupation $k = 1$ from the age of $t^*_2(B)$, in comparison with the optimal policy.

– At point **C** (worker with age t and ability $a \in [\tilde{a}_{t,2}, a^\star_{t,1}[$), the worker is able for training in occupation $k = 2$ but not in $k = 1$, both from the firm and the social planer point of view. As a worker with ability C hired in occupation $k = 1$ is not trained, he has no occupation-specific human capital, and he accepts any offer from a most productive occupation ($k = 2$). If the worker receive an outside offer, he can be trained until the age of $t^{\sim}_2(C)$ at equilibrium, and $t^*_2(C)$ at optimum (with $t^{\sim}_2(C) < t^*_2(C)$). Future training opportunities are lower at equilibrium than at optimum, but there is no difference concerning occupational mobility.

– At point **D** (worker with age t and ability $a \in [\tilde{a}_{t,2}, a^\star_{t,2}[$), the worker is not trained by firms but can be trained in the most productive occupation by the social planer. In the same way as for point C, a worker employed in occupation $k = 1$ accepts any offer from $k = 2$, both from the equilibrium and the optimum point of view, so that there is a training externality but no occupation mobility externalities.

– At point **E** (worker with age t and ability $a < a^\star_{t,2}$), the worker has a level of ability too low to be trained, even from the social planer point of view. For this type of worker, there is no training or occupation mobility externalities.

Figure 2: Ability thresholds required to be trained



All in all, using a model where occupational training and mobility decisions over the life cycle are endogenous, we show that occupational mobility is inefficient. More precisely, as the social planner perfectly internalised poaching and unemployment externalities, the social return of training is higher than the firms' private return. As a result, the social planner is less selective than firms in terms of ability thresholds required to be trained. Some workers untrained at equilibrium (but trained at optimum) choose to leave their employer for a more productive one because they can not benefit from the additional output related to training with their current employer. At the opposite, some workers decide, contrary to what optimal policy suggest to do, to stay with their current employer because they don't have outside training opportunities at equilibrium (whereas they can be trained outside at optimum). Finally, this simplified model with two occupations allows to underline that the under-investment in OSHC also leads to inefficient occupational mobility decisions.

5 Simulations

5.1 Calibration

The model is simulated at a quaterly frequency. A first set of parameters is calibrated in a fairly standard way, $\phi_1 = \{T, \beta, b, \alpha\}$. Note that $T = 160$ leads to consider $t = [1, 160]$ by referring to workers from 20 to 60 years old (corresponding to the average retirement age).

A second set of parameters concerns the productivity of workers and firms, $\phi_2 = \{\underline{a}, p_1, p_K\}$. To be consistent with a long right tailed distribution of individual productivities, we consider the following Pareto distribution of abilities:

$$F(a) = \frac{1 - (\underline{a}/a)^\tau}{1 - (\underline{a}/\bar{a})^\tau}$$

where $\underline{a}$ and $\bar{a}$ are respectively the lowest and the highest level of ability. In accordance with the range of estimates of Bontemps et al. [2000], we set $\underline{a} = 0.7$, $\bar{a} = 7$ and $\tau = 2$ such that when computing quantiles related to the distribution $F(a)$, we get $Q3/Q1 = 1.72$ and $P90/P10 = 3.1$. As we consider that each occupation is represented by a firm, we set $f(k) = 1/K$. Lastly, productivity of firms varies from 1 to 1.5 in order to reproduce the inter-industry dispersion of value-added per worker.

A third set of parameters concerns the labour market features, $\phi_3 = \{\delta, \lambda^{u_0}, \lambda^u, \lambda^{e_0}, \lambda^e\}$. Contact probability for unemployed persons with OSHC is $\lambda^{u_0}\frac{K-1}{K} + \lambda^u\frac{1}{K}$. We set $\lambda^u = \lambda^{u_0}(K+1)$, so that the total contact rate for unemployed persons with OSHC is twice larger than for individuals without OSHC. We consider a job destruction probability $\delta = 3.56\%$ per quarter which is in accordance with Hairault et al. [2015]. λ^{u_j} and λ^{e_j} are chosen to be consistent with an average employment rate for workers aged 25-54 of 81.43%, a probability that a previously unemployed person switches occupation upon finding a job of 71.22% (at the 3-digit level), and an average rate of workers who switch occupation of 1.8% per quarter (at the 3-digit level), in accordance with Lalé [2012]. This calibration is in line with the work of Lynch [1991], Dearden et al. [1997] and Parent [1999] (see Chéron et al. [2010] in the French case) who show that untrained workers are more likely to leave their jobs. Although trained workers receive more outside offers with our calibration, they have a greater probability to refuse them, so that their occupational mobility is lower.

Finally, the last set of parameters concerns vocational training, $\phi_4 = \{\Delta, \gamma_f\}$. The impact of employer-provided training on productivity has been the subject of a broad debate. Konings and Vanormelingen [2015] estimate with belgian data the productivity premium for a trained worker at 23%, while the wage premium of training is estimated at 12%. In France, Chéron et al. [2010] evaluate that training participation increases wages by 7%. We adopt a precautionary approach by considering that the wage premium represents half of the productivity premium and set $\Delta = 14\%$. Lastly, we set $\gamma_f = 4.8$ in order to reproduce the total expenditure for firms-provided training (0.55% of GDP), according to the data provided by Chéron et al. [2015].

Table 1: Model parameters

Parameter	Description	Value
T	Retirement age	160
β	Discount factor	0.99
b	Home production	0.7
α	Bargaining power of workers	0.5
$\underline{a}$	Lowest ability level	0.7
$\bar{a}$	Highest ability level	7
p_1	Lowest firm productivity	1
p_K	Highest firm productivity	1.5
δ	Job separation probability	0.0356
λ^{u_0}	Contact rate for unemployed workers without OSHC	0.116
λ^{u}	Contact rate for unemployed workers with OSHC	$\lambda^{u_0}(K+1)$
λ^{e_0}	On-the-job contact rate for workers without OSHC	0.1
λ^{e}	On-the-job contact rate for workers with OSHC	0.8
Δ	Additional output related to OSHC	0.14
γ_f	Training cost	8

5.2 Results

Table 2 presents the fit of the model. The results of the model are close to the observed data in terms of average employment rate, occupational mobility and training expenditure. The model is also able to reproduce the dynamics of human capital investments and mobility decisions over the life cycle.

Table 2: Model fit

Moment	Target	Model
% Employment rate	81.43	81.44
% Previously unemployed persons who switch occupation upon finding a job	71.22	71.76
% Workers who switch occupation per quarter	1.80	1.82
% Training expenditure (as a percentage of GDP)	0.55	0.55

Figure 3 illustrates the training dynamics of employed workers over the life-cycle. The average ability threshold required to be trained is increasing with age, since the return on investment period is shorter at the end of the working life. In particular, firms are highly selective from the age of 50 because the average job duration declines sharply with proximity to retirement age. The right panel of Figure 3 plots the average gap between equilibrium and efficient training selection, with a gap between average ability thresholds expressed as a percentage of the lowest threshold $\underline{a}$. As a result of social externalities related to training (poaching, unemployment and occupational mobility externalities), the efficient training policy is less selective, but as already shown by Chéron and Terriau [2016, 2017], the gap between equilibrium and efficient training policies is not monotonous. As social externalities combine each other, it comes indeed that firms increase too far from retirement the selection into training programs with respect to what it would be optimal to do.

Figure 3: Training policy

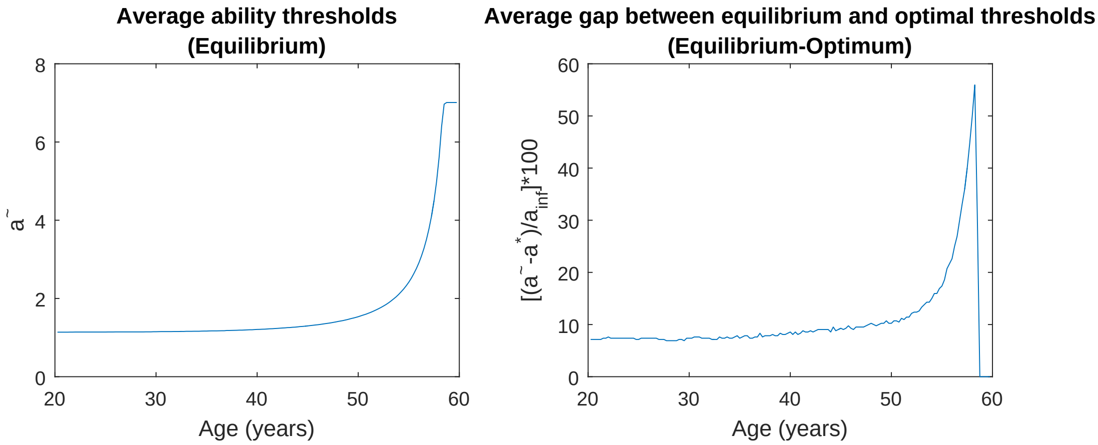


Figure 4 plots the distribution of training expenditure over the life cycle. As expected, most of the investments focus on young workers. Firstly, workers enter the labor market without occupation-specific human capital. As a result, any worker with a sufficient level of ability is trained at the time of hiring. Secondly, ability thresholds required to be trained are lower at the beginning of the life cycle, so that young workers have a greater probability to be trained. Consequently, spending is mostly concentrated on early career workers.

Figure 4: Distribution of training expenditure over the life cycle

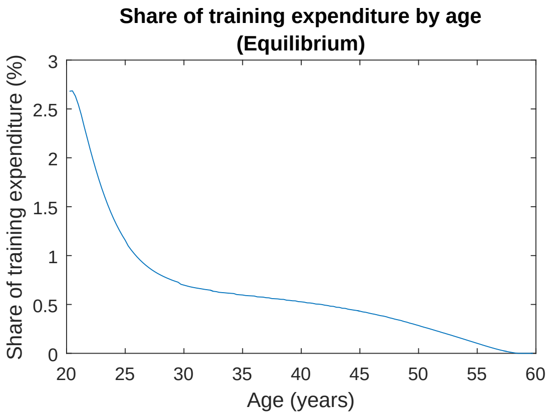


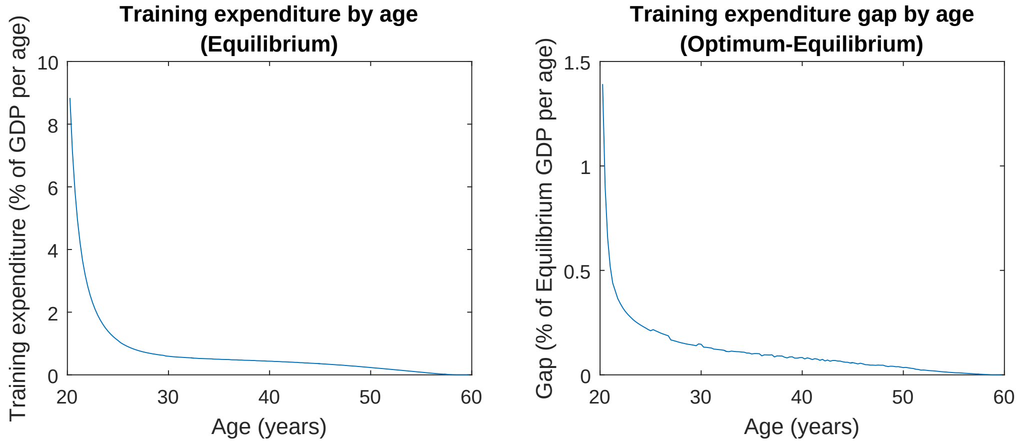
Figure 5 plots the training expenditure by age as a percentage of GDP per age. Training expenditure represents a large part of the production at early stage of the career, in particular before the age of 25. The right panel of Figure 5 focus on the training expenditure gap by age, expressed as a percentage of Equilibrium GDP per age, *ie.*:

$$\left[\frac{\text{Optimal training expenditure (t) - Equilibrium training expenditure (t)}}{\text{Equilibrium GDP (t)}}\right] \times 100$$

Overall, the additional expenditure at optimum represents 0.10% of the equilibrium GDP, which means that to achieve efficiency, training expenditure should represent 0.65% of equilibrium GDP (instead of 0.55%). From a macro-economic point of view, this represents an additional cost of €1.83 billion. It sould be noted that the training expenditure gap is about 0.20% at the age of 25 and converges to zero at the end of the life-cycle.

