Growth in an overlapping generation economy with a polluting non-renewable resource

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Abstract

This paper analyses the effects of flow pollution implied by the use of non-renewable resources on an overlapping economy. Notably, it shows that, on the balanced growth path, flow pollution reduces the resource use and thus increases the ability of an economy to sustain a non-decreasing consumption path. In addition, the paper analyses decentralization strategies that are able to put the decentralized economy on an optimal balanced growth path. If the resource stock is privately owned by households, resource taxation is necessary and the optimal tax level is characterized. If the state owns the resource stock, it should entrust the resource management to a fund constraint by an extraction rule which is also defined.

Keywords: Non-renewable Resources; Growth; Pollution; Overlapping Generations

JEL Codes: Q32; Q38; Q53

1 Introduction

Since the 70’s, one of the most discussed question among economists is essentially: “Can economies endowed with a finite stock of an essential non-renewable resource may grow indefinitely?” If the Club of Rome’s answer was pessimistic, this view has been challenged by neoclassical economists. Notably, Stiglitz (1974), Solow (1974), Dasgupta & Heal (1974, 1979) have highlighted the importance of (exogenous) technical progress, increasing returns and substitution possibilities between man-made and natural capital in order to surpass the problem of resource depletion.

Those authors use the infinitely-lived agents (ILA) framework. This choice is not without consequences. In ILA models, agents are implicitly assumed to be intergenerationally altruistic. More precisely, the ILA framework assumes a dynastic altruism where parents are able to maximize the welfare of their children. If such an altruism is not supported by empirical results (Altonji et al. (1992)), there also exist other types of altruism. The paternalist altruism represents a biased altruism:1 agents take care of some constituents of their children consumption, not equated with their children welfare, but with believes

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1This form of altruism seems empirically supported (see for example Jacobsson et al. (2007) or Rodríguez & León (2004) )
on what is good for them (McConnell, 1997; Lazo et al., 1997). In any case altruism tends to make a sustainable management more probable: since current generations take care of future ones, they are more likely to preserve the resource stock or to invest the rent in order to promote a decent standard of living for future generations following the Hartwick’s rule.

This observation has led to the work of Agnani et al. (2005). Those authors use the overlapping generation (OLG) framework, in which each generation is supposed to be perfectly selfish, to study the sustainability of growth with natural resources. In an OLG framework, agents are prone to consume a larger share of the natural rent than in the ILA framework: the economy is more likely to contract.\(^2\) Formally, Agnani et al. (2005) show that the labor share has to be large enough to allow for a positive balanced growth rate. A high labor share allows savings to compensate for resource depletion and makes the economy sustainable. If the labor share is not high enough the economy will contract.

One important feature of natural resources neglected by Agnani et al. (2005) is the pollution resulting from their extraction or use in the production process. Indeed, many real world environmental problems come from natural resources: resource extraction is a polluting activity, especially for metals (gold, silver, nickel...), while it is the resource use in production which is polluting for other non-renewable resources (oil, coal...). Thus, we will introduce an environmental externality in their model in order to analyze the impact of a pollution resulting from natural resources on the economy. If there exist numerous paper highlighting that human activities are sources of pollution, this pollution is often modeled as a stock increasing with total output (Jouvet et al., 2010), consumption (John & Pecchenino, 1994), or capital stock (Gradus & Smulders, 1993). In opposition, few papers have introduced pollution as resulting from resource extraction and/or resource use in the production process. Notable exception are Babu et al. (1997) and Schou (2000, 2002).\(^3\) While the literature usually concentrates on stock pollutants, we will introduce pollution as a flow affecting current factor productivity. This simplifying assumption shouldn’t be shown as restrictive. Indeed, there exists numerous pollutants with short lifetime (sulphur, black carbon, fine air particulate, nitrogen dioxide, tropospheric ozone...) which may be considered as flow pollutants, especially in the OLG framework where a period accounts for 25-30 years. Those pollutants are known to have effects on health (and thus worker productivity), land productivity, and may affect negatively plants (through acid rain for example). Flow emissions are often associated with stock emissions. Nevertheless the impact of flow emissions on the economy has known little attention from economists while they can lead to interesting results. In this framework, it will be shown that the existence of a pollution resulting from natural resources enhances the ability of the economy to sustain a non-decreasing consumption path. More precisely, the more the pollution affects factor productivity, the less the resource will be used in the production, releasing the level of investment necessary to compensate for resource depletion.\(^4\)

This work also analyzes the impact of resource dependence on sustainability. Not

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\(^2\) If one cannot infer that agents are selfish, using OLG model without altruism in order to study sustainability issues allows to obtain strong results: the introduction of altruism in the model should relax constraints on sustainability.

\(^3\) We focus here on general equilibrium literature. There also exists a deep literature that focuses on the exhaustible-resource/pollution problem (see for example Withagen, 1994).

\(^4\) In the very long run and abstracting from emission ceiling, this results stay true with stock pollutants: emissions are caused by a natural resource which tends to be exhausted so that emissions tends to zero and the natural absorption prevents the stock of pollutant to grow.
surprisingly, a resource dependent economy is less likely to exhibit positive long-run growth, because it needs more savings for capital accumulation to compensate for a faster resource depletion.

Finally, the last contribution of this paper is to surpass the Pareto efficiency criterion used by Agnani et al. (2005). While in the ILA framework decentralization of the Ramsey allocation does not require public intervention because the flow pollution is not distortive (Schou, 2000), this is not ever true in the OLG framework: the discrete time formulation with finite lifetime exhibits limitation of the market. Indeed, in presence of rational selfish agents, the market is not able to preserve the welfare of unborn generations. We thus propose two alternative instruments which may be used in order to decentralize a Ramsey optimal equilibrium. If the economy follows the usual assumption that the resource initially belongs to the first generation of agents, the Ramsey optimal equilibrium may be decentralized using a tax which is characterized. If we assumes that the resource is shared by all generations of agents, we are able to characterize the optimal policy that should be followed by an independent trust fund.

The rest of the paper is organized as follow: section 2 presents the decentralize economy, section 3 develops the Ramsey economy, section 4 is devoted to the decentralization of the Ramsey optimal allocation while section 5 concludes.

2 The decentralized economy

2.1 The Model

We use the two-period OLG model with one representative good. Agents are alive for two periods. For the sake of simplicity, no demographical growth is assumed and the size of the working force is normalized to one.\(^5\)

2.1.1 The Non-Renewable Resource

Following Agnani et al. (2005) the economy is initially endowed with a quantity \(m_{-1}\) of a necessary exhaustible resource held by the first generation of aged agents. At each date \(t\), elderly agents sell their resource share to the young generation and a quantity \(x_t\) of the resource is used in the production process and generates an environmental externality. The resource stock in \(t\) is thus denoted by \(m_t = m_{t-1} - x_t\) and it belongs to the generation \(t\). The rate of exhaustion of the natural asset is:

\[
q_t = \frac{x_t}{m_{t-1}} \quad (1)
\]

The dynamics of the per worker resource stock is thus \(^6\):

\[
m_t = (1 - q_t)m_{t-1} \quad (2)
\]

It leads, associated with the non renewability of the resource, to the exhaustibility condition

\[
1 \geq \sum_{t=0}^{\infty} q_t \prod_{j=1}^{t} (1 - q_{j-1}) \quad (3)
\]

\(^5\)Lowercases represent per worker variables.

