Does public debt secure social peace? A diversionary theory of public debt management

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Abstract

This paper develops a new analysis of the strategic use of public debt. Contrary to the usual view that politicians can use public debt to tie the hands of their successors, we show that an incumbent government can take advantage of having tied his own hands before the election by the means of public debt. By so doing, he reduces the base for future social conflicts, beneficiates from social peace during his term and possibly enhances his chances to be reelected. In addition, in the case with foreign or external public debt, the incumbent can strategically divert future social conflicts toward a common enemy (the foreign creditors). Thus, by increasing public debt before the election, the incumbent can strengthen social cohesion during the mandate, both by reducing the base of internal conflicts and by diverting citizens from internal toward external rent-seeking activities.

Keywords: Public debt, Election, Conflict, Rent-seeking.

1. Introduction

The long-lasting tradition of Political Budget Cycle suggest that election cycles in public spending and taxes can result from the strategic behavior of incumbent politicians who seek reelection. Opportunistic governments will undertake pre-electoral expansionary fiscal policies because short-sighted electors appreciate low taxes and high expenditures (Nordhaus, 1975), or, in modern probabilistic voting models (see, e.g. Lindbeck and Weibull, 1987; Persson and Tabellini, 2000) because rational forward-looking electors face informational problems. In this literature, voters are imperfectly informed about candidates' abilities or about the environment, and a fiscal expansion that boosts economic activity may signal the incumbent's competencies. The latter is then tempted to stimulate activity to be viewed as more competent (see Rogoff and Sibert, 1988).

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A major shortcoming of this approach is that pre-electoral public deficits gives rise to increases in public debt, and it is difficult to believe that such increases signal politicians' competence. On the contrary, forward-looking electors are aware of the intertemporal government budget constraint and know that post-electoral adjustments will follow pre-election fiscal expansions. By observing large public debt issuance before the election, they may consider, in contrast, that the incumbent is relatively incompetent in public finance management and should not reward politicians who engage electoral fiscal manipulations. If voters are "fiscally conservative", for example, pre-electoral increases of deficits can damage rather than improve the incumbent's reputation (Peltzman, 1992).[2]

In this paper, we develop a new explanation for why an incumbent increases public debt before the election: by generating a large debt burden, he can prevent social conflicts about distributional issues during his future mandate. There is a considerable number of examples showing that favorable economic conditions, generating windfall gains for public finance, are likely to generate claims for increasing public spending or tax-cuts, to the benefits of some social groups or lobbies defending particular interests. As the social rent increases, protests and disputes appear everywhere. By issuing public debt before the election, the incumbent can credibly commit to future austerity measures that will discipline his constituents during the term, thus escaping such rent-seeking activities. In other words, pre-electoral increases in public debt will secure future social peace, because citizens know that the elected government will not be able to handle all claims.[3]

Of course, post-electoral austerity measures can be a source of dissatisfaction that undermines the popularity of the elected government. But in the case of external public debt (e.g., federal or foreign debt), a strategic government may attempt to establish internal cohesion by creating a common enemy (the creditors in the federation or abroad). By diverting the discontent caused by austerity policies toward outside scapegoats, he can escape from unpopularity. Public debt management then can serve

[2]In the United States, e.g., Peltzman (1992) shows that electors punish politicians who let public spending increase. The explanation is related to the progressive fiscal systems: in average, voters are wealthier than nonvoters, and penalize spender incumbents because they will pay the price of future fiscal adjustments. Such a fiscal "conservatism" of electors is also found by Bertola and Drazen (1993) and Brender and Drazen (2008), who show that, using a sample of 74 countries from 1960 to 2003, preelection deficits reduce the chances of reelection of the incumbent. Garmann (2017) shows that, if voters are highly fiscally conservative, incumbents even decrease spending before elections

[3]The intuition that debt can be used to manage social conflicts is not new. Capital structure theories (see, e.g. Grossman and Hart, 1982; Jensen, 1986; Stulz, 1990, among others) have emphasized the role played by corporate debt in reducing agency conflicts between managers and shareholders. Specifically, by increasing the probability of failure, private debt exerts a threat that can discipline the workers by forcing them to accept low wages and working conditions. In this paper, we show that the same type of argument can apply to public debt, which can be used as a disciplining device in order to limit rent-seeking behavior during the term.

to manage future internal and external social conflicts for electoral purposes.