However, a higher level of training expenditure for young workers does not necessarily mean that the effect of the optimal policy on employment is greater at the beginning of the life cycle. Indeed, Figure 6 shows that young workers are characterized by a very high occupational mobility rate. Thus, a large share of additional expenditures does

Figure 5: Training expenditure



not lead to further social returns because social externalities related to training are lost when workers switch occupation. The right panel of Figure 6 also shows the occupational mobility rate gap by age. Our simulations suggest that the under-invesment of firms in occupation-specific human capital leads, on average, to an excessive occupational mobility of workers compared with what it would be optimal to do. As the social planner choose to train more workers, the efficient training policy leads to a retention effect which reduces occupational mobility. On the one hand, as the gap between equilibrium and efficient selection into training programs is increasing with age, the additional retention effect is stronger for older workers. On the other hand, occupational mobility declines over the life cycle, so that the impact of training on mobility is limited at the end of the career. As a consequence, the occupational mobility rate gap is hump-shaped over the life-cycle.

Figure 6: Occupational mobility

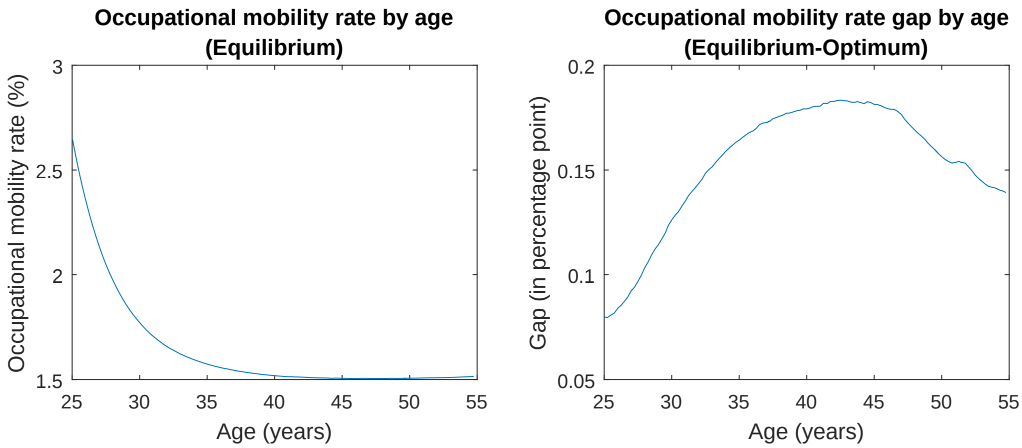


Figure 7 shows the employment rate by age and the employment rate gap between optimum and equilibrium. According to our simulations, implementing the optimal policy would account for substantial increase in employment, in particular for middle-career workers. At the beginning of the life cycle, the additional training expenditure leads to a low gain in employment because young workers have a high mobility rate. As they switch occupation more often, a large part of the social return from training is lost. At the end of the life-cycle, the training policy is more selective. Although the gap between equilibrium and optimum threshold is greater, the density of workers who are between this two thresholds is very low (cf. the distribution of abilities $F(a)$). As a result, the impact of efficient policy on employment is maximal for workers aged 35-45 (about +1.5 percentage point).

Figure 7: Employment

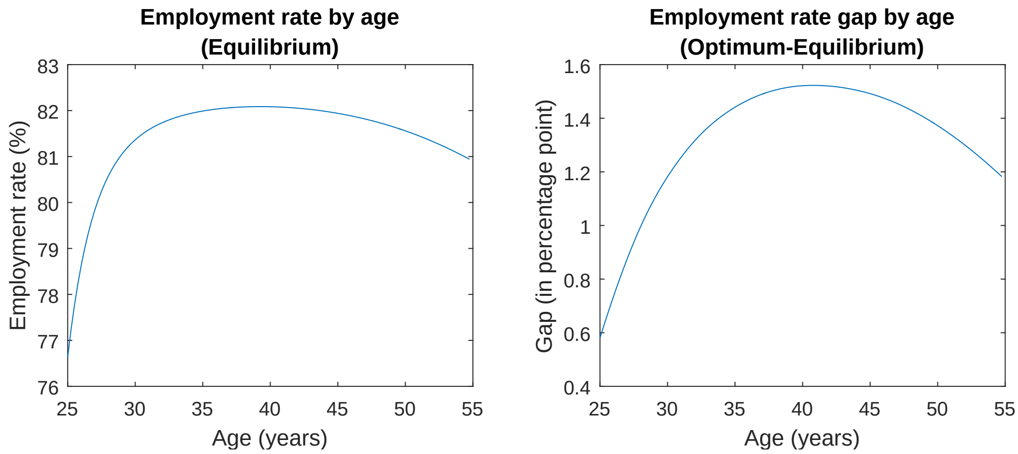


Table 3 shows some aggregated results of the simulations. As we can see, the optimal policy is associated with a higher level of production per capita (+0.86%) which can be explained through improvements in production per worker and gains in employment (+1.62%). First, as the social planner internalise all the returns from training contrary to the firms, the optimal policy training is less selective than the equilibrium one in terms of ability required to be trained. In other words, more workers are trained at optimum and benefit from the additional output related to training, which leads to an increase in individual productivity and employment.

Secondly, training decisions also affect occupational mobility by modifying the value of outside options. Some untrained workers, who have little chance to be trained outside at equilibrium, may have more external training opportunities at optimum, so that they have an incentive to move to a better job (according to the social planner perspective). This is reflected in a greater occupational mobility of untrained workers at optimum. At the opposite, workers who are unable to be trained by firms in their current job but able to be trained from the social planner point of view, may take some external offers which are not acceptable according to the efficient occupational mobility policy. This effect appears to be dominant, since total occupational mobility is lower at optimum. In other words, workers are too mobile from the social planner point of view because the firms' training policy is too selective, so that the only way to have a better salary consists in moving to a more productive occupation.

Lastly, training policy has an impact on employment. As unemployed persons with OSHC benefit from a higher job finding rate, the optimal training policy, which is less selective than the equilibrium one, leads to lower unemployment.

Table 3: Model results

	Equilibrium	Optimum	Gap (%)
Production per capita (net of training costs)	1.6405	1.6545	+0.86%
% Employment rate 25-54	81.44	82.76	+1.63%
% Training expenditure (as a percentage of Equilibrium GDP)	0.55	0.65	+17.31%
% Previously unemployed persons who switch occupation upon finding a job	71.76	67.95	-5.32%
% Workers who switch occupation per quarter (all workers)	1.82	1.66	-8.61%
% Untrained workers who switch occupation per quarter	2.78	3.32	+19.52%

6 Sensitivity analysis

6.1 On-the-job contact rate

We now propose to examine the sensitivity of our results to some calibrated parameters. We first analyse the impact of an increase of 10% of the on-the-job contact rate for workers with OSHC, denoted λ^e. This increase provides an incentive for workers without OSHC to finance a larger part of training (through wage cuts) in order to benefit from a greater probability to switch toward a high productivity firm after training. Consequently, the training policy is less selective and more individuals are trained, which leads to an increase in production and employment both at equillibrium and at optimum. Similarly, we consider an increase of 10% of the on-the-job contact rate for workers without OSHC, denoted λ^{e_0}. As the gap $(\lambda^e - \lambda^{e_0})$ is reduced, the value of training for workers without OSHC decreases, so that they are less likely to be trained. As a result, firms are more selective and training expenditure decreases, which contributes to lower production and unemployment both from the equilibrium and the optimal point of view. We note slight effects of a rise in λ^e or λ^{e_0} on the gap between equilibrium and optimum, which confirms the robustness of our results.

Table 4: Sensitivity analysis (On-the-job contact rate)

	$\lambda^e = 0.8$ (Ref.) $\lambda^{e_0} = 0.08$ (Ref.)			$\lambda^e = 0.8(1+10\%)$ $\lambda^{e_0} = 0.08$ (Ref.)			$\lambda^e = 0.8$ (Ref.) $\lambda^{e_0} = 0.08(1+10\%)$		
	Eq.	Opt.	Gap	Eq.	Opt.	Gap	Eq.	Opt.	Gap
Production per capita (net of training costs)	1.6405	1.6545	0.86%	1.6420	1.6561	0.86%	1.6400	1.6539	0.85 %
% Employment rate	81.44	82.76	1.63%	81.46	82.78	1.63%	81.44	82.74	1.60%
% Training expenditure (as a percentage of Equilibrium GDP)	0.55	0.65	17.31%	0.55	0.65	17.23%	0.54	0.63	16.69%
% Previously unemployed persons who switch occupation upon finding a job	71.76	67.95	-5.32%	71.71	67.88	-5.33%	71.76	68.02	-5.21%
% Workers who switch occupation per quarter (all workers)	1.82	1.66	-8.61%	1.82	1.66	-8.60%	1.89	1.73	-8.43%
% Untrained workers who switch occupation per quarter	2.78	3.32	19.52%	2.77	3.31	19.53%	2.97	3.57	20.09%

6.2 Bargaining power

An additional sensitivity analysis consists in modifying the value of the worker's bargaining power. The value of this parameter is essential to measure the social externalities related to training because equilibrium decisions converge towards optimal ones if employees (who perfectly internalize the social externalities related to training) have the full bargaining power, $\alpha = 1$. In the benchmark calibration, we first consider $\alpha = 0.5$, as is often the case when the worker's bargaining power is not estimated. Using a matched employer-employee panel of French data collected by the French National Statistical Institute (INSEE), Cahuc et al. [2006] estimate a very low bargaining power for "unskilled" workers, between 0% and 20%, depending on the particular industry considered, and a somewhat higher value for "skilled" workers, between 20% and 40%. As can be seen in table 5, our results are reinforced if we consider this range of values. Indeed, as employers do not take into account the social externalities related to training, a larger employer's bargaining power leads to increase the gap between equilibrium and optimal decisions, and thus to greater inefficiencies.