\(^6\)It may also be interpreted as a resource market clearing condition as in Agnani et al. (2005).
2.1.2 Consumers

As in Agnani et al. (2005), agents are alive for two periods and maximize the following utility function:

\[ u_t(c_t; d_{t+1}) = \ln(c_t) + \frac{1}{1+\rho} \ln(d_{t+1}) \]  

where \( c \) represents the consumption while young, \( d \) the consumption while old and \( \rho \) the individual rate of time preference.

In the first period of life, the representative agent works to earn a wage \( w_t \), which may be consumed, saved as physical capital \( s_t \), or used to buy rights on the resource stock \( m_t \) at a price \( p_t \) in terms of the representative good. His first period budget constraint is:

\[ w_t = c_t + s_t + p_t m_t \]  

While old, he gets his savings increased at the interest rate, and he sells his resource rights at a price \( p_{t+1} \). His second period budget constraint is

\[ d_{t+1} = (1 + r_{t+1})s_t + p_{t+1}m_t \]  

Combining 5 and 6, we obtain the following inter-temporal budget constraint (IBC hereafter):

\[ w_t = c_t + \frac{d_{t+1}}{1 + r_{t+1}} - \frac{p_{t+1}m_t}{1 + r_{t+1}} + p_t m_t \]  

The maximization of utility with respect to \( m_t, c_t, d_{t+1} \) subject to the IBC leads to the following first order conditions:

\[ \frac{d_{t+1}}{c_t} = \frac{1 + r_{t+1}}{1 + \rho} \]  
\[ \frac{p_{t+1}}{p_t} = 1 + r_{t+1} \]  

(8) is the standard Euler equation while (9) is an arbitrage condition between the two assets in this economy, capital and resource.

2.1.3 Firms

Firms produce the representative good \( Y_t \) using a Cobb-Douglas technology. They use capital \( K_t \), labor \( N_t \), and resources \( X_t \), with constant returns to scale for a given level of technology \( A_t \) which grows at a rate \( a \) such that:

\[ A_{t+1} = (1 + a)A_t \]  

The extraction and use of the resource in the production process generate a flow of pollution \( e_t \) such that:

\[ e_t = \phi x_t \]  

Pollution resulting from the use of the resource in the production process generates a productivity loss. The flow pollution considered here is not so reductive. There exists a wide variety of pollutant with short lifetime that may cause productivity losses. For example sulfur dioxide resulting from the burning of fossil fuel is a major cause of acid
rain. Tropospheric ozone resulting from fossil fuel burning\(^7\) generates health issues that may affect directly the productivity of workers (Zivin & Neidell, 2012)\(^8\). \(\theta\) captures the detrimental impact of pollution on the level of production. Nielsen \textit{et al.} (1995) and Schou (2000) model the detrimental effect from pollution on productivity in the same way. The production function is thus:

\[ y_t = A_t k_t^{\alpha} x_t^{\nu} e_t^{-\theta} \]  

with \(\alpha + \beta + \nu = 1\).\(^9\) Thus, the model assumes constant return to scale from the firm point of view. Indeed, the environmental externality is not taken into account by individual firms but they consider the aggregated level of pollution as given while they decide their production plan.

Capital is remunerated at the interest rate \(r_t\) and depreciates at a rate \(0 < \delta < 1\). Firms pay a wage \(w_t\) to their workers and buy the natural input at its price \(p_t\). The profit of the representative firm is thus:

\[ \Pi_t = A_t k_t^{\alpha} x_t^{\nu} e_t^{-\theta} - (r_t + \delta)k_t - w_t - p_t x_t \]  

We focus on the case \(\theta < \nu\). That is, for an identical amount of resource and emissions, we assume that the positive impact of resources on income outweighs its negative one. A similar assumption may be found in Schou (2000), where pollution is also seen as a flow. The standard profits maximization leads to the following first order conditions:

\[ r_t = \alpha A_t k_t^{\alpha-1} x_t^{\nu} e_t^{-\theta} - \delta \]  

\[ w_t = \beta A_t k_t^{\alpha} x_t^{\nu} e_t^{-\theta} \]  

\[ p_t = v A_t k_t^{\alpha} x_t^{\nu-1} e_t^{-\theta} \]  

Each factor is thus paid at its marginal productivity.\(^{10}\)

### 2.2 The Balanced Growth Path

The economy produces a representative good which may be consumed or saved as physical capital. Following Diamond (1965), the good market clearing condition is

\[ s_t = k_{t+1} \]  

\[ 5 \]

**Definition 1.** An inter-temporal competitive equilibrium is a solution of the system

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\(^7\)More precisely, ozone is produced through the interaction of nitrogen dioxide with dioxygen and sun.  
\(^8\)An effect of pollution on health also affects agents utility and would be better modeled if also introduced in the utility function. For the sake of simplicity, we do not consider the impact of pollution on utility which could be analyzed in future research.  
\(^9\)\(\beta\) represents the elasticity of production with respect to labor which is normalized to one.  
\(^{10}\)This paper concentrates on interior solutions.
formed by the following equations:

\[
\beta (1 + a)^t A_0 k_t (m_{t-1} - m_t)^{v-\theta} \phi^{-\theta} = (2 + \rho) [k_{t+1} + v(1 + a)^t A_0 k_t^\alpha (m_{t-1} - m_t)^{v-1-\theta} \phi^{-\theta} m_t]
\]

(18)

\[
(1 + a) \frac{k_t^\alpha (m_t - m_{t+1})^{v-1-\theta}}{k_t (m_{t-1} - m_t)^{v-1-\theta}} = 1 + \alpha (1 + a)^{t+1} A_0 k_{t+1}^{\alpha-1} (m_t - m_{t+1})^{v-\theta} \phi^{-\theta} - \delta
\]

(19)

This system is a system of second order equations. Assuming a constant extraction rate \( q \), the system may be rewritten in terms of first order equations:

\[
\beta (1 + a)^t A_0 k_t x_t^{v-\theta} \phi^{-\theta} = (2 + \rho) [k_{t+1} + v(1 + a)^t A_0 k_t^\alpha x_t^{v-1-\theta} \phi^{-\theta} x_{t+1}/q]
\]

(20)

\[
(1 + a) \frac{k_{t+1}^\alpha x_{t+1}^{v-1-\theta}}{k_t^\alpha x_t^{v-1-\theta}} = 1 + \alpha (1 + a)^{t+1} A_0 k_{t+1}^{\alpha-1} x_{t+1}^{v-\theta} \phi^{-\theta} - \delta
\]

(21)

Then, remarking that \( p_0 \) is a jump variable which is not given by history, we can assume that \( p_0 \) jumps instantaneously to a value that ensures a constant extraction path.

The rest of the paper focuses on balanced growth path because they constitute the only case where long-run positive growth is possible, as noted in Agnani et al. (2005). Moreover, it is in accordance with stylized facts of growth literature.

**Definition 2.** An inter-temporal equilibrium where all variables grow at a constant rate is defined as a balanced growth path (BGP hereafter).

Let \( \mu_h \) be the BGP notation of the ratio \( h_{t+1}/h_t \). According to definition 2, \( \mu_m \) should be constant. Thus (2) implies a constant rate of extraction along the BGP, i.e. \( q_t = q_{t+1} = q \).