To develop these ideas, we build an original probabilistic voting model with rent-seeking activities. We consider a two-period game between N districts (indexed by i) belonging to a federal union (in our stylized setup, a district can describe any local authority –such as a municipality, a federated state or a particular country– belonging to a wider group –such as, respectively, a region a federal state or a supra-national union). In each district, the local government provides a flow of public goods that can be financed using federal debt, i.e. debt issued by the federal authority in the first period (from a particular district point of view, public debt is then assimilable to foreign debt). At the end of the first period, an election is held in district i, which opposes the incumbent to a challenger. In the second period, the newly elected government will have to repay the debt burden, which reduces available resources to finance the public good. However, there is a conflict between districts about debt repayment (a particular district can formally or informally default, negotiate to obtain debt alleviations, etc, in which case the other districts will have to pay the bill).

Citizens are voters, workers, and rent-seekers. There are two type of conflicts: an internal conflict among citizens of district i (to capture the highest share of the local public good) and an external conflict that opposes district i as a whole to other districts (to escape debt repayment). In symmetric equilibrium, these two conflicts generate unproductive rent-seeking efforts that reduce the amount of taxable income (hence the level of public good).

Our main result is that, through public debt, a strategic incumbent can manage internal and external conflicts for his benefit. By increasing public debt today, the incumbent generates a debt burden tomorrow that reduces internal conflict (the size of the feasible public good decreases) but strengthens external conflict (citizens provide more external rent-seeking effort to escape from the repayment of this burden). Depending on his comparative advantages over the challenger in managing internal and external conflicts, the incumbent will be able to choose the level of public debt that maximizes his objective (the inter-temporal expected value of power-rents he beneficiates from holding office).

Our model is at the interface of two strands of literature.

On the one hand, our results can be compared with the so-called diversionary theory of war, whereby *"it appears to be a general law that human groups react to external pressure by increased internal coherence"* (Dahrendorf 1964, cited by Levine and Campbell 1972, p.31). According to Levy (1989), a diversionary war occurs when a leader begins a foreign conflict to divert attention from an ongoing domestic issue. In our model, by increasing debt, the incumbent creates a common enemy that serves to strengthen internal cohesion and limit social conflicts about the repartition of public goods.[4]

The diversionary mechanism has been introduced in a formal model of rent-seeking by Münster and Staal (2011). Their innovative paper exposes one of the first economic model with simultaneous conflicts at different levels (see also Katz et al., 1990). However, Münster and Staal (2011) use a static setup and do not study electoral competition, while our model is dynamic and mixes rent-seeking and probabilistic voting approaches. Moreover, in their model, there is conflict either between or within groups, but not both at the same time (in the basic case where all players decide simultaneously and independently how to allocate their resources); while both conflicts can appear simultaneously in our setup.[5] This is an important point because public debt is an instrument in the hands of the incumbent to orient internal and external conflicts according to his preferences. In other words, public debt allows the incumbent to create diversion.

On the other hand, our model belongs to the abundant literature on the strategic use of public debt. Building on the pioneer works of Alesina and Tabellini (1990) and Persson and Svensson (1989), a number of works have developed and tested the idea that, if a government anticipates the possibility of defeat, he will try to use the debt strategically in order to constraint future policies of his opponents.[6] By bequeathing a high debt burden to his possible successor, the incumbent can force his newly elected challenger to pay the bill and prevent him from carrying out his own policies.[7] In all this literature, public debt serves to tie the hands of possible successors. Our model develops the complementary point of view that the incumbent himself can beneficiate of having tied his own hands. By so doing, he can credibly commit to not meet the claims of citizens during his term, and discourages rent-seeking activities. Therefore, in our model, public debt will be used strategically even in the absence of partisan preferences (as Alesina and Tabellini, 1990), or disagreement between politicians about the desired level or composition of public goods (as Persson and Svensson, 1989). Effectively, it is in the incumbent's interest to constraint its own room for manoeuvre rather than his

[4]This group cohesion effect has been the subject of a large literature in the social sciences, in particular in sociology, psychology, and anthropology. To describe this feature, Sumner (1906) first introduced the term *ethnocentrism*, and developed the idea that *"the exigencies of war with outsiders are what make peace inside. (...) These exigencies also make government and law in the in-group, in order to prevent quarrels and enforce discipline"*.