Table 5: Sensitivity analysis (Bargaining power)

	Optimum	Equilibrium $\alpha = 0.5$ (Ref.)	Equilibrium $\alpha = 0.4$	Equilibrium $\alpha = 0.3$	Equilibrium $\alpha = 0.2$
Production per capita (net of training costs)	1.6545	1.6405	1.6345	1.6271	1.6164
% Employment rate	82.76	81.44	80.98	80.44	79.76

7 Conclusion

This paper examines how training and occupational mobility interact each other over the life cycle and lead to some inefficiency issues. As such, he contributes to the litterature that highlight the existence of social externalities related to transferable human capital investments which justify a public intervention to support vocational training. Acemoglu [1997], Acemoglu and Pischke [1999a] underlined that transferable human capital can benefit, with some probability, to some future employers (poaching externality). Belan and Chéron [2011, 2014] also argued that training may improve employability and reduce the risk of a human capital depreciation during unemployment (unemployment externality). As a result, firm's private return of training investments is lower than its social return, so that there is a room for an optimal policy to promote vocational training investments. Chéron and Terriau [2016, 2017] developed a life cycle model to examine the inefficiency of vocational training in a frictional labor market context. However, this model does not allow for on-the-job search and human capital investments are considered as purely general. Recent empirical evidence suggests that human capital may be industry- (Neal [1995], Parent [2000]) or occupation- (Kambourov and Manovskii [2009b]) specific, so that a substiantial amount of human capital may be destroyed upon occupational switches. Moreover, occupation-specific human capital (OSHC) investments may lead to a retention effect, so that training decisions also impact occupational mobility.

In this paper, we propose to examine the inefficiency issues of OSHC investments in a model with search frictions and endogenous occupational mobility over the life cycle. Firstly, we show that ability thresholds required to be trained at the time of hiring depends on worker's age and occupation's productivity, so that a worker unable to be trained in his current occupation can have a sufficient level of ability to be trained in another occupation, and vice versa. Secondly, we underline that the social planner, who perfectly internalise the social externalities related to training, is less selective than firms in terms of ability thresholds required to be trained. In other words, the optimal policy leads to train more workers in their current occupation but also provides more outside training opportunities than at equilibrium, for a given level of ability. As a result, some untrained workers at equilibrium (but trained at optimum) choose to leave their employer for a more productive one because they can not benefit from the additional output related to training with their current employer. At the opposite, some workers decide, contrary to what optimal policy suggest to do, to stay with their current employer because they don't have outside training opportunities at equilibrium (whereas they can be trained outside at optimum). All in all, we show not only that occupational training investments are sub-optimal but also that this leads to inefficient occupational mobility. Our quantitative investigation suggests that implementing the optimal allocation would account for significant increase in employment and productivity.

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A Value of unemployment - Without OSHC

The expected value of unemployment for a worker with age t, ability a and without OSHC is given by:

$$U_{0,t,a} = b + \beta \Bigg[\lambda^{u_0} \Bigg(\int_{\omega \in M_{t+1,a}^{U_0 \to E_0}} E_{0,t+1,a}(\omega, u)\, \mathrm{d}F(\omega) + \int_{\omega \in M_{t+1,a}^{U_0 \to E_1}} E_{1,t+1,a}(\omega, u)\, \mathrm{d}F(\omega) \Bigg) + \Bigg(1 - \lambda^{u_0} \int_{\omega \in M_{t+1,a}^{U_0 \to E_0} \cup M_{t+1,a}^{U_0 \to E_1}} \mathrm{d}F(\omega) \Bigg) U_{0,t+1,a} \Bigg]$$

With:

$$\begin{aligned} E_{0,t+1,a}(\omega, u) - U_{0,t+1,a} &= \alpha S_{0,t+1,a}(\omega) \\ E_{1,t+1,a}(\omega, u) - U_{0,t+1,a} &= \alpha S_{1,t+1,a}(\omega) \end{aligned}$$

This implies the following value of unemployment for a worker without OSHC :

$$U_{0,t,a} = b + \beta \Bigg[\alpha \lambda^{u_0} \Bigg(\int_{\omega \in M_{t+1,a}^{U_0 \to E_0}} S_{0,t+1,a}(\omega)\, \mathrm{d}F(\omega) + \int_{\omega \in M_{t+1,a}^{U_0 \to E_1}} S_{1,t+1,a}(\omega)\, \mathrm{d}F(\omega) \Bigg) + U_{0,t+1,a} \Bigg]$$

B Value of unemployment - With OSHC

The expected value of unemployment for a worker with age t, ability a and with OSHC is given by:

$$U_{t,a}(k) = b + \beta\Bigg[\lambda^{u_0}\bigg(\int_{\omega \in M_{t+1,a}^{U\to E_0}(k)} E_{0,t+1,a}(\omega,u)\,\mathrm{d}F(\omega) + \int_{\omega \in M_{t+1,a}^{U\to E_1}(k)} E_{1,t+1,a}(\omega,u)\,\mathrm{d}F(\omega)\bigg)$$
$$+\lambda^{u}\bigg(\int_{\omega \in k} E_{2,t+1,a}(\omega,u)\,\mathrm{d}F(\omega)\bigg)$$
$$+\bigg(1-\lambda^{u_0}\int_{\omega \in M_{t+1,a}^{U\to E_0}(k)\cup M_{t+1,a}^{U\to E_1}(k)} \mathrm{d}F(\omega) - \lambda^{u}\int_{\omega\in k}\mathrm{d}F(\omega)\bigg)U_{t+1,a}(k)\Bigg]$$

With:

$$E_{0,t+1,a}(\omega,u) - U_{t+1,a}(k) = \alpha S_{0,t+1,a}(\omega) - (U_{t+1,a}(k) - U_{0,t+1,a})$$

$$E_{1,t+1,a}(\omega,u) - U_{t+1,a}(k) = \alpha S_{1,t+1,a}(\omega) - (U_{t+1,a}(k) - U_{0,t+1,a})$$

$$E_{2,t+1,a}(\omega,u) - U_{t+1,a}(k) = \alpha S_{2,t+1,a}(\omega)$$

This implies the following value of unemployment for a worker without OSHC :

$$U_{t,a}(k) = b + \beta\Bigg[\alpha\lambda^{u_0}\bigg(\int_{\omega \in M_{t+1,a}^{U\to E_0}(k)} S_{0,t+1,a}(\omega)\,\mathrm{d}F(\omega) + \int_{\omega \in M_{t+1,a}^{U\to E_1}(k)} S_{1,t+1,a}(\omega)\,\mathrm{d}F(\omega)\bigg)$$
$$+\alpha\lambda^{u}\int_{\omega\in k}\mathrm{d}F(\omega)S_{2,t+1,a}(\omega)$$
$$-\lambda^{u_0}\bigg(\int_{\omega \in M_{t+1,a}^{U\to E_0}(k)\cup M_{t+1,a}^{U\to E_1}(k)} \big(U_{t+1,a}(k) - U_{0,t+1,a}\big)\,\mathrm{d}F(\omega)\bigg) + U_{t+1,a}(k)\Bigg]$$

C Joint surplus - Type 0

$$E_{0,t,a}(k,NB) = w_{0,t,a}(k,NB) + \beta\Bigg[(1-\delta)\Bigg[\lambda^{e_0}\Bigg(\int_{\omega\in M^{R_0}_{t+1,a}(k,NB)} E_{0,t+1,a}(k,\omega)\,\mathrm{d}F(\omega)$$
$$+\int_{\omega\in M^{E_0\to E_0}_{t+1,a}(k)} E_{0,t+1,a}(\omega,k)\,\mathrm{d}F(\omega) + \int_{\omega\in M^{E_0\to E_1}_{t+1,a}(k)} E_{1,t+1,a}(\omega,k)\,\mathrm{d}F(\omega)\Bigg)$$
$$+\Bigg(1-\lambda^{e_0}\int_{\omega\in M^{R_0}_{t+1,a}(k,NB)\cup M^{E_0\to E_0}_{t+1,a}(k)\cup M^{E_0\to E_1}_{t+1,a}(k)} \mathrm{d}F(\omega)\Bigg)E_{0,t+1,a}(k,NB)\Bigg] + \delta U_{0,t+1,a}\Bigg]$$

$$U_{0,t,a} = b + \beta\Bigg[\lambda^{u_0}\Bigg(\int_{\omega\in M^{U_0\to E_0}_{t+1,a}} E_{0,t+1,a}(\omega,u)\,\mathrm{d}F(\omega) + \int_{\omega\in M^{U_0\to E_1}_{t+1,a}} E_{1,t+1,a}(\omega,u)\,\mathrm{d}F(\omega)\Bigg)$$
$$+\Bigg(1-\lambda^{u_0}\int_{\omega\in M^{U_0\to E_0}_{t+1,a}\cup M^{U_0\to E_1}_{t+1,a}} \mathrm{d}F(\omega)\Bigg)U_{0,t+1,a}\Bigg]$$

$$J_{0,t,a}(k,NB) = ap_k - w_{0,t,a}(k,NB) + \beta(1-\delta)\Bigg[\lambda^{e_0}\int_{\omega\in M^{R_0}_{t+1,a}(k,NB)} J_{0,t+1,a}(k,\omega)\,\mathrm{d}F(\omega)$$
$$+\Bigg(1-\lambda^{e_0}\int_{\omega\in M^{R_0}_{t+1,a}(k,NB)\cup M^{E_0\to E_0}_{t+1,a}(k)\cup M^{E_0\to E_1}_{t+1,a}(k)} \mathrm{d}F(\omega)\Bigg)J_{0,t+1,a}(k,NB)\Bigg]$$

$$S_{0,t,a}(k) \quad = \quad E_{0,t,a}(k,NB) - U_{0,t,a} + J_{0,t,a}(k,NB)$$

$$
\begin{aligned}
&S_{0,t,a}(k) = \\
&ap_k - b + \beta\Bigg[(1-\delta)\Bigg[S_{0,t+1,a}(k) + \alpha\lambda^{e_0}\Bigg(\int_{\omega\in M_{t+1,a}^{E_0\to E_0}(k)} \Big(S_{0,t+1,a}(\omega) - S_{0,t+1,a}(k)\Big)\,\mathrm{d}F(\omega) \\
&+ \int_{\omega\in M_{t+1,a}^{E_0\to E_1}(k)} \Big(S_{1,t+1,a}(\omega) - S_{0,t+1,a}(k)\Big)\,\mathrm{d}F(\omega)\Bigg)\Bigg] \\
&-\alpha\lambda^{u_0}\Bigg(\int_{\omega\in M_{t+1,a}^{U_0\to E_0}} S_{0,t+1,a}(\omega)\,\mathrm{d}F(\omega) + \int_{\omega\in M_{t+1,a}^{U_0\to E_1}} S_{1,t+1,a}(\omega)\,\mathrm{d}F(\omega)\Bigg)\Bigg]
\end{aligned} \tag{21}
$$