**Proposition 1.** This overlapping economy is characterized by the following growth rates:

\[
\begin{align*}
\mu_k &= \mu_y = \mu_s = \mu_w = \mu_c = \mu \\
\mu_x &= \mu_e = \mu_m = 1 - q \\
\mu_p &= \mu/\mu_x \\
\mu_a &= 1 + a \\
\mu_r &= 1 \\
\mu &= (1 + a)^{\frac{1}{1-\alpha}} (1 - q)^{\frac{v-\theta}{1-\alpha}}
\end{align*}
\]

**Proof.** Proof is reported in appendix A.1 .

From Proposition 1, it may be established that a necessary condition for a long run positive growth is \( q < 1 - (1 + a)^{\frac{1}{1-\alpha}} \). This threshold will be referred as a positive growth threshold (PGT hereafter). To analyze how the balanced growth path is affected by a change in \( \theta \), it is necessary to characterize the constant extraction rate. The market clearing condition (17) may be written, using (5), (6), (8), (15), (16), as:

\[
k_{t+1} = \left[ \frac{\beta}{2 + \rho} - \frac{(1 - q) v}{q} \right] A_t k_t^\alpha x_t^{v-\theta} e_t^{\theta}
\]

(22)
Evaluating this equation at the BGP, we can obtain:

\[
\frac{(1 + a) \frac{1}{1-\alpha} (1 - q)^{\frac{1}{1-\alpha}} \alpha (2 + \rho)q}{\beta q - v(1 - q)(2 + \rho)} = \frac{(1 + a) \frac{1}{1-\alpha} (1 - q)^{\frac{1}{1-\alpha}}}{1 - q} - (1 - \delta)
\]  

(23)

It may now be established that \(q^*\) is solution to the preceding non linear equation. We denote LHS and RHS the left and right hand side of (23). \(RHS(q)\) is defined on \([0; 1]\) with \(RHS(0) = (1 + a) \frac{1}{1-\alpha} - (1 - \delta)\). Since \(\lim_{q\to1} RHS(q) = +\infty\), \(RHS(q)\) admits a vertical asymptote in \(q = 1\). Moreover, \(\frac{\partial RHS(q)}{\partial q} > 0\) and \(\frac{\partial^2 RHS(q)}{\partial^2 q} > 0\) imply an increasing and convex function. \(LHS(q)\) is defined on \([0; \hat{q}[U][\hat{q}; 1]\) with \(\hat{q} = \frac{v(2+\rho)}{\beta+c(2+\rho)}\). \(LHS(q) < 0 \forall q < \hat{q}\) which do not allow the possibility of an equilibrium extraction rate given that \(RHS(q) > 0 \forall q \in [0; 1]\). Since \(\lim_{q\to\hat{q}(+)} = +\infty\) and \(\lim_{q\to1} = 0\) it exists a unique \(q^*\) such that (23) is satisfied. This situation is represented in Figure 1.

Figure 1: Characterization of the competitive equilibrium extraction rate

The threshold \(\hat{q}\) is called the growth area threshold (GAT hereafter). Indeed, an increase in the GAT reduces the set of \(q\) such that the economy grows at the BGP, because the economy contracts if \(PGT < \hat{q}\).

From Proposition 1 it appears that an higher extraction rate is associated with a lower growth while looking at (1) and (12), an increase in \(q\) implies an higher income. It is thus necessary to distinguish between short and long run impacts of a higher extraction rate. In the short run, an increase in \(q\), ceteris paribus, implies an increase of one input in the production process. Current production thus increases. Nevertheless, it will be harder to maintain this level of production, because less natural resources are available to produce. In the long run, the higher is the extraction, the more the economy needs capital to compensates for resource depletion. The pressure on natural resources thus limits future growth possibilities.
2.3 The impact of flow pollution on sustainability

In this work, we define sustainability as the ability of the economy to sustain a non-declining balanced consumption path. The economy will contract if $PGT < GAT$ i.e. if $\beta < \frac{(1+a)\theta^{-\rho}(2+\rho)v}{1-(1+a)\theta^{-\rho}}$. This condition is less likely to be satisfied when $\theta$ increases. Figure 2 represents the results of a numerical simulation in order to see how the sustainability of the economy is affected when $\theta$ increases, keeping the constant returns to scale assumption. The simulation is performed for the following annual values: $a = 0.028$, $\delta = 0.027$, $\rho = 0.016$. The space over the curve represents the set of capital, resource and labor shares such that the economy will contract.

![Figure 2: Effect of an increase of $\theta$ on sustainability](image)

**Proposition 2.** When pollution hurts severely productivity, the contraction area is reduced. The detrimental effect of pollution on production thus enhance sustainability.

**Proof.**

$$\frac{\partial}{\partial \theta} \left( \frac{(1+a)\theta^{-\rho}(2+\rho)v}{1-(1+a)\theta^{-\rho}} \right) < 0$$

This effect may seems puzzling, it is nevertheless quite intuitive. If sustainability is enhanced while the detrimental impact of pollution on production increases, this is due to the specificity of pollution which is considered here. Firstly, we consider the category of flow pollutants that hurt the current period productivity, and disappear in the next period. Secondly, we do not use the standard assumption that pollution is an externality resulting from production. Here, pollution is an externality coming from the extraction and/or the use of a non-renewable resource in the production. Along the balanced growth path, emissions thus evolve as the resource do, and disappear asymptotically. When time goes by, the resource is used in smaller and smaller amount. The negative impact that natural resources have on productivity through pollution also decreases. The higher

\footnote{The choice of parameter values comes from Agnani et al. (2005).}

\footnote{As previously exposed, those assumptions are realistic for some pollutants (tropospheric ozone, methane, black carbon...).}
is the detrimental effect of pollution on growth, the stronger will be this effect, and the higher is growth on the balanced growth path.\textsuperscript{13} That is, when $\theta$ increases, it diminishes the adverse effect on growth imposed by the necessary decreasing resource extraction. To say it differently, $\theta$ diminishes the net resource’s contribution to growth, and thus its implicit negative contribution. Moreover, when $\theta$ increases, the BGP extraction rate is lower. It allows the economy to save some resources for the future. In the long run, a lower level of savings is needed to compensate for resource depletion and the condition on the labor share thus eases.

2.4 The impact of resource dependence on growth

In this economy, $v$ is the income share of natural resources. Indeed, $w_t/y_t = \beta$, $(r_t - \delta)k_t/y_t = \alpha$ and $p_t x_t/y_t = v$.

Indeed, we can calibrate the model in order to show that an increase in $v$ is associated with a lower balanced growth rate. The derivative of growth with respect to the resource share is:

$$\frac{\partial \mu}{\partial v} = \mu \left[ \frac{\ln(1-q)}{1-\alpha} - \frac{\partial q}{\partial v} \frac{v-\theta}{(1-q)(1-\alpha)} \right]$$

(24)

Restricting our analysis to positive growth area, we can represent in the $(v,q,\frac{\partial \mu}{\partial v})$ space this derivative with respect to all possible $v$ and $q$ values.\textsuperscript{14}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3}
\caption{\(\frac{\partial \mu}{\partial v}\) for $a = 0.028$, $\delta = 0.027$, $\rho = 0.016$, $\theta = 0.01$, $\alpha = 0.3$}
\end{figure}

The numerical exercise represented in figure 3 shows that an increase in resource dependence (the income share of natural resources) is associated with a lower growth rate.

\textsuperscript{13}Using a numerical approach, appendix A.2 shows that an increase in $\theta$ increases the rate of growth for reasonable parameter values.

\textsuperscript{14}A sensitivity analysis has been performed. This result is robust to variation of parameter values in a reasonable range.
Because of the exhaustibility of the resource stock, a given growth rate implies to save more and more on the resource. In itself, it implies that natural resources have a negative contribution to growth. It is not surprising that when the importance of the resource in production increases, this negative contribution increases. However, the constant returns to scale assumption has been lost. To keep the constant returns to scale assumption, an increase in $v$ necessarily implies a decrease in either $\beta$ or $\alpha$ by the same order (or both obviously). The effect of a change in either $\beta$ or $\alpha$ are reported in appendix A.3. Results show that an increase in $v$ compensated by a decrease in $\alpha$ or a decrease in $\beta$ is associated with a lower balanced growth for reasonable parameter values. Keeping constant returns to scale does not change the result. Note that if an increase in resource dependence depresses growth, an increase in resource abundance (i.e. higher initial endowment) affect positively the level of income reached at each date.