[5]Münster and Staal (2011) use exponential contest functions, with constant marginal returns of conflicts. Thus, if marginal returns to intra-group rent-seeking are higher than those to inter-group rent-seeking, there is only internal conflict, and vice versa. In contrast, we use a more general gain function with varying marginal returns, such that an interior solution with both conflicts can emerge in equilibrium.

[6]References

[7]In the same vein, Aghion and Bolton (1990) present an interesting model in which the incumbent has an incentive to excessive accumulation of public debt because he can more credibly commit not to default than his opponent.

challenger's one.

Furthermore, usual works on the strategic use of the public debt rest on exogenous probability of reelection.[8] Yet, even if his chances are very low, the incumbent will probably not undertake actions that reduce these chances, even if they strongly hinders the acts of his opponents.[9] In this way, a crucial feature of our model is that the inducement to public indebtedness is not related to the probability to lose the election. On the contrary, the incumbent strategically manage public debt to maximize the expected power-rents he will receive, which positively depend on the probability of being renewed.

The rest of the paper is organized as follows. Section 2 presents the baselines of the model, Section 3 describes the political competition, while Sections 4 and 5 computes the second period and the first period equilibrium, respectively. Finally, Section 6 presents our main findings about public debt and social cohesion, and Section 7 concludes.

2. The Model

We consider a federal union including N districts, indexed by $i \in \{1, \cdots, N\}$, each populated by $M_i > 1$ citizens. In district i, there is an incumbent (denoted by I) who seeks reelection and a challenger (denoted by O). Building on a standard probabilistic voting setup, we model a two-period game (before and after the election) between the incumbent and voters.

2.1. The multi-district setup

In each period $t \in \{1, 2\}$, the incumbent provides a public good ($g_{t,i}$) and levies taxes from output ($\tau_i Y_{t,i}$, with τ_i the constant flag tax rate).

In the first period, the deficit must be financed by borrowing from the federal union $D_{1,i} = g_{1,i} - \tau_i Y_{1,i}$. At the end of period 1, an election takes place in district i and the incumbent is renewed or his challenger takes office.

In the second (and last) period, the newly elected politician has to repay public debt and interests, since he cannot borrow in the last period ($D_{2,i} = 0$). However, the sharing of the debt burden is subject to conflicts within the federation, since each district attempts to escape reimbursement.[10] In this line, district i will have to repay

[8] In Alesina and Tabellini (1990) the incumbent has a chance to get reelected, albeit its exact chances are uncertain; in Persson and Svensson (1989) the current government is certain that it is unable to hold onto power.

[9] For example, Hodler (2011) finds that the incumbent never manipulates his opponent public spending, if he can ensure his own reelection.

[10] After the election, the debt burden can be renegotiated inside the federation. Some districts can obtain debt alleviations, bailouts, or specific assistance from the federal budget, depending on their bargaining power. Since federal budget must be balanced, others districts will have to pay for these measures, hence producing inter-district conflicts about the repayment of public debt.

a part ψ_i of the debt burden $(1+r)D_{1,i}$ ($r \geq 0$ is the constant interest rate). This part will be endogenously determined in the following sections. Thus, the government's budget constraint is, in period 2

$$D_{2,i} = g_{2,i} + \psi_i(1+r)D_{1,i} - \tau_i Y_{2,i} = 0. \tag{1}$$

If $\psi_i < 1$, district i citizens benefit from debt alleviation (or partial default), while they must pay for their neighbors in the opposite case ($\psi_i > 1$). In equilibrium, however, all public debt issued by the federation has to be repaid; hence, $\sum_{i=1}^{N} \psi_i D_{1,i} = \sum_{i=1}^{N} D_{1,i}$, and symmetric equilibrium will require $\psi_i = 1, \forall i$.

The inter-temporal utility of citizen n, $n \in \{1, \cdots, M_i\}$, denoted by $U_{i,n}$, is assumed to be linear, namely

$$U_{i,n} = g_{1,i,n} + \beta g_{2,i,n}, \tag{2}$$

where $\beta \in (0,1)$ is the discount factor and $g_{t,i,n}$ denotes the amount of public good received by citizen n belonging to district i in period t, with $g_{t,i} = \sum_{n=1}^{M_i} g_{t,i,n}$.