D Joint surplus - Type 1

$$E_{1,t,a}(k,NB) = w_{1,t,a}(k,NB) + \beta\Bigg[(1-\delta)\Bigg[\lambda^e\Bigg(\int_{\omega\in M^{R_1}_{t+1,a}(k,NB)} E_{1,t+1,a}(k,\omega)\,\mathrm{d}F(\omega)$$
$$+\int_{\omega\in M^{E_1\to E_0}_{t+1,a}(k)} E_{0,t+1,a}(\omega,k)\,\mathrm{d}F(\omega) + \int_{\omega\in M^{E_1\to E_1}_{t+1,a}(k)} E_{1,t+1,a}(\omega,k)\,\mathrm{d}F(\omega)\Bigg)$$
$$+\Bigg(1-\lambda^e\int_{\omega\in M^{R_1}_{t+1,a}(k,NB)\cup M^{E_1\to E_0}_{t+1,a}(k)\cup M^{E_1\to E_1}_{t+1,a}(k)} \mathrm{d}F(\omega)\Bigg)E_{1,t+1,a}(k,NB)\Bigg] + \delta U_{t+1,a}(k)\Bigg]$$

$$U_{0,t,a} = b + \beta\Bigg[\lambda^{u_0}\Bigg(\int_{\omega\in M^{U_0\to E_0}_{t+1,a}} E_{0,t+1,a}(\omega,u)\,\mathrm{d}F(\omega) + \int_{\omega\in M^{U_0\to E_1}_{t+1,a}} E_{1,t+1,a}(\omega,u)\,\mathrm{d}F(\omega)\Bigg)$$
$$+\Bigg(1-\lambda^{u_0}\int_{\omega\in M^{U_0\to E_0}_{t+1,a}\cup M^{U_0\to E_1}_{t+1,a}} \mathrm{d}F(\omega)\Bigg)U_{0,t+1,a}\Bigg]$$

$$J_{1,t,a}(k,NB) = (1+\Delta)ap_k - w_{1,t,a}(k,NB) + \beta(1-\delta)\Bigg[\lambda^e\int_{\omega\in M^{R_1}_{t+1,a}(k,NB)} J_{1,t+1,a}(k,\omega)\,\mathrm{d}F(\omega)$$
$$+\Bigg(1-\lambda^e\int_{\omega\in M^{R_1}_{t+1,a}(k,NB)\cup M^{E_1\to E_0}_{t+1,a}(k)\cup M^{E_1\to E_1}_{t+1,a}(k)} \mathrm{d}F(\omega)\Bigg)J_{1,t+1,a}(k,NB)\Bigg]$$

$$S_{1,t,a}(k) \quad = \quad E_{1,t,a}(k,NB) - U_{0,t,a} + J_{1,t,a}(k,NB) - \gamma_f$$

$$
\begin{aligned}
&S_{1,t,a}(k) = \qquad (22)\\
&(1+\Delta)ap_k - b - \gamma_f + \beta\Bigg[\delta\Big[U_{t+1,a}(k) - U_{0,t+1,a}\Big] + (1-\delta)\Big[S_{1,t+1,a}(k) + \gamma_f\\
&+\alpha\lambda^e\Bigg(\int_{\omega\in M_{t+1,a}^{E_1\to E_0}(k)} \Big(S_{0,t+1,a}(\omega) - (S_{1,t+1,a}(k)+\gamma_f)\Big)\,\mathrm{d}F(\omega)\\
&+\int_{\omega\in M_{t+1,a}^{E_1\to E_1}(k)} \Big(S_{1,t+1,a}(\omega) - (S_{1,t+1,a}(k)+\gamma_f)\Big)\,\mathrm{d}F(\omega)\Bigg)\Big]\\
&-\alpha\lambda^{u_0}\Bigg(\int_{\omega\in M_{t+1,a}^{U_0\to E_0}} S_{0,t+1,a}(\omega)\,\mathrm{d}F(\omega) + \int_{\omega\in M_{t+1,a}^{U_0\to E_1}} S_{1,t+1,a}(\omega)\,\mathrm{d}F(\omega)\Bigg)\Bigg]
\end{aligned}
$$

E Joint surplus - Type 2

$$E_{2,t,a}(k,NB) = w_{2,t,a}(k,NB) + \beta\Bigg[(1-\delta)\Bigg[\lambda^e\Bigg(\int_{\omega\in M^{R_2}_{t+1,a}(k,NB)} E_{2,t+1,a}(k,\omega)\,\mathrm{d}F(\omega)$$

$$+\int_{\omega\in M^{E_2\to E_0}_{t+1,a}(k)} E_{0,t+1,a}(\omega,k)\,\mathrm{d}F(\omega) + \int_{\omega\in M^{E_2\to E_1}_{t+1,a}(k)} E_{1,t+1,a}(\omega,k)\,\mathrm{d}F(\omega)\Bigg)$$

$$+\Bigg(1-\lambda^e\int_{\omega\in M^{R_2}_{t+1,a}(k,NB)\cup M^{E_2\to E_0}_{t+1,a}(k)\cup M^{E_2\to E_1}_{t+1,a}(k)} \mathrm{d}F(\omega)\Bigg)E_{2,t+1,a}(k,NB)\Bigg] + \delta U_{t+1,a}(k)\Bigg]$$

$$U_{t,a}(k) = b + \beta\Bigg[\lambda^{u_0}\Bigg(\int_{\omega\in M^{U\to E_0}_{t+1,a}(k)} E_{0,t+1,a}(\omega,u)\,\mathrm{d}F(\omega) + \int_{\omega\in M^{U\to E_1}_{t+1,a}(k)} E_{1,t+1,a}(\omega,u)\,\mathrm{d}F(\omega)\Bigg)$$

$$+\lambda^u\Bigg(\int_{\omega\in k} E_{2,t+1,a}(\omega,u)\,\mathrm{d}F(\omega)\Bigg)$$

$$+\Bigg(1-\lambda^{u_0}\int_{\omega\in M^{U\to E_0}_{t+1,a}(k)\cup M^{U\to E_1}_{t+1,a}(k)} \mathrm{d}F(\omega) - \lambda^u\int_{\omega\in k}\mathrm{d}F(\omega)\Bigg)U_{t+1,a}(k)\Bigg]$$

$$J_{2,t,a}(k,NB) = (1+\Delta)ap_k - w_{2,t,a}(k,NB) + \beta(1-\delta)\Bigg[\lambda^e\int_{\omega\in M^{R_2}_{t+1,a}(k,NB)} J_{2,t+1,a}(k,\omega)\,\mathrm{d}F(\omega)$$

$$+\Bigg(1-\lambda^e\int_{\omega\in M^{R_2}_{t+1,a}(k,NB)\cup M^{E_2\to E_0}_{t+1,a}(k)\cup M^{E_2\to E_1}_{t+1,a}(k)} \mathrm{d}F(\omega)\Bigg)J_{2,t+1,a}(k,NB)\Bigg]$$

$$S_{2,t,a}(k) = E_{2,t,a}(k,NB) - U_{t,a}(k) + J_{2,t,a}(k,NB)$$

$$
\begin{aligned}
&S_{2,t,a}(k) = \qquad (23)\\
&(1+\Delta)ap_k - b + \beta\Bigg[(1-\delta)\Bigg[S_{2,t+1,a}(k)\\
&+\alpha\lambda^e\Bigg(\int_{\omega\in M_{t+1,a}^{E_2\to E_0}(k)}\Big(S_{0,t+1,a}(\omega) - S_{2,t+1,a}(k) - \frac{1}{\alpha}\big(U_{t+1,a}(k) - U_{0,t+1,a}\big)\Big)\,\mathrm{d}F(\omega)\\
&+\int_{\omega\in M_{t+1,a}^{E_2\to E_1}(k)}\Big(S_{1,t+1,a}(\omega) - S_{2,t+1,a}(k) - \frac{1}{\alpha}\big(U_{t+1,a}(k) - U_{0,t+1,a}\big)\Big)\,\mathrm{d}F(\omega)\Bigg)\Bigg]\\
&-\alpha\Bigg[\lambda^{u_0}\Bigg(\int_{\omega\in M_{t+1,a}^{U\to E_0}(k)}\Big(S_{0,t+1,a}(\omega) - \frac{1}{\alpha}\big(U_{t+1,a}(k) - U_{0,t+1,a}\big)\Big)\,\mathrm{d}F(\omega)\\
&+\int_{\omega\in M_{t+1,a}^{U\to E_1}(k)}\Big(S_{1,t+1,a}(\omega) - \frac{1}{\alpha}\big(U_{t+1,a}(k) - U_{0,t+1,a}\big)\Big)\,\mathrm{d}F(\omega)\Bigg) + \lambda^u\Bigg(\int_{\omega\in k} S_{2,t+1,a}(\omega)\,\mathrm{d}F(\omega)\Bigg)\Bigg]\Bigg]
\end{aligned}
$$

where :