This results directly come from the Euler equation and the non-arbitrary condition between the capital and the resource. Indeed, combining (8) and (9) and evaluating the right hand side on the BGP, we can write:

$$(1 + \rho)^{d_{t+1}} = \mu_k^\alpha \mu_x^{v-\theta}$$

For a given extraction rate, an increase in $v$ (or a decrease in $\theta$) implies a decrease in the return of capital (or resource). Households then choose to increase their consumption while young and they reduce their savings and thus capital accumulation. Nevertheless, a change in those parameter causes a change in the extraction rate that may overcompensate the present result. The numerical simulation shows however that this is unlikely for reasonable parameter values since growth decreases with $v$.

### 3 The Ramsey Economy

#### 3.1 The Model

Let’s consider a benevolent social planner assumed to solve the following Ramsey problem:

$$\max_{\{c_t; d_t; k_t; x_t\}_{t=0}^{\infty}} \frac{1}{1 + \rho} \ln(d_0) + \sum_{t=0}^{\infty} \frac{1}{(1 + \gamma)^{t+1}} \left[ \ln(c_t) + \frac{1}{1 + \rho} \ln(d_{t+1}) \right]$$

subject to:

$$y_t = A_t k_t^\alpha x_t^{v-\theta}$$  \hspace{1cm} (25)

$$y_t = c_t + d_t + k_{t+1} - (1 - \delta) k_t$$  \hspace{1cm} (26)

$$A_{t+1} = (1 + a) A_t$$  \hspace{1cm} (27)

$$e_t = \phi x_t$$  \hspace{1cm} (28)

$$m_t = (1 - q_t) m_{t-1}$$  \hspace{1cm} (29)

$$x_t = q_t m_{t-1}$$  \hspace{1cm} (30)

$$m_{-1} = \sum_{t=0}^{\infty} q_t m_{t-1}$$  \hspace{1cm} (31)

$$k_0, \ m_{-1}, \ e_{-1}, \ A_0 > 0 \text{ given,}$$  \hspace{1cm} (32)
where $\gamma$ is the social discount rate. (25) represents the production function. (26) established that the economy consumes or invests exactly its net production in each period. (27) represents the exogenous technological progress. (28) is the emissions implied by the resource use while (29) and (30) represents the dynamics of the resource. (31) is a total exhaustibility condition for the resource while (32) represents initial endowments.

The FOC of the previous problems may be reduced to:

$$
1 + \frac{\gamma}{1 + \rho} = \frac{d_t}{c_t}
$$

(33)

$$
(1 + \rho) \frac{d_{t+1}}{c_t} = \alpha A_{t+1} k_{t+1}^{\alpha - 1} x_{t+1}^{\nu} e_t^\theta + 1 - \delta
$$

(34)

$$
\frac{A_{t+1} k_{t+1}^{\alpha} x_{t+1}^{\nu} e_t^\theta}{A_t k_t^{\alpha} x_t^{\nu} e_t^\theta} = \frac{(v x_t^{\nu - 1} + \phi \theta e_t^{-1})}{(v x_{t+1}^{\nu - 1} + \phi \theta e_{t+1}^{-1})}
$$

(35)

$$
\lim_{t \to \infty} \left( \frac{1}{1 + \gamma} \right)^t \frac{k_{t+1}}{c_t} = 0
$$

(36)

(33) is an intergenerational optimality condition establishing that the marginal rate of substitution between consumption of young and old has to be equal to one. (34) is an intragenerational optimality condition which states that the marginal rate of substitution between consumption while young and consumption while old has to be equal to the marginal product of physical capital net of depreciation. (35) characterize the optimal inter-temporal resource allocation which indicates that the depletion of the resource stock implies a implicit return equal to the physical capital return. This condition implies that the economy should satisfy the Hotelling rule. (36) is the transversality condition associated with the planner problem.

Combining (25)-(36) we can define the balanced growth path of this Ramsey economy.

**Proposition 3.** The optimal balanced growth path is defined by:

$$
\tilde{\mu}_k = \tilde{\mu}_y = \tilde{\mu}_c = \tilde{\mu}
$$

(37)

$$
\tilde{\mu}_x = \tilde{\mu}_e = \tilde{\mu}_m = 1 - \tilde{q}
$$

(38)

$$
\tilde{\mu}_a = 1 + a
$$

(39)

$$
\tilde{\mu} = (1 + a) \tilde{\mu}_a (1 - \tilde{q}) \tilde{\mu}_m^{\tilde{a}}
$$

(40)

*Proof. *Proof is reported in appendix A.4.

Using (33), (34), (35), we have $(1 + a) \tilde{\mu}_k^{\tilde{a}} \tilde{\mu}_x^{\tilde{a}} \tilde{\mu}_e^{\tilde{a}} = (1 + \gamma) \tilde{\mu}_c$. Since $\tilde{\mu}_c = \tilde{\mu}_k$, it can be established that $\tilde{q} = \frac{\gamma}{1 + \gamma}$. Thus, the optimal extraction rate only depends on the social rate of time preference. A higher social preference for the present implies a higher depletion rate of the resource stock, and a lower growth. To put it differently, a society which strongly cares about future generations is more conservative and achieve a larger rate of growth.

To decentralize the optimal balanced growth path, a government should find an instrument able to put the extraction rate at the optimal level $\frac{\gamma}{1 + \gamma}$, keeping the rate of growth of other variables at their optimal level. In section 4, we will compare the ability of two scenarios of resource attribution in decentralizing the optimal allocation.

It appears that the optimal policy crucially depends on the social rate of time preference. Indeed, since pollution is modeled as a flow, it is not distortive on the balanced
growth path. The only market failure is linked to the demographic structure imposed by the OLG framework: the existence of future generations is not taken into account by the current market economy. Thus, a critical parameter for policy making is the social rate of time preference. The amplitude and existence of a positive social discount rate is widely discussed in the literature. In our OLG framework, the social planner takes into account the fact that agents discount their own utility. The social rate of time preference thus reflects solely the weight that the central planner attaches to each generation. One may argue that the central planner shouldn’t favor closer in time generation. In such a case, the policy maker should choose \( \gamma = 0 \) (Ramsey, 1928; Pigou, 1932; Solow, 1986). Nevertheless, the uncertainty about future economic conditions argue for positive discount rate. For example, our framework is not robust to the existence of a backstop technology that may appear in the future and which will modify the production function (resource may becomes unnecessary in the future). Since this is likely to happen in the very long run, a positive social discount rate allow to avoid an overweighting of distant generations’ welfare. Moreover, the existence of far distant generations is not guaranteed. Before looking at the optimum decentralization procedures, one should analyze how changes in important parameters affects the optimal balanced growth path.

3.2 Comparative Statics

This section analysis how movements in \( \theta \) and \( v \) impact the optimal rate of growth. We begin by looking at the impact of \( \theta \) on the rate of growth.

\[
\frac{\partial \tilde{\mu}}{\partial \theta} = -\frac{(1 + a)^{\frac{1}{1-\alpha}} (1 - \bar{q})^{\frac{-\beta}{1-\alpha}} \log(1 - q)}{1 - \alpha} > 0
\]  

**Proposition 4.** The optimal long run rate of growth increases with the detrimental effect of pollution.

\( \theta \) diminishes the net resource contribution to production. Mechanisms are similar to those previously exposed. When time goes by, the resource, which is not reproducible, will be used in ever small amount. Thus, when \( \theta \) increases, it decreases the detrimental effect caused by the need to keep diminishing resource extraction. That leads to a higher growth rate of the economy. Note that contrary to what happens in the market equilibrium, an increase in \( \theta \) does not decrease the extraction rate in the Ramsey economy. The increase in growth is uniquely due to a diminishing resource contribution to growth. While a higher \( \theta \) causes a higher growth rate, the level of production may decrease for some instant of time when \( \theta \) increases due to a contemporaneous adverse effect on production.

Let’s consider now a variation of resource dependence. In such a model, resource dependence is characterized by the resource share of output \( v \). In order to keep constant returns to scale, an increase in \( v \) should be compensated by an equivalent decrease of either \( \beta \) or \( \alpha \), or both.

Let’s consider that an increase in \( v \) is compensated by a decrease in \( \beta \) such that returns to scale are kept constant.

\[
\frac{\partial \tilde{\mu}}{\partial \beta} = 0
\]  

\[
\frac{\partial \tilde{\mu}}{\partial v} = \frac{(1 + a)^{\frac{1}{1-\alpha}} (1 - \bar{q})^{\frac{-\beta}{1-\alpha}} \log(1 - q)}{1 - \alpha} < 0
\]
Now, imagine that an increase $dv$ of $v$ is compensated by an equivalent decrease $d\alpha$ of $\alpha$ such that $dv = d\alpha$. The total impact on the balanced growth rate will be:

$$
\frac{\partial \tilde{\mu}}{\partial v} dv - \frac{\partial \tilde{\mu}}{\partial \alpha} d\alpha = \frac{(1 + a) \frac{1}{1 - \alpha} (1 - \hat{q})^{\frac{1}{1 - \alpha}} \log(1 - q)}{1 - \alpha} dv
$$

$$
- \frac{(1 + a) \frac{1}{1 - \alpha} (1 - \hat{q})^{\frac{1}{1 - \alpha}} (v - \theta) \log(1 - q)}{(1 - \alpha)^2} d\alpha
$$

$$
- \frac{(1 + a) \frac{1}{1 - \alpha} (1 - \hat{q})^{\frac{1}{1 - \alpha}} \log(1 + a)}{(1 - \alpha)^2} d\alpha < 0
$$

Proposition 5. The optimal long run rate of growth decreases with resource dependence.

This result comes from the scarcity of the resource which imposes to the economy to save more and more on the resource stock in order to avoid complete exhaustion in finite time. In itself, this gives a negative resource contribution to growth. When $v$ increases, the importance of the resource in the production increases, the negative contribution of the resource exhaustibility to growth is higher.

Those results are direct outcomes of equation (34) and (35). Combining those equations, we can write

$$(1 + \rho) \frac{d_{t+1}}{c_t} = \frac{A_{t+1}k_{t+1}^\alpha x_t^v \epsilon_t^{v-\theta}(v x_{t+1}^{1-\epsilon} + \phi \theta e_{t+1}^{1-\epsilon})}{A_t k_t^\alpha x_t^{v-\theta}(v x_t^{1-\epsilon} + \phi \theta e_t^{1-\epsilon})}$$

Evaluating the right hand side of this equation at the BGP leads to:

$$(1 + \rho) \frac{d_{t+1}}{c_t} = (1 + a) \tilde{\mu}_x^\alpha \mu_x^{v-\theta-1}$$

Since $\tilde{\mu}_x \in (0, 1)$, is not affected by a change in either $v$ or $\theta$, we can infer that an increase in $v$ or a decrease in $\theta$ cause a decrease in the right hand side of the last equation, which represents the rate of return of capital (and resource) on the BGP. Present consumption thus increases at the expense of future consumption, and capital accumulation decreases, leading to a lower growth.

4 Decentralizing the Ramsey Optimal Balanced Growth Path

OLG model with a pollution externality are often associated with two market failures. The first one linked to the pollution, the second one linked to the demographic structure of the OLG economy. Concentrating on BGP, it appears from propositions 1 and 3 that pollution doesn’t disturb the economy because the flow pollution we are considering evolves at the extraction rate. OLG economies are also known to allow for possible capital over-accumulation, which enables the implementation of Pareto improving policies. Nevertheless, Rhee (1991), and Gerlagh & Keyzer (2001) have shown that OLG economies endowed with a finite non-renewable resource are efficient in the Pareto sense, and this results is robust in the model we are using here (Agnani et al., 2005).

The present section aims at surpassing the Pareto efficiency criterion previously used in the literature. We are thus interesting in policies that are able to decentralize the
Ramsey optimal allocation for a given social discount rate calibrated by the policy maker to reflect intergenerational fairness. Looking at proposition 1 and 3, it immediately appears that decentralization of the BGP requires to put the market extraction rate at its optimal $\tilde{q}$ level letting other growth determinants unchanged.\footnote{Results obtained in this section only apply to the BGP and should not be translated outside.} We propose two ways of decentralization: a tax on resource use and extraction, and to let resource management to a public fund constrained by an extraction rule.

4.1 Tax on the private resource

A way that may be used to decentralize the optimal Ramsey equilibrium is to tax the resource used in the production, and to redistribute this income to the young generation as a transfer. Since the resource is taxed, equation (16) becomes:

$$p_t + \tau_t = v A_t k_t^\alpha x_t^{1-\delta} e_t^{-\theta}$$

where $\tau_t$ is the tax rate set by the government in $t$. The tax is redistributed to the young generation as a transfer $g_t$ such that the government budget is balanced: \footnote{Considering that the tax is invested and then redistributed to old households leads to the same results.}

$$\tau_t x_t = g_t$$

The budgetary constraint of the young agent (5) is thus modified as follow:

$$g_t + w_t = c_t + s_t + p_t m_t$$

**Proposition 6.** If the tax and the resource price increase at the same rate, the balanced growth path of the economy is not distorted by the tax.

**Proof.** Proof is reported in Appendix A.5.

Proceeding as in section A.1 we can write the market clearing condition as

$$k_{t+1} = A_t k_t^\alpha x_t^{1-\delta} e_t^{-\theta} \left[ \frac{\beta}{2 + \rho} - \frac{v(1-q)}{q} \right] + \frac{\tau_t x_t}{2 + \rho}$$

Dividing both side by $k_t$, it leads to

$$\mu_k - \frac{\tau_t x_t}{(2 + \rho) k_t} = A_t k_t^{\alpha-1} x_t^{\nu-\theta} e_t^{-\theta} \left[ \frac{\beta}{2 + \rho} - \frac{v(1-q)}{q} \right]$$

Since the tax and the resource price increase at the same rate, the fiscal revenues grow at a rate, $\mu_{xT} = \mu_k$, and $\frac{\tau_t x_t}{k_t}$ is constant on the BGP. Let $\xi$ denote this constant. Evaluating (25) on the BGP leads to

$$\left(1 + a\right)^{\frac{1}{1-\gamma}} \left(1 - q\right)^{\frac{\nu-\theta}{1-\gamma}} \left(\alpha(2 + \rho)q - \xi \alpha q\right) = (1 + a)^{\frac{1}{1-\gamma}} (1 - q)^{\frac{\nu-\theta}{1-\gamma} - 1} - (1 - \delta)$$

Comparing (50) with its counterpart without tax (23) it appears that RHS has not been impacted by the tax. Nevertheless, the LHS of equation (23) has been modified by the
existence of the resource tax. LHS of equation (50) is defined for \( q \in [0, \hat{q}[U]\hat{q}, 1] \) with 
\[ \hat{q} = \frac{\gamma}{\beta + \gamma(2 + \rho)}. \] 
Since \( \hat{q} = \frac{\gamma}{1 + \gamma} \), the Ramsey equilibrium is not decentralizable for \( \gamma = \frac{\alpha(2 + \rho)}{\beta}. \)

Below the threshold \( \hat{q} \), the derivative of LHS(\( q \)) \( >0 \) for a sufficient level of resource taxation \( \hat{\xi} \), \( \lim_{q \to 0}(q) = 0 \) and \( \lim_{q \to 1}(q) = +\infty \), where
\[ \hat{\xi} = v^{-1}(1 + a)^{1/\rho} (1 - q)^{-\alpha/\rho^2} [\beta q(\nu - \theta) - v(2 + \rho)(1 - q)(\nu - \theta) + v(2 + \rho)] > 0 \] (51)
Thus, if \( 0 < \tilde{q} < \hat{q} \), there exists a level \( \xi > \hat{\xi} \) such that the Ramsey equilibrium is decentralizable.