- $\omega \in M_{t,a}^{R_0}(k, NB)$ if $S_{0,t,a}(k) > S_{j,t,a}(\omega) > S_{j,t,a}(NB)\ \forall\ j \in 0, 1$
- $\omega \in M_{t,a}^{R_1}(k, NB)$ if $S_{1,t,a}(k) > S_{j,t,a}(\omega) > S_{j,t,a}(NB)\ \forall\ j \in 0, 1$
- $\omega \in M_{t,a}^{R_2}(k, NB)$ if $S_{2,t,a}(k) > S_{j,t,a}(\omega) > S_{j,t,a}(NB)\ \forall\ j \in 0, 1$
- $\omega \in M_{t,a}^{U_0 \to E_0}$ if $E_{0,t,a}(k, NB) > U_{0,t,a}$ and $S_{0,t,a}(\omega) > S_{1,t,a}(\omega)$
- $\omega \in M_{t,a}^{U_0 \to E_1}$ if $E_{1,t,a}(k, NB) > U_{0,t,a}$ and $S_{1,t,a}(\omega) \geq S_{0,t,a}(\omega)$
- $\omega \in M_{t,a}^{U \to E_0}(k)$ if $E_{0,t,a}(k, NB) > U_{t,a}(k)$, $S_{0,t,a}(\omega) > S_{1,t,a}(\omega)$ and $\omega \neq k$
- $\omega \in M_{t+1,a}^{U \to E_1}(k)$ if $E_{1,t,a}(k, NB) > U_{t,a}(k)$, $S_{1,t,a}(\omega) \geq S_{0,t,a}(\omega)$ and $\omega \neq k$
- $\omega \in M_{t,a}^{E_0 \to E_0}(k)$ if $S_{0,t,a}(\omega) > S_{0,t,a}(k)$, $S_{0,t,a}(\omega) > S_{1,t,a}(\omega)$ and $\omega \neq k$
- $\omega \in M_{t,a}^{E_0 \to E_1}(k)$ if $S_{1,t,a}(\omega) > S_{0,t,a}(k)$, $S_{1,t,a}(\omega) \geq S_{0,t,a}(\omega)$ and $\omega \neq k$
- $\omega \in M_{t,a}^{E_1 \to E_0}(k)$ if $S_{0,t,a}(\omega) > S_{1,t,a}(k) + \gamma_f$, $S_{0,t,a}(\omega) > S_{1,t,a}(\omega)$ and $\omega \neq k$
- $\omega \in M_{t,a}^{E_1 \to E_1}(k)$ if $S_{1,t,a}(\omega) > S_{1,t,a}(k) + \gamma_f$, $S_{1,t,a}(\omega) \geq S_{0,t,a}(\omega)$ and $\omega \neq k$
- $\omega \in M_{t,a}^{E_2 \to E_0}(k)$ if $S_{0,t,a}(\omega) > S_{2,t,a}(k)$, $S_{0,t,a}(\omega) > S_{1,t,a}(\omega)$ and $\omega \neq k$
- $\omega \in M_{t,a}^{E_2 \to E_1}(k)$ if $S_{1,t,a}(\omega) > S_{2,t,a}(k)$, $S_{1,t,a}(\omega) \geq S_{0,t,a}(\omega)$ and $\omega \neq k$

F Equilibrium training policy

The firm's training policy consists in determining, for a given age t and a given occupation k, an ability threshold $\tilde{a}_{t,k}$ above which any worker without occupation-specific human capital is trained by the employed at the time of hiring. Hence, $\tilde{a}_{t,k}$ satisfies the following condition :

$$S_{1,t,\tilde{a}_{t,k}}(k) = S_{0,t,\tilde{a}_{t,k}}(k) \tag{24}$$

With the following joint surplus (see Equations 22 and 23) and $a = \tilde{a}_{t,k}$:

$$\begin{aligned} S_{0,t,a}(k) &= ap_k - b + \beta\Bigg[(1-\delta)\Bigg[S_{0,t+1,a}(k) \\ &+\alpha\lambda^{e_0}\Bigg(\int_{\omega\in M^{E_0\to E_0}_{t+1,a}(k)}\left(S_{0,t+1,a}(\omega) - S_{0,t+1,a}(k)\right)\mathrm{d}F(\omega) \\ &+\int_{\omega\in M^{E_0\to E_1}_{t+1,a}(k)}\left(S_{1,t+1,a}(\omega) - S_{0,t+1,a}(k)\right)\mathrm{d}F(\omega)\Bigg)\Bigg] \\ &-\alpha\lambda^{u_0}\Bigg(\int_{\omega\in M^{U_0\to E_0}_{t+1,a}} S_{0,t+1,a}(\omega)\,\mathrm{d}F(\omega) + \int_{\omega\in M^{U_0\to E_1}_{t+1,a}} S_{1,t+1,a}(\omega)\,\mathrm{d}F(\omega)\Bigg)\Bigg] \end{aligned}$$

$$\begin{aligned} S_{1,t,a}(k) &= (1+\Delta)ap_k - b - \gamma_f + \beta\Bigg[\delta\Big[U_{t+1,a}(k) - U_{0,t+1,a}\Big] + (1-\delta)\Big[S_{1,t+1,a}(k) + \gamma_f \\ &+\alpha\lambda^{e}\Bigg(\int_{\omega\in M^{E_1\to E_0}_{t+1,a}(k)}\Big(S_{0,t+1,a}(\omega) - (S_{1,t+1,a}(k)+\gamma_f)\Big)\mathrm{d}F(\omega) \\ &+\int_{\omega\in M^{E_1\to E_1}_{t+1,a}(k)}\Big(S_{1,t+1,a}(\omega) - (S_{1,t+1,a}(k)+\gamma_f)\Big)\mathrm{d}F(\omega)\Bigg)\Bigg] \\ &-\alpha\lambda^{u_0}\Bigg(\int_{\omega\in M^{U_0\to E_0}_{t+1,a}} S_{0,t+1,a}(\omega)\,\mathrm{d}F(\omega) + \int_{\omega\in M^{U_0\to E_1}_{t+1,a}} S_{1,t+1,a}(\omega)\,\mathrm{d}F(\omega)\Bigg)\Bigg] \end{aligned}$$

G Social values

We denote per worker social values, according to type, age, ability and occupation as follows:

<u>**Social value of an unemployed without OSHC:**</u>

$$Y^{u_0}_{t,a} = b + \beta\Bigg[\lambda^{u_0}\Bigg(\int_{\omega\in M^{Y^{u_0}\to\hat{Y}}_{t+1,a}} \hat{Y}_{t+1,a}(\omega)\,\mathrm{d}F(\omega) + \int_{\omega\in M^{Y^{u_0}\to\tilde{Y}}_{t+1,a}} (\tilde{Y}_{t+1,a}(\omega)-\gamma_f)\,\mathrm{d}F(\omega)\Bigg)$$
$$+\Bigg(1-\lambda^{u_0}\int_{\omega\in M^{Y^{u_0}\to\hat{Y}}_{t+1,a}\cup M^{Y^{u_0}\to\tilde{Y}}_{t+1,a}} \mathrm{d}F(\omega)\Bigg)Y^{u_0}_{t+1,a}\Bigg] \tag{25}$$

<u>**Social value of an unemployed with OSHC:**</u>

$$Y^{u}_{t,a}(k) = b + \beta\Bigg[\lambda^{u_0}\Bigg(\int_{\omega\in M^{Y^{u}\to\hat{Y}}_{t+1,a}(k)} \hat{Y}_{t+1,a}(\omega)\,\mathrm{d}F(\omega) + \int_{\omega\in M^{Y^{u}\to\tilde{Y}}_{t+1,a}(k)} (\tilde{Y}_{t+1,a}(\omega)-\gamma_f)\,\mathrm{d}F(\omega)\Bigg)$$
$$+\lambda^{u}\Bigg(\int_{\omega\in k} \tilde{Y}_{t+1,a}(\omega)\,\mathrm{d}F(\omega)\Bigg)$$
$$+\Bigg(1-\lambda^{u_0}\int_{\omega\in M^{Y^{u}\to\hat{Y}}_{t+1,a}(k)\cup M^{Y^{u}\to\tilde{Y}}_{t+1,a}(k)} \mathrm{d}F(\omega) - \lambda^{u}\int_{\omega\in k}\mathrm{d}F(\omega)\Bigg)Y^{u}_{t+1,a}(k)\Bigg] \tag{26}$$

<u>**Social value of an employed worker without OSHC:**</u>

$$\hat{Y}_{t,a}(k) = ap_k + \beta\Bigg[(1-\delta)\Bigg[\lambda^{e_0}\Bigg(\int_{\omega\in M^{\hat{Y}\to\hat{Y}}_{t+1,a}(k)} \hat{Y}_{t+1,a}(\omega)\,\mathrm{d}F(\omega)$$
$$+\int_{\omega\in M^{\hat{Y}\to\tilde{Y}}_{t+1,a}(k)} (\tilde{Y}_{t+1,a}(\omega)-\gamma_f)\,\mathrm{d}F(\omega)\Bigg)$$
$$+\Bigg(1-\lambda^{e_0}\int_{\omega\in M^{\hat{Y}\to\hat{Y}}_{t+1,a}(k)\cup M^{\hat{Y}\to\tilde{Y}}_{t+1,a}(k)} \mathrm{d}F(\omega)\Bigg)\hat{Y}_{t+1,a}(k)\Bigg] + \delta Y^{u_0}_{t+1,a}\Bigg] \tag{27}$$

Social value of an employed worker with OSHC:

$$\tilde{Y}_{t,a}(k) = (1+\Delta)ap_k + \beta\Bigg[(1-\delta)\Bigg[\lambda^e\Bigg(\int_{\omega\in M_{t+1,a}^{\tilde{Y}\to\hat{Y}}(k)} \hat{Y}_{t+1,a}(\omega)\,\mathrm{d}F(\omega) + \int_{\omega\in M_{t+1,a}^{\tilde{Y}\to\tilde{Y}}(k)} (\tilde{Y}_{t+1,a}(\omega)-\gamma_f)\,\mathrm{d}F(\omega)\Bigg) + \Bigg(1-\lambda^e\int_{\omega\in M_{t+1,a}^{\tilde{Y}\to\hat{Y}}(k)\cup M_{t+1,a}^{\tilde{Y}\to\tilde{Y}}(k)} \mathrm{d}F(\omega)\Bigg)\tilde{Y}_{t+1,a}(k)\Bigg] + \delta Y_{t+1,a}^{u}(k)\Bigg] \tag{28}$$

where :

- $\omega \in M_{t,a}^{Y^{u_0}\to\hat{Y}}$ if $\hat{Y}_{t,a}(\omega) > Y_{t,a}^{u_0}$ and $\hat{Y}_{t,a}(\omega) > \left(\tilde{Y}_{t,a}(\omega)-\gamma_f\right)$
- $\omega \in M_{t,a}^{Y^{u_0}\to\tilde{Y}}$ if $\left(\tilde{Y}_{t,a}(\omega)-\gamma_f\right) > Y_{t,a}^{u_0}$ and $\left(\tilde{Y}_{t,a}(\omega)-\gamma_f\right) \geq \hat{Y}_{t,a}(\omega)$
- $\omega \in M_{t,a}^{Y^{u}\to\hat{Y}}(k)$ if $\hat{Y}_{t,a}(\omega) > Y_{t,a}^{u}(k)$, $\hat{Y}_{t,a}(\omega) > \left(\tilde{Y}_{t,a}(\omega)-\gamma_f\right)$ and $\omega \neq k$
- $\omega \in M_{t,a}^{Y^{u}\to\tilde{Y}}(k)$ if $\left(\tilde{Y}_{t,a}(\omega)-\gamma_f\right) > Y_{t,a}^{u}(k)$, $\left(\tilde{Y}_{t,a}(\omega)-\gamma_f\right) \geq \hat{Y}_{t,a}(\omega)$ and $\omega \neq k$
- $\omega \in M_{t,a}^{\hat{Y}\to\hat{Y}}(k)$ if $\hat{Y}_{t,a}(\omega) > \hat{Y}_{t,a}(k)$, $\hat{Y}_{t,a}(\omega) > \left(\tilde{Y}_{t,a}(\omega)-\gamma_f\right)$ and $\omega \neq k$
- $\omega \in M_{t,a}^{\hat{Y}\to\tilde{Y}}(k)$ if $\left(\tilde{Y}_{t,a}(\omega)-\gamma_f\right) > \hat{Y}_{t,a}(k)$, $\left(\tilde{Y}_{t,a}(\omega)-\gamma_f\right) \geq \hat{Y}_{t,a}(\omega)$ and $\omega \neq k$
- $\omega \in M_{t,a}^{\tilde{Y}\to\hat{Y}}(k)$ if $\hat{Y}_{t,a}(\omega) > \tilde{Y}_{t,a}(k)$, $\hat{Y}_{t,a}(\omega) > \left(\tilde{Y}_{t,a}(\omega)-\gamma_f\right)$ and $\omega \neq k$
- $\omega \in M_{t,a}^{\tilde{Y}\to\tilde{Y}}(k)$ if $\left(\tilde{Y}_{t,a}(\omega)-\gamma_f\right) > \tilde{Y}_{t,a}(k)$, $\left(\tilde{Y}_{t,a}(\omega)-\gamma_f\right) \geq \hat{Y}_{t,a}(\omega)$ and $\omega \neq k$