Above the threshold \( \hat{q} \), the derivative of LHS(\( q \)) \( <0 \) for a sufficiently low level of resource taxation \( \xi \), \( \lim_{q \to 0}(q) = +\infty \) and \( \lim_{q \to 1}(q) = -\frac{\xi_0}{\beta} \). Thus, if \( \hat{q} < \tilde{q} < 1 \), there exists a level of \( \xi < \hat{\xi} \) which decentralizes the Ramsey optimal equilibrium.

From equation (50) the optimal taxation may be obtained:
\[ \xi = -\frac{[(1 + a)^{1/\rho} (1 - \tilde{q})^{-\alpha/\rho^2} - (1 - \delta)][\beta \tilde{q} - v(2 + \rho)(1 - \tilde{q}) - (1 + a)^{1/\rho} (1 - \hat{q})^{-\alpha/\rho^2} \alpha(2 + \rho)\tilde{q}]}{a\tilde{q}} \] (52)
where it should be recalled that \( \tilde{q} = \frac{\gamma}{1 + \gamma} \). Calibrating the model with annual rates of \( a = 0.028 \), \( \delta = 0.027 \), \( \rho = 0.016 \), \( \nu = 0.05 \), \( \alpha = 0.3 \), \( \beta = 0.65 \), and \( \theta = 0.01 \), \( e \) can characterize \( \xi \) for different level of \( \gamma \). The level of the tax decreases with the social rate of time preference. With the above calibrated values, a negative taxation (i.e. subvention) for resource extraction is required for an annual discount rate of \( \gamma > 0.0202 \).

As previously discussed, a reasonable value for \( \gamma \) is such that \( 0 < \gamma < \rho \). A subvention becomes necessary when the social planner preferences are oriented in favor of close in time generations, such that he considers that current generations are too conservative. In such a case, the policy maker should subvention the resource use which will decrease the rate of growth. In case of very strong social preference in favor of present, the rate of growth may become negative, leading to (optimal) extinction. This little simulation exercises show that for reasonable annual social discount rate values (i.e. between 0 and 0.016 ), policy makers should implement a tax on fossil resource use. In this model with flow pollutants, this tax also helps to fight against pollution and may also be qualified as an environmental tax.

### 4.2 Public property rights on the resource stock

Another way to decentralize the Ramsey optimal BGP is to modify the property rights on the resource stock. Considering that the resource is a common shared by all generations, one may decide that the resource should belong to a trust fund which cares about all generations. On the BGP, this fund should follow an extraction rule such that the resource exhaustion is done at the optimal rate \( \hat{q} \). This fund is the sole supplier of natural resources and thus knows the productive firms demand for this input. We assume that the fund invests the rent from resource extraction and gives a transfer to the old generation as a retirement pension such that its budget is balanced.\(^ {17} \)

\(^{17}\)It is known that letting the resource management to a fund which maximizes the present value of the resource rent is often a way to decentralize the optimal equilibrium (see Conrad, 2010). However, it can be shown that in this model, such a policy is able to decentralize the market equilibrium only if there is no growth, i.e. if the technical progress increases at a rate \( a = ((1 + \gamma)/\gamma)^{\nu - \theta} \). This is because utility is not derived directly from resource use in the model which takes into account the productive structure of the economy.
In the first period of life, the household works and earn a wage which is used for savings and consumption. The first period budget constraint becomes:

\[ w_t = c_t + s_t \]  \hspace{1cm} (53)

The second budget constraint becomes:

\[ d_{t+1} = (1 + r_{t+1})s_t + (1 + r_{t+1})g_t \]  \hspace{1cm} (54)

We can then derive the inter-temporal budget constraint:

\[ w_t = c_t + \frac{d_{t+1}}{1 + r_{t+1}} - g_t \]  \hspace{1cm} (55)

Maximization of utility with subject to the IBC leads to the Euler equation (8), which established that the marginal rate of substitution between the two consumptions is equal to their relative price.

Competitive productive firms are profit maximizing and their behavior is not modified with respect to previous sections. Thus the demand function from productive firms is thus characterized by the following equation:

\[ p_t = vA_t k_t^\alpha \tilde{x}_t^{\alpha - 1} \epsilon_t^{-\theta} \]  \hspace{1cm} (56)

The trust fund mission is to manage optimally the resource stock according to a rule of extraction. Each period, the resource stock should be exhausted at a rate \( \tilde{q} = \gamma/(1 + \gamma) \). Since it is the sole which can offer the non-renewable resource to firms, he knows the demand function that emerge from the firms. Thus it will sold to firms a quantity \( \tilde{x}_t = \tilde{q}m_{t-1} \) at each date. Since it is a monopoly, the funds optimal policy is to choose \( \tilde{p} \) such that:

\[ \tilde{p}_t = vA_t k_t^\alpha \tilde{x}_t^{\alpha - 1} \epsilon_t^{-\theta} \]  \hspace{1cm} (57)

This allows market to be balanced and do not create distortion in the rest of the economy. Since the budget of the fund should be balanced, we have:

\[ \tilde{p}_t \tilde{x}_t = \tilde{g}_t \]  \hspace{1cm} (58)

The new market clearing condition is:

\[ \tilde{g}_t + s_t = k_{t+1} \]  \hspace{1cm} (59)

**Proposition 7.** The trust fund is able to decentralize the Ramsey optimal BGP.

**Proof.** Proof is reported in appendix A.6.

As shown in appendix A.6, this policy doesn’t distort the balanced growth path and put the growth rate at its optimal level. The reason is simple: the rule implies that the extraction rate is calibrated at its optimal level. The optimal transfer \( \tilde{g}_t \), which is equal to the fund revenues resulting from the implementation of the optimal policy, grows at the same rate than the capital stock. Thus, on the balanced growth path, the market clearing condition is not affected. Wages also grow at the capital stock, so consumption while young should grow at this rate (otherwise, the economy is not on a balanced growth
path.) Then the Euler equation implies that consumption while old also grows at this rate. Thus, all variable grow at their optimal rate.

### 4.3 Discussion

I previously provide two solutions in order to decentralize the optimal balanced growth path. In the first case, one can see that a good tax implementation may allow to decentralize the optimal BGP. Nevertheless, implementation of such a tax may face a technical difficulty: if the social rate of time preference is \( \gamma = \frac{v(2+\rho)}{\beta} \), a market economy could never reach the optimal extraction rate \( \tilde{q} = \frac{v(2+\rho)}{\beta+v(2+\rho)} \). Moreover, we propose to implement a tax by unit extracted. Of course, this proposition is not independent on the modeling framework and should be taken carefully. Smith (2013) remarks that there do not exist two countries which own similar resource tax regimes and that economists’ conclusions on those regimes strongly depend on the modeling framework.

The other possibility that we propose is to declare public property rights on the resource stock and then to manage the latter optimally. This policy is likely to be not implementable if property rights on the resource stock are currently privately defined because it necessitates to expropriate one generation. Thus, depending on the current law, countries may choose to implement one or another policy. For example in United States, the owner of a land also own its subsoil assets. In France, the mining code declares that the state own property rights on the subsoil assets. Thus the trust fund policy is easier to implement in France than in United States where the tax policy seems to be a better solution.

The model established that investing the tax revenues and give it to old generations or give it directly to young agents is equivalent. Nevertheless, this is less likely to be true in the real world where young agents are more likely to consume a share of this rent. A safer policy would thus consist in investing the resource rent and give it to agents while retired. This policy is notably followed by The Government Pension Fund Global of Norway. Oil fiscal revenues as well as dividend from the national oil company are placed in the fund which invest them in order to save welfare for future generations.\(^\text{18}\) The present work thus provides some theoretical insight that this policy is both ethical and fruitful since it preserves the future of unborn generations and promotes growth.

### 5 Conclusion and general discussion

In this paper, the model of Agnani et al. (2005) is reused and extended. i) We show that a flow pollution resulting from resource extraction may help the economy to reach a non-declining consumption path, because it decreases the rate of resource extraction. Thus, pollution diminishes pressure on natural resources. This analytical result is obtained due to the quite restrictive but convenient assumption that pollutants have short lifetime. With stock pollutants coming from the resource use, pollution would follow an inverse U-shaped with respect to time, increasing in a first step due to resource extraction and then decreasing when the extraction rate is very low, due to natural absorption. Thus, the answer of the economy to an increase in the elasticity of production with respect to

\(^{18}\)The funds declare on its homepage that “The Government Pension Fund Global is saving for future generations in Norway. One day the oil will run out, but the return on the fund will continue to benefit the Norwegian population.” [https://www.nbim.no/en/the-fund/](https://www.nbim.no/en/the-fund/)
pollution will be uncertain. Intuition suggests that our results stay true when natural absorption is sufficiently strong to prevent an increase of the stock of pollution. ii) Resource dependence reduces growth possibilities, because resource scarcity imposes the economy to save more and more on the resource stock. When resource dependence is high, saving on the resource is harder. iii) I analyze the ability of two resource allocation schemes in decentralizing the optimal equilibrium. If the resource is held by the first generation of agents, a resource tax redistributed to the young as a transfer generally allow to decentralize the Ramsey optimal equilibrium. If property rights on the resource stock are shared by all generations, an independent trust fund may be able to decentralize the optimal equilibrium. To summarize, the policy that should be implemented depends on the preexisting distribution of property rights in the economy.

Further research may include a model with stock pollution and a ceiling of emission in order to give more accurate policy recommendation. In first stages, pollution will increase due to high resource extraction. While extraction diminishes, the stock of pollution will begin to decrease in some point in time due to natural absorption. The aim of the work will thus be to keep pollution below the threshold. This project may be seen as an extension of Stern et al. (2006) where climate change is a byproduct of resource extraction and use only, while in the latter climate change is driven by population, production and consumption. If the present framework is conserved, the work should rely on simulation methods.

A.1 Proof of Proposition 1

The proof uses continuously definitions 1 and 2.

- A BGP implies a constant extraction rate. Thus equation (2) implies $\mu_m = 1 - q$.
- Equation (1) implies $\frac{n_{t+1}}{q_t} = \frac{m_{t+1}}{x_t} \frac{m_t}{m_e} \mu$. On the BGP we have $1 = \mu_x / \mu_m$ so $\mu_x = \mu_m$.
- Since emissions are modeled as a linear function of resource use (14) $\mu_e = \mu_x$.
- The market clearing condition $s_t = k_{t+1}$ implies $\mu_s = \mu_k$ on the BGP.
- By definition, the technological progress increases at a rate $a$ such that $\mu_a = 1 + a$.
- Evaluating the ratio of production (12) in $t+1$ and in $t$ on the BGP: $\mu_y = (1 + a)\mu_k^\alpha \mu_x^\beta \mu_e^{-\theta}$.
- From the firms FOC for resource use (16) we can compute $\mu_p$.

$$\mu_p = (1 + a)\mu_k^\beta \mu_x^{\alpha - 1} \mu_e^{-\theta} = \frac{\mu_y}{\mu_x}$$

- From the firms FOC for labor (15) $\mu_w = (1 + a)\mu_k^\alpha \mu_x^\alpha \mu_e^{-\theta} = \mu_y$
- The non-arbitrary condition between the two assets in the economy (9) tells us that $\mu_p = 1 + r_{t+1}$. Thus the interest rate should be constant on the BGP: $\mu_r = 1$
- Since the interest is constant, evaluating the ratio of the Euler equation in $t+1$ and in $t$ gives $\mu_c = \mu_d$
Taking the ratio of the firms FOC for capital in $t+1$ and in $t$ gives:

$$\frac{r_{t+1} - \delta}{r_t - \delta} = \frac{\alpha A_{t+1} k_{t+1}^{\alpha-1} x_{t+1} e_t^{\theta}}{\alpha A_t k_t^{\alpha-1} x_t e_t^{\theta}}$$

Because the interest rate is constant, evaluating this ratio on the BGP leads to:

$$1 = (1 + a) \mu_k^{\alpha-1} \mu_x^{v-\theta}$$

Thus:

$$\mu_k = (1 + a) \mu_k \mu_x^{v-\theta} = \mu_y$$

and:

$$\mu_k = (1 + a) \frac{1}{1-\alpha} \mu_x^{v-\theta}$$

Reintroducing the Euler equation (8) in the IBC (7) we obtain:

$$w_t = \frac{c_t (2 + \rho)}{1 + \rho}$$

On the BGP, taking the ratio of the last expression in $t+1$ and in $t$, we obtain $\mu_c = \mu_w$.

To summarize:

$$\mu_c = \mu_d = \mu_y = \mu_s = \mu_k = \mu_w$$

$$\mu_p = \mu_y / \mu_x$$

$$\mu_{a} = (1 + a)$$

$$\mu_r = 1$$

$$\mu_x = \mu_m = \mu_e$$

### A.2 Effect of an increase of $\theta$ on growth

The effect of an increase in $\theta$ is:

$$\frac{\partial \mu}{\partial \theta} = \mu \left[ -\frac{\ln(1-q)}{1-\alpha} - \frac{\partial q}{\partial \theta} \frac{v-\theta}{(1-q)(1-\alpha)} \right]$$

Restricting our analysis to the positive growth area, we can represent this derivative with respect to all possible values of $\theta$ and $q$ in the space $(\theta, q, \frac{\partial q}{\partial \theta})$

**Proposition 8.** When pollution hurts severely productivity, growth is higher.

This effect may seems puzzling, it is nevertheless quite intuitive. $\theta$ diminishes the net resource’s contribution to GDP. The BGP extraction rate is thus lower and that implies that less savings are needed to reach a given growth rate.
A.3 Keeping the constant returns to scale hypothesis

The effect of a change in the labor share on growth is determined by:

\[
\frac{\partial \mu}{\partial \beta} = -(1 + a)^{1/\alpha} (\nu - \theta)^{v - \theta} \left( \frac{1}{1 - \alpha} \right) \frac{\partial q}{\partial \beta}
\]

(61)

Since the sign of this expression appears ambiguous, we restrict the analysis on the positive growth area. We then calibrate the model and we show that all possible combinations \((\beta, q(\beta))\) on the space \(GAT < q < PGT\) lead to a positive sign for \(\frac{\partial \mu}{\partial \beta}\). The simulation is performed for a wide variety of parameter values and always lead to a positive sign for \(\frac{\partial \mu}{\partial \beta}\). Sensitivity analysis has been performed and the result is robust to variation in parameters values. The growth rate of the economy increases with the labor share. An increase in \(\nu\) compensated by a decrease in \(\beta\) leads to a lower growth rate.