H Social surplus - Worker without OSHC unable to be trained

The social surplus for a worker without OSHC unable to be trained can be written as follows:

$$\hat{Y}_{t,a}(k) - Y^{u_0}_{t,a} = ap_k - b + \beta\Bigg[(1-\delta)\Bigg[\left(\hat{Y}_{t+1,a}(k) - Y^{u_0}_{t+1,a}\right) \tag{29}$$

$$+\lambda^{e_0}\Bigg(\int_{\omega\in M^{\hat{Y}\to\hat{Y}}_{t+1,a}(k)} \left(\hat{Y}_{t+1,a}(\omega) - Y^{u_0}_{t+1,a}\right) - \left(\hat{Y}_{t+1,a}(k) - Y^{u_0}_{t+1,a}\right)\,\mathrm{d}F(\omega)$$

$$+\int_{\omega\in M^{\hat{Y}\to\tilde{Y}}_{t+1,a}(k)} \left(\tilde{Y}_{t+1,a}(\omega) - \gamma_f - Y^{u_0}_{t+1,a}\right) - \left(\hat{Y}_{t+1,a}(k) - Y^{u_0}_{t+1,a}\right)\,\mathrm{d}F(\omega)\Bigg)\Bigg]$$

$$-\lambda^{u_0}\Bigg(\int_{\omega\in M^{Y^{u_0}\to\hat{Y}}_{t+1,a}} \left(\hat{Y}_{t+1,a}(\omega) - Y^{u_0}_{t+1,a}\right)\,\mathrm{d}F(\omega) + \int_{\omega\in M^{Y^{u_0}\to\tilde{Y}}_{t+1,a}} \left(\tilde{Y}_{t+1,a}(\omega) - \gamma_f - Y^{u_0}_{t+1,a}\right)\,\mathrm{d}F(\omega)\Bigg)\Bigg]$$

I Social surplus - Worker without OSHC able to be trained

The social surplus for a worker without OSHC able to be trained can be written as follows:

$$\tilde{Y}_{t,a}(k) - \gamma_f - Y^{u_0}_{t,a} = (1+\Delta)ap_k - b - \gamma_f + \beta\Bigg[\delta\Big[Y^{u}_{t+1,a}(k) - Y^{u_0}_{t+1,a}\Big] \tag{30}$$

$$+(1-\delta)\Bigg[\Big(\tilde{Y}_{t+1,a}(k) - Y^{u_0}_{t+1,a}\Big)$$

$$+\lambda^e\Bigg(\int_{\omega\in M^{\tilde{Y}\to\hat{Y}}_{t+1,a}(k)} \Big(\hat{Y}_{t+1,a}(\omega) - Y^{u_0}_{t+1,a}\Big) - \Big(\tilde{Y}_{t+1,a}(k) - Y^{u_0}_{t+1,a}\Big)\,\mathrm{d}F(\omega)$$

$$+\int_{\omega\in M^{\tilde{Y}\to\tilde{Y}}_{t+1,a}(k)} \Big(\tilde{Y}_{t+1,a}(\omega) - \gamma_f - Y^{u_0}_{t+1,a}\Big) - \Big(\tilde{Y}_{t+1,a}(k) - Y^{u_0}_{t+1,a}\Big)\,\mathrm{d}F(\omega)\Bigg)\Bigg]$$

$$-\lambda^{u_0}\Bigg(\int_{\omega\in M^{Y^{u_0}\to\hat{Y}}_{t+1,a}} \Big(\hat{Y}_{t+1,a}(\omega) - Y^{u_0}_{t+1,a}\Big)\,\mathrm{d}F(\omega) + \int_{\omega\in M^{Y^{u_0}\to\tilde{Y}}_{t+1,a}} \Big(\tilde{Y}_{t+1,a}(\omega) - \gamma_f - Y^{u_0}_{t+1,a}\Big)\,\mathrm{d}F(\omega)\Bigg)\Bigg]$$

J Social surplus - Worker with OSHC

The social surplus for a worker with OSHC can be written as follows:

$$
\begin{aligned}
&\tilde{Y}_{t,a}(k) - Y^{u}_{t+1,a}(k) = (1+\Delta)ap_k - b + \beta\Bigg[(1-\delta)\Bigg[\Big(\tilde{Y}_{t+1,a}(k) - Y^{u}_{t+1,a}(k)\Big) \\
&+\lambda^{e}\Bigg(\int_{\omega \in M^{\tilde{Y}\to\hat{Y}}_{t+1,a}(k)} \Big(\hat{Y}_{t+1,a}(\omega) - Y^{u_0}_{t+1,a}\Big) - \Big(\tilde{Y}_{t+1,a}(k) - Y^{u_0}_{t+1,a}\Big)\,\mathrm{d}F(\omega) \\
&+\int_{\omega \in M^{\tilde{Y}\to\tilde{Y}}_{t+1,a}(k)} \Big(\tilde{Y}_{t+1,a}(\omega) - \gamma_f - Y^{u_0}_{t+1,a}\Big) - \Big(\tilde{Y}_{t+1,a}(k) - Y^{u_0}_{t+1,a}\Big)\,\mathrm{d}F(\omega)\Bigg)\Bigg] \\
&-\lambda^{u_0}\Bigg(\int_{\omega \in M^{Y^u\to\hat{Y}}_{t+1,a}(k)} \Big((\hat{Y}_{t+1,a}(\omega) - Y^{u_0}_{t+1,a}) - (Y^{u}_{t+1,a}(k) - Y^{u_0}_{t+1,a})\Big)\,\mathrm{d}F(\omega) \\
&+\int_{\omega \in M^{Y^u\to\tilde{Y}}_{t+1,a}(k)} \Big((\tilde{Y}_{t+1,a}(\omega) - \gamma_f - Y^{u_0}_{t+1,a}) - (Y^{u}_{t+1,a}(k) - Y^{u_0}_{t+1,a})\Big)\,\mathrm{d}F(\omega)\Bigg) \\
&+\lambda^{u}\Bigg(\int_{\omega \in k} \Big(\tilde{Y}_{t+1,a}(\omega) - Y^{u}_{t+1,a}(k)\Big)\,\mathrm{d}F(\omega)\Bigg)\Bigg]\Bigg]
\end{aligned} \tag{31}
$$

K Efficient training policy

The efficient training policy is now derived by determining, for a given age t and a given occupation k, an ability threshold $a^{\star}_{t,k}$ above which any worker without occupation-specific human capital is trained by the social planner at the time of hiring. Hence, $\tilde{a}_{t,k}$ satisfies the following condition :

$$\tilde{Y}_{t,a}(k) - \gamma_f - Y^{u_0}_{t,a} = \hat{Y}_{t,a}(k) - Y^{u_0}_{t,a} \tag{32}$$

With the following social surplus (see Equations 29 and 30) and $a = a^{\star}_{t,k}$:

$$\begin{aligned}
\hat{Y}_{t,a}(k) - Y^{u_0}_{t,a} &= ap_k - b + \beta\Bigg[(1-\delta)\Bigg[\left(\hat{Y}_{t+1,a}(k) - Y^{u_0}_{t+1,a}\right) \\
&+\lambda^{e_0}\Bigg(\int_{\omega\in M^{\hat{Y}\to\hat{Y}}_{t+1,a}(k)} \left(\hat{Y}_{t+1,a}(\omega) - Y^{u_0}_{t+1,a}\right) - \left(\hat{Y}_{t+1,a}(k) - Y^{u_0}_{t+1,a}\right) \mathrm{d}F(\omega) \\
&+ \int_{\omega\in M^{\hat{Y}\to\tilde{Y}}_{t+1,a}(k)} \left(\tilde{Y}_{t+1,a}(\omega) - \gamma_f - Y^{u_0}_{t+1,a}\right) - \left(\hat{Y}_{t+1,a}(k) - Y^{u_0}_{t+1,a}\right) \mathrm{d}F(\omega)\Bigg)\Bigg] \\
&-\lambda^{u_0}\Bigg(\int_{\omega\in M^{Y^{u_0}\to\hat{Y}}_{t+1,a}} \left(\hat{Y}_{t+1,a}(\omega) - Y^{u_0}_{t+1,a}\right) \mathrm{d}F(\omega) + \int_{\omega\in M^{Y^{u_0}\to\tilde{Y}}_{t+1,a}} \left(\tilde{Y}_{t+1,a}(\omega) - \gamma_f - Y^{u_0}_{t+1,a}\right) \mathrm{d}F(\omega)\Bigg)\Bigg]
\end{aligned}$$

$$\begin{aligned}
\tilde{Y}_{t,a}(k) - \gamma_f - Y^{u_0}_{t,a} &= (1+\Delta)ap_k - b - \gamma_f + \beta\Bigg[\delta\left[Y^{u}_{t+1,a}(k) - Y^{u_0}_{t+1,a}\right] + (1-\delta)\Bigg[\left(\tilde{Y}_{t+1,a}(k) - Y^{u_0}_{t+1,a}\right) \\
&+\lambda^{e}\Bigg(\int_{\omega\in M^{\tilde{Y}\to\hat{Y}}_{t+1,a}(k)} \left(\hat{Y}_{t+1,a}(\omega) - Y^{u_0}_{t+1,a}\right) - \left(\tilde{Y}_{t+1,a}(k) - Y^{u_0}_{t+1,a}\right) \mathrm{d}F(\omega) \\
&+ \int_{\omega\in M^{\tilde{Y}\to\tilde{Y}}_{t+1,a}(k)} \left(\tilde{Y}_{t+1,a}(\omega) - \gamma_f - Y^{u_0}_{t+1,a}\right) - \left(\tilde{Y}_{t+1,a}(k) - Y^{u_0}_{t+1,a}\right) \mathrm{d}F(\omega)\Bigg)\Bigg] \\
&-\lambda^{u_0}\Bigg(\int_{\omega\in M^{Y^{u_0}\to\hat{Y}}_{t+1,a}} \left(\hat{Y}_{t+1,a}(\omega) - Y^{u_0}_{t+1,a}\right) \mathrm{d}F(\omega) + \int_{\omega\in M^{Y^{u_0}\to\tilde{Y}}_{t+1,a}} \left(\tilde{Y}_{t+1,a}(\omega) - \gamma_f - Y^{u_0}_{t+1,a}\right) \mathrm{d}F(\omega)\Bigg)\Bigg]
\end{aligned}$$

L Inefficiency of the equilibrium training policy

Comparing the firms' training policy with the social planner's policy and using equations 21, 22, 23, 29, 30 and 31, we can note that if employees have the full bargaining power, $\alpha = 1$, equilibrium training policy and mobility are efficient.