The effect of a change in the man-made capital share on growth is determined by:

\[
\frac{\partial \mu}{\partial \alpha} = \frac{(1 + a)^{1/\alpha} (1 - q)^{v - \theta}}{(1 - \alpha)^2} \left[ ln(1 + a) + (\nu - \theta)ln(1 - q) - \frac{(v - \theta)(1 - \alpha)\frac{\partial q}{\partial \alpha}}{1 - q} \right]
\]

(62)

Since the sign of this expression appears ambiguous, we restrict the analysis on the positive growth area. We then calibrate the model and we show that all possible combinations \((\alpha, q(\alpha))\) on the space \(GAT < q < PGT\) leads to a positive sign for \(\frac{\partial \mu}{\partial \alpha}\). The simulation is performed for a wide variety of parameter values and always lead to a positive sign for \(\frac{\partial \mu}{\partial \alpha}\). Figure 5 represents an example for given parameter values. Sensitivity analysis has been performed and the result is robust to variation in parameters values. We have characterized possible derivative of \(\mu\) with respect to \(\alpha\), for all feasible \(q\). Nevertheless \(\tau\) is endogenously determined by \(\alpha\). Each possible level of \(\alpha\) thus lead to one level of
Figure 5: $\frac{\partial \mu}{\partial \alpha}$ for $a = 0.028$, $\delta = 0.027$, $\rho = 0.016$, $\theta = 0.01$, $v = 0.02$

$q$. This combination $(\alpha, q)$ belongs to the plotted surface so it makes no doubt that for reasonable parameter values, growth increases with the capital share. Since a increase in $v$ may imply a decrease in $\alpha$ to keep constant returns, the proposition 4 remains true if the increase in $v$ is compensated by a decrease in $\alpha$.

A.4 **Proof of Proposition 3**

- A BGP implies a constant extraction rate. Thus, equations (29) and (30) imply $\tilde{\mu}_x = \tilde{\mu}_m = 1 - \tilde{q}$.
- Since emissions are modeled as a linear function of extracted resources $\tilde{\mu}_e = \tilde{\mu}_x$.
- By definition, $\mu_a = 1 + a$.
- The ratio of the intergenerational optimality condition (33) evaluated in $t + 1$ and in $t$ gives on the BGP $\tilde{\mu}_e = \tilde{\mu}_d$.
- The BGP ratio of the production function (12) in $t + 1$ and $t$ implies $\tilde{\mu}_y = (1 + a)\tilde{\mu}_k \tilde{\mu}_x^{\nu - \theta}$.
- The inter-temporal resource allocation optimality condition (35) evaluated on the BGP gives:

$$
(1 + a)\tilde{\mu}_k^{\alpha - 1} \tilde{\mu}_x^{\nu - \theta - 1} - 1 + \delta = \alpha A_{t+1}k_{t+1}^{\alpha - 1} x_{t+1}^{\nu - \theta} e_{t+1}^{-\theta}
$$

Taking the ratio of the last expression in $t + 1$ and in $t$ and evaluating on the BGP, we obtain:

$$
1 = (1 + a)\tilde{\mu}_k^{\alpha - 1} \tilde{\mu}_x^{\nu - \theta}
$$

Thus:

$$
\tilde{\mu}_k = (1 + a)\tilde{\mu}_k^{\alpha - \nu} \tilde{\mu}_x^{\nu - \theta} = \tilde{\mu}_y
$$
and:

$$\hat{\mu}_k = (1 + a)^{\frac{1}{1-\alpha}} (1 - \tilde{q})^{\frac{v^\theta}{1-\alpha}}$$

### A.5 Proof of proposition 6

Equations (1), (2), (4) (10) (11) (12) (14) (15) and (17) are not modified by the tax. Thus, we can conclude from appendix A.1 that

$$\mu_x = \mu_m = \mu_e = 1 - q, \mu_a = 1 + a, \mu_w = \mu_x = \mu_y = \mu_k$$

and $\mu_c = 1$. The introduction of the tax and the transfer don’t modify the Euler equation or the arbitrary condition between capital and resource in household investment decisions. Thus $\mu_p = 1 + r_{t+1}$ and $\mu_c = \mu_d$.

- If the resource price and the tax level increase at the same rate, taking the ratio of equation (45) in $t+1$ and in $t$ gives

$$\frac{\mu_p (p_t + \tau_t)}{p_t + \tau_t} = (1 + a) \mu_k \mu_x^{v^\theta-1}$$

Thus, we can conclude that $\mu_p = \mu_y / \mu_x$.

- The IBC with the transfer writes:

$$w_t + g_t = c_t + \frac{d_{t+1}}{1 + r_{t+1}} - \frac{p_{t+1} m_t}{1 + r_{t+1}} + p_t m_t$$

Substituting equations (8) (15), (46) and taking into account that the resource price increase at the interest rate, we can write:

$$\beta A k_t^\alpha x_t^v e^{-\theta} + \tau_t x_t = \frac{c_t (2 + \rho)}{1 + \rho}$$

Taking the ratio of the last expression and evaluating on the BGP, we obtain:

$$\mu_c = \frac{\mu_k / \beta A k_t^\alpha x_t^v e^{-\theta} + \mu_p \mu_x \tau_t x_t}{\beta A k_t^\alpha x_t^v e^{-\theta} + \tau_t x_t}$$

Since $\mu_p \mu_x = \mu_k$, we have $\mu_c = \mu_k$.

### A.6 Proof of proposition 7

- The resource management fund follows a rule and should extract a quantity $\tilde{q}$ at each period. Thus, we have $\mu_x = \mu_e = \mu_m = 1 - q$.

- As in appendix A.1 , $\mu_a = 1 + a, \mu_y = \mu_w = (1 + a) \mu_k \mu_x^{v^\theta}$.  

- The FOCs of the household problem are not affected by the tax. Thus, the resource price $\hat{p}$ should grow at the interest rate, and the growth rate of young and old consumption is the same. Thus we have $\mu_{\hat{p}} = (1 + r_{t+1})$ which implies that $\mu_r = 1$ and $\mu_c = \mu_d$. 

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• The firm FOC for resource use is now $\tilde{p}_t = vA_t k_t^\alpha \tilde{x}_t^{1-\alpha} e_t^{-\theta}$. As in appendix A.1, we can deduce that $\mu_{\tilde{p}} \mu_{\tilde{x}} = \mu_y$.

• Equation (58) implies $\mu_{\tilde{q}} = \mu_{\tilde{p}} \mu_{\tilde{x}} = \mu_y$.

• The firm FOC for capital leads to $\mu_k = \mu_y = (1 + a)^{\frac{1}{1-\alpha}} (1 - \tilde{q})^{\frac{\tilde{x}}{1-\alpha}}$.

• The new market clearing condition (59) thus implies $\mu_s = \mu_k$.

• Since savings and consumption grow at the same rate, consumption of young should also grow at this rate on the BGP. Thus, equation (53) implies $\mu_c = \mu_y$. 

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References


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