Proof:

$$\begin{aligned}
S_{0,t,a}(k) &= ap_k - b + \beta\Bigg[(1-\delta)\Bigg[S_{0,t+1,a}(k) \\
&+\alpha\lambda^{e_0}\Bigg(\int_{\omega\in M_{t+1,a}^{E_0\to E_0}(k)}\Big(S_{0,t+1,a}(\omega)-S_{0,t+1,a}(k)\Big)\,\mathrm{d}F(\omega) \\
&+\int_{\omega\in M_{t+1,a}^{E_0\to E_1}(k)}\Big(S_{1,t+1,a}(\omega)-S_{0,t+1,a}(k)\Big)\,\mathrm{d}F(\omega)\Bigg)\Bigg] \\
&-\alpha\lambda^{u_0}\Bigg(\int_{\omega\in M_{t+1,a}^{U_0\to E_0}} S_{0,t+1,a}(\omega)\,\mathrm{d}F(\omega)+\int_{\omega\in M_{t+1,a}^{U_0\to E_1}} S_{1,t+1,a}(\omega)\,\mathrm{d}F(\omega)\Bigg)\Bigg]
\end{aligned}$$

$$\begin{aligned}
S_{1,t,a}(k) &= (1+\Delta)ap_k - b - \gamma_f + \beta\Bigg[\delta\Big[U_{t+1,a}(k)-U_{0,t+1,a}\Big]+(1-\delta)\Bigg[S_{1,t+1,a}(k)+\gamma_f \\
&+\alpha\lambda^{e}\Bigg(\int_{\omega\in M_{t+1,a}^{E_1\to E_0}(k)}\Big(S_{0,t+1,a}(\omega)-\big(S_{1,t+1,a}(k)+\gamma_f\big)\Big)\,\mathrm{d}F(\omega) \\
&+\int_{\omega\in M_{t+1,a}^{E_1\to E_1}(k)}\Big(S_{1,t+1,a}(\omega)-\big(S_{1,t+1,a}(k)+\gamma_f\big)\Big)\,\mathrm{d}F(\omega)\Bigg)\Bigg] \\
&-\alpha\lambda^{u_0}\Bigg(\int_{\omega\in M_{t+1,a}^{U_0\to E_0}} S_{0,t+1,a}(\omega)\,\mathrm{d}F(\omega)+\int_{\omega\in M_{t+1,a}^{U_0\to E_1}} S_{1,t+1,a}(\omega)\,\mathrm{d}F(\omega)\Bigg)\Bigg]
\end{aligned}$$

$$\begin{aligned}
S_{2,t,a}(k) &= (1+\Delta)ap_k - b + \beta\Bigg[(1-\delta)\Bigg[S_{2,t+1,a}(k) \\
&+\alpha\lambda^{e}\Bigg(\int_{\omega\in M_{t+1,a}^{E_2\to E_0}(k)}\Big(S_{0,t+1,a}(\omega)-S_{2,t+1,a}(k)-\frac{1}{\alpha}\big(U_{t+1,a}(k)-U_{0,t+1,a}\big)\Big)\,\mathrm{d}F(\omega) \\
&+\int_{\omega\in M_{t+1,a}^{E_2\to E_1}(k)}\Big(S_{1,t+1,a}(\omega)-S_{2,t+1,a}(k)-\frac{1}{\alpha}\big(U_{t+1,a}(k)-U_{0,t+1,a}\big)\Big)\,\mathrm{d}F(\omega)\Bigg)\Bigg] \\
&-\alpha\Bigg[\lambda^{u_0}\Bigg(\int_{\omega\in M_{t+1,a}^{U\to E_0}(k)}\Big(S_{0,t+1,a}(\omega)-\frac{1}{\alpha}\big(U_{t+1,a}(k)-U_{0,t+1,a}\big)\Big)\,\mathrm{d}F(\omega) \\
&+\int_{\omega\in M_{t+1,a}^{U\to E_1}(k)}\Big(S_{1,t+1,a}(\omega)-\frac{1}{\alpha}\big(U_{t+1,a}(k)-U_{0,t+1,a}\big)\Big)\,\mathrm{d}F(\omega)\Bigg) \\
&+\lambda^{u}\Bigg(\int_{\omega\in k} S_{2,t+1,a}(\omega)\,\mathrm{d}F(\omega)\Bigg)\Bigg]\Bigg]
\end{aligned}$$

$$
\hat{Y}_{t,a}(k) - Y^{u_0}_{t,a} = ap_k - b + \beta\Bigg[(1-\delta)\Bigg[\Big(\hat{Y}_{t+1,a}(k) - Y^{u_0}_{t+1,a}\Big)
$$

$$
+\lambda^{e_0}\Bigg(\int_{\omega\in M^{\hat{Y}\to\hat{Y}}_{t+1,a}(k)} \Big(\hat{Y}_{t+1,a}(\omega) - Y^{u_0}_{t+1,a}\Big) - \Big(\hat{Y}_{t+1,a}(k) - Y^{u_0}_{t+1,a}\Big)\ \mathrm{d}F(\omega)
$$

$$
+\int_{\omega\in M^{\hat{Y}\to\tilde{Y}}_{t+1,a}(k)} \Big(\tilde{Y}_{t+1,a}(\omega) - \gamma_f - Y^{u_0}_{t+1,a}\Big) - \Big(\hat{Y}_{t+1,a}(k) - Y^{u_0}_{t+1,a}\Big)\ \mathrm{d}F(\omega)\Bigg)\Bigg]
$$

$$
-\lambda^{u_0}\Bigg(\int_{\omega\in M^{Y^{u_0}\to\hat{Y}}_{t+1,a}} \Big(\hat{Y}_{t+1,a}(\omega) - Y^{u_0}_{t+1,a}\Big)\ \mathrm{d}F(\omega) + \int_{\omega\in M^{Y^{u_0}\to\tilde{Y}}_{t+1,a}} \Big(\tilde{Y}_{t+1,a}(\omega) - \gamma_f - Y^{u_0}_{t+1,a}\Big)\ \mathrm{d}F(\omega)\Bigg)\Bigg]
$$

$$
\tilde{Y}_{t,a}(k) - \gamma_f - Y^{u_0}_{t,a} = (1+\Delta)ap_k - b - \gamma_f + \beta\Bigg[\delta\Big[Y^{u}_{t+1,a}(k) - Y^{u_0}_{t+1,a}\Big] + (1-\delta)\Bigg[\Big(\tilde{Y}_{t+1,a}(k) - Y^{u_0}_{t+1,a}\Big)
$$

$$
+\lambda^{e}\Bigg(\int_{\omega\in M^{\tilde{Y}\to\hat{Y}}_{t+1,a}(k)} \Big(\hat{Y}_{t+1,a}(\omega) - Y^{u_0}_{t+1,a}\Big) - \Big(\tilde{Y}_{t+1,a}(k) - Y^{u_0}_{t+1,a}\Big)\ \mathrm{d}F(\omega)
$$

$$
+\int_{\omega\in M^{\tilde{Y}\to\tilde{Y}}_{t+1,a}(k)} \Big(\tilde{Y}_{t+1,a}(\omega) - \gamma_f - Y^{u_0}_{t+1,a}\Big) - \Big(\tilde{Y}_{t+1,a}(k) - Y^{u_0}_{t+1,a}\Big)\ \mathrm{d}F(\omega)\Bigg)\Bigg]
$$

$$
-\lambda^{u_0}\Bigg(\int_{\omega\in M^{Y^{u_0}\to\hat{Y}}_{t+1,a}} \Big(\hat{Y}_{t+1,a}(\omega) - Y^{u_0}_{t+1,a}\Big)\ \mathrm{d}F(\omega) + \int_{\omega\in M^{Y^{u_0}\to\tilde{Y}}_{t+1,a}} \Big(\tilde{Y}_{t+1,a}(\omega) - \gamma_f - Y^{u_0}_{t+1,a}\Big)\ \mathrm{d}F(\omega)\Bigg)\Bigg]
$$

$$
\tilde{Y}_{t,a}(k) - Y^{u}_{t+1,a}(k) = (1+\Delta)ap_k - b + \beta\Bigg[(1-\delta)\Bigg[\Big(\tilde{Y}_{t+1,a}(k) - Y^{u}_{t+1,a}(k)\Big)
$$

$$
+\lambda^{e}\Bigg(\int_{\omega\in M^{\tilde{Y}\to\hat{Y}}_{t+1,a}(k)} \Big(\hat{Y}_{t+1,a}(\omega) - Y^{u_0}_{t+1,a}\Big) - \Big(\tilde{Y}_{t+1,a}(k) - Y^{u_0}_{t+1,a}\Big)\ \mathrm{d}F(\omega)
$$

$$
+\int_{\omega\in M^{\tilde{Y}\to\tilde{Y}}_{t+1,a}(k)} \Big(\tilde{Y}_{t+1,a}(\omega) - \gamma_f - Y^{u_0}_{t+1,a}\Big) - \Big(\tilde{Y}_{t+1,a}(k) - Y^{u_0}_{t+1,a}\Big)\ \mathrm{d}F(\omega)\Bigg)\Bigg]
$$

$$
-\lambda^{u_0}\Bigg(\int_{\omega\in M^{Y^{u}\to\hat{Y}}_{t+1,a}(k)} \Big((\hat{Y}_{t+1,a}(\omega) - Y^{u_0}_{t+1,a}) - (Y^{u}_{t+1,a}(k) - Y^{u_0}_{t+1,a})\Big)\ \mathrm{d}F(\omega)
$$

$$
+\int_{\omega\in M^{Y^{u}\to\tilde{Y}}_{t+1,a}(k)} \Big((\tilde{Y}_{t+1,a}(\omega) - \gamma_f - Y^{u_0}_{t+1,a}) - (Y^{u}_{t+1,a}(k) - Y^{u_0}_{t+1,a})\Big)\ \mathrm{d}F(\omega)\Bigg)
$$

$$
+\lambda^{u}\Bigg(\int_{\omega\in k} \Big(\tilde{Y}_{t+1,a}(\omega) - Y^{u}_{t+1,a}(k)\Big)\ \mathrm{d}F(\omega)\Bigg)\Bigg]\Bigg]
$$

M Worker flows by k (Equilibrium)

Worker flows are defined as a function of age t and ability level a distributed over the interval $[\underline{a},\bar{a}]$, according to p.d.f. $f(a)$. Let $u_{0,t,a}$, $u_{t,a}$, $e_{0,t,a}$, $e_{1,t,a}$, $e_{2,t,a}$ be respectively the density of workers unemployed without OSHC, unemployed with OSHC, employed without OSHC and not trained, employed without OSHC and trained, and employed with OSHC.

M.1 For t=0

Initially, all individuals have no OSHC and enter on the labor market as unemployed, so that:

- $\forall\ a$:

$u_{0,t,a} = f(a)$

M.2 $\forall\ t \in [1, T-1]$

- $\forall\ a$:

$$u_{0,t,a} = u_{0,t-1,a}\Big(1 - \lambda^{u_0}\int_{\omega \in M_{t,a}^{U_0 \to E_0} \cup M_{t,a}^{U_0 \to E_1}} \mathrm{d}F(\omega)\Big) + \delta \int e_{0,t-1,a}(\omega)\,\mathrm{d}\omega$$

$$u_{t,a}(k) = u_{t-1,a}(k)\Big(1 - \lambda^{u_0}\int_{\omega \in M_{t,a}^{U \to E_0} \cup M_{t,a}^{U \to E_1}} \mathrm{d}F(\omega) - \lambda^{u}\int_{\omega \in k} \mathrm{d}F(\omega)\Big) + \delta\Big(e_{1,t-1,a}(k) + e_{2,t-1,a}(k)\Big)$$

• $\forall\ a < \tilde{a}_{t,k}$:

$$
\begin{aligned}
e_{0,t,a}(k) &= u_{0,t-1,a} \times f(k) \times \lambda^{u_0} \int_{\omega \in M_{t,a}^{U_0 \to E_0}} \mathrm{d}F(\omega) \\
&+ \int u_{t-1,a}(\omega)\,\mathrm{d}\omega \times f(k) \times \lambda^{u_0} \int_{\omega \in M_{t,a}^{U \to E_0}(k)} \mathrm{d}F(\omega) \\
&+ \Big(1-\delta\Big) f(k) \Big(\lambda^{e_0} \int_{k \in M_{t,a}^{E_0 \to E_0}(\omega)} e_{0,t-1,a}(\omega)\,\mathrm{d}F(\omega) + \lambda^{e} \int_{k \in M_{t,a}^{E_1 \to E_0}(\omega)} e_{1,t-1,a}(\omega)\,\mathrm{d}F(\omega) \\
&+ \lambda^{e} \int_{k \in M_{t,a}^{E_2 \to E_0}(\omega)} e_{2,t-1,a}(\omega)\,\mathrm{d}F(\omega)\Big) \\
&+ e_{0,t-1,a}(k)\Big(1-\delta\Big)\Big(1-\lambda^{e_0} \int_{\omega \in M_{t,a}^{E_0 \to E_0}(k) \cup M_{t,a}^{E_0 \to E_1}(k)} \mathrm{d}F(\omega)\Big)
\end{aligned}
$$

$$
e_{1,t,a}(k) = e_{1,t-1,a}(k)\Big(1-\delta\Big)\Big(1-\lambda^{e} \int_{\omega \in \cup M_{t,a}^{E_1 \to E_0}(k) \cup M_{t,a}^{E_1 \to E_1}(k)} \mathrm{d}F(\omega)\Big)
$$

$$
\begin{aligned}
e_{2,t,a}(k) &= u_{t-1,a}(k) \times f(k) \times \lambda^{u} \\
&+ e_{2,t-1,a}(k)\Big(1-\delta\Big)\Big(1-\lambda^{e} \int_{\omega \in \cup M_{t,a}^{E_2 \to E_0}(k) \cup M_{t,a}^{E_2 \to E_1}(k)} \mathrm{d}F(\omega)\Big)
\end{aligned}
$$

• $\forall\ a \geq \tilde{a}_{t,k}$:

$$
\begin{aligned}
e_{1,t,a}(k) &= u_{0,t-1,a} \times f(k) \times \lambda^{u_0} \int_{\omega \in M_{t,a}^{U_0 \to E_1}} \mathrm{d}F(\omega) \\
&+ \int u_{t-1,a}(\omega)\,\mathrm{d}\omega \times f(k) \times \lambda^{u_0} \int_{\omega \in M_{t,a}^{U \to E_1}(k)} \mathrm{d}F(\omega) \\
&+ \Big(1-\delta\Big) f(k) \Big(\lambda^{e_0} \int_{k \in M_{t,a}^{E_0 \to E_1}(\omega)} e_{0,t-1,a}(\omega)\,\mathrm{d}F(\omega) + \lambda^{e} \int_{k \in M_{t,a}^{E_1 \to E_1}(\omega)} e_{1,t-1,a}(\omega)\,\mathrm{d}F(\omega) \\
&+ \lambda^{e} \int_{k \in M_{t,a}^{E_2 \to E_1}(\omega)} e_{2,t-1,a}(\omega)\,\mathrm{d}F(\omega)\Big) \\
&+ e_{1,t-1,a}(k)\Big(1-\delta\Big)\Big(1-\lambda^{e} \int_{\omega \in M_{t,a}^{E_1 \to E_0}(k) \cup M_{t,a}^{E_1 \to E_1}(k)} \mathrm{d}F(\omega)\Big)
\end{aligned}
$$

$$
\begin{aligned}
e_{2,t,a}(k) &= u_{t-1,a}(k) \times f(k) \times \lambda^{u} \\
&+ e_{2,t-1,a}(k)\Big(1-\delta\Big)\Big(1-\lambda^{e} \int_{\omega \in M_{t,a}^{E_2 \to E_0}(k) \cup M_{t,a}^{E_2 \to E_1}(k)} \mathrm{d}F(\omega)\Big)
\end{aligned}
$$

N Worker flows by k (Optimum)

Worker flows are defined as a function of age t and ability level a distributed over the interval $[\underline{a},\bar{a}]$, according to p.d.f. $f(a)$. Let $u^\star_{0,t,a}$, $u^\star_{t,a}$, $e^\star_{0,t,a}$, $e^\star_{t,a}$ be respectively the density of workers unemployed without OSHC, unemployed with OSHC, employed without OSHC and employed with OSHC.

N.1 For t=0

Initially, all individuals have no OSHC and enter on the labor market as unemployed, so that:

• $\forall\ a$:

$u^\star_{0,t,a} = f(a)$

N.2 $\forall\ t \in [1, T-1]$

• $\forall\ a$:

$$u^\star_{0,t,a} = u^\star_{0,t-1,a}\Big(1 - \lambda^{u_0} \int_{\omega \in M^{Y^{u_0} \to \hat{Y}}_{t,a} \cup M^{Y^{u_0} \to \tilde{Y}}_{t,a}} \mathrm{d}F(\omega)\Big) + \delta \int e^\star_{0,t-1,a}(\omega)\,\mathrm{d}\omega$$

$$u^\star_{t,a}(k) = u^\star_{t-1,a}(k)\Big(1 - \lambda^{u_0} \int_{\omega \in M^{Y^{u} \to \hat{Y}}_{t,a}(k) \cup M^{Y^{u} \to \tilde{Y}}_{t,a}(k)} \mathrm{d}F(\omega) - \lambda^{u} \int_{\omega \in k} \mathrm{d}F(\omega)\Big)$$
$$+ \delta e^\star_{t-1,a}(k)$$

- $\forall\ a < a^{\star}_{t,k}$:

$$
\begin{aligned}
e^{\star}_{0,t,a}(k) = {} & u^{\star}_{0,t-1,a} \times f(k) \times \lambda^{u_0} \int_{\omega \in M_{t,a}^{Y^{u_0} \to \hat{Y}}} \mathrm{d}F(\omega) \\
& + \int u^{\star}_{t-1,a}(\omega)\,\mathrm{d}\omega \times f(k) \times \lambda^{u_0} \int_{\omega \in M_{t,a}^{Y^{u} \to \hat{Y}}(k)} \mathrm{d}F(\omega) \\
& + \Big(1-\delta\Big) f(k) \Big(\lambda^{e_0} \int_{k \in M_{t,a}^{\hat{Y} \to \hat{Y}}(\omega)} e^{\star}_{0,t-1,a}(\omega)\,\mathrm{d}F(\omega) + \lambda^{e} \int_{k \in M_{t,a}^{\tilde{Y} \to \hat{Y}}(\omega)} e^{\star}_{t-1,a}(\omega)\,\mathrm{d}F(\omega)\Big) \\
& + e^{\star}_{0,t-1,a}(k)\Big(1-\delta\Big)\Big(1-\lambda^{e_0} \int_{\omega \in M_{t,a}^{\hat{Y} \to \hat{Y}}(k) \cup M_{t,a}^{\hat{Y} \to \tilde{Y}}(k)} \mathrm{d}F(\omega)\Big)
\end{aligned}
$$

$$
\begin{aligned}
e^{\star}_{t,a}(k) = {} & u^{\star}_{t-1,a}(k) \times f(k) \times \lambda^{u} \\
& + e^{\star}_{t-1,a}(k)\Big(1-\delta\Big)\Big(1-\lambda^{e} \int_{\omega \in \cup M_{t,a}^{\tilde{Y} \to \hat{Y}}(k) \cup M_{t,a}^{\tilde{Y} \to \tilde{Y}}(k)} \mathrm{d}F(\omega)\Big)
\end{aligned}
$$

- $\forall\ a \geq a^{\star}_{t,k}$:

$$
\begin{aligned}
e^{\star}_{t,a}(k) = {} & u^{\star}_{0,t-1,a} \times f(k) \times \lambda^{u_0} \int_{\omega \in M_{t,a}^{Y^{u_0} \to \tilde{Y}}} \mathrm{d}F(\omega) + u^{\star}_{t-1,a}(k) \times f(k) \times \lambda^{u} \\
& + \int u^{\star}_{t-1,a}(\omega)\,\mathrm{d}\omega \times f(k) \times \lambda^{u_0} \int_{\omega \in M_{t,a}^{Y^{u} \to \tilde{Y}}(k)} \mathrm{d}F(\omega) \\
& + \Big(1-\delta\Big) f(k) \Big(\lambda^{e_0} \int_{k \in M_{t,a}^{\hat{Y} \to \tilde{Y}}(\omega)} e^{\star}_{0,t-1,a}(\omega)\,\mathrm{d}F(\omega) + \lambda^{e} \int_{k \in M_{t,a}^{\tilde{Y} \to \tilde{Y}}(\omega)} e^{\star}_{t-1,a}(\omega)\,\mathrm{d}F(\omega)\Big) \\
& + e^{\star}_{t-1,a}(k)\Big(1-\delta\Big)\Big(1-\lambda^{e} \int_{\omega \in M_{t,a}^{\tilde{Y} \to \hat{Y}}(k) \cup M_{t,a}^{\tilde{Y} \to \tilde{Y}}(k)} \mathrm{d}F(\omega)\Big)
\end{aligned}
$$