Owing to imperfect property rights, the public good is rival but non-excludable, and the amount $g_{t,i,n}$ that citizen n can appropriate is subject to conflicts. Thus, citizens are induced to provide rent-seeking efforts to capture the biggest share of total public good $g_{t,i}$ (yet to be derived) that defines the rent available in district i. This rent is, in the two periods,

$$g_{1,i} = \tau_i Y_{1,i} + D_{1,i}, \tag{3}$$

$$g_{2,i} = \tau_i Y_{2,i} - \psi_i(1+r)D_{1,i}. \tag{4}$$

Additionally, as we discussed above, property rights on the federal debt burden are imperfectly defined. Thus, by engaging a collective post-electoral fighting effort against other districts, district i citizens can expect reducing the debt burden they will face, hence producing an extra incentive to rent-seeking activities.

Consequently, two kinds of conflicts can be distinguished: *internal conflicts* (among citizens belonging to district i), and *external conflicts* (between citizens of district i and citizens of other districts), which are respectively detailed in the following subsections.

2.2. Internal Conflicts

In district i, citizen n provides a rent-seeking effort $x_{t,i,n}$ aiming at catching a part of the public good $g_{t,i}$. Let $j \in \{I, O\}$ denotes the politician who holds power, and $\alpha_{i,n}^{j}$ defines the share of captured public good (that depends on the type of office-holder, as we will show), namely

$$g_{t,i,n} = \alpha_{i,n}^{j}(\mathbf{x}_{t,i})g_{t,i}. \tag{5}$$

where $\mathbf{x}_{t,i} := (x_{t,i,1}, \cdots, x_{t,i,M_i})$ is the vector of citizens' effort. We assume that $\alpha^j_{i,n}$ is a member of the family of functions $\mathscr{F}$, defined as follows.

Definition 1. The class function $\alpha^j_{i,n} \in \mathscr{F}$ if $\alpha^j_{i,n}$ respects

(i) $\alpha^j_{i,n}$ is a twice differentiable mapping from $[0,1]^{M_i} \mapsto [0,1]$.
(ii) $\alpha^j_{i,n}$ in increasing convex in $x_{t,i,n}$.
(iii) $\alpha^j_{i,n} = 1/M_i$ in symmetric equilibrium.
(iv) $a^j_i := E(\alpha^j_{i,n})$ in symmetric equilibrium. For notational convenience, we define the elasticity of a CES function f by $E(f) = xf'(x)/f(x)$.

Assumption (i) states that the rent-seeking technology is smooth. By assumption (ii), citizens expect capturing more public goods by increasing effort, with decreasing returns. Assumption (iii) ensures that, in symmetric equilibrium, these efforts will be ineffective (citizens can not expect to get anything other than average public good in equilibrium).[11] From assumption (iv), in symmetric equilibrium, the rent-seeking technology has a constant elasticity $a^j_i \geq 0$.[12]

As usual in the literature (see, e.g. Hirshleifer, 1989, 1991), a^j_i measures the *decisiveness* of the internal contest. Indeed, if a^j_i is small, rent-seeking activities low impact the expected allocation of rent, whereas for high value of a^j_i, small differences in rent-seeking efforts are expected to be decisive. The higher a^j_i, the higher the expected efficiency of rent-seeking individual effort. In this respect, the degree of contest of internal conflict is inversely related to the decisiveness parameter a^j_i.

Following Hirshleifer (2008) or Münster and Staal (2011), a^j_i is determined both by institutional and political factors such as, e.g., police technology or protection of property rights. Therefore, a^j_i depends on the type of politician (j) who holds office since the decisiveness of contest depends on political characteristics. Indeed, politicians can differ in competencies to secure property rights inside their jurisdiction (Hirshleifer, 2008, p.21). Besides, even without gap in politicians competencies, the mere fact of being reelected (for the incumbent) or newly acceding to power (for the challenger) may change citizens' incentive to fight and make the decisiveness parameter dependent upon the type of politician, as we will detail below.[13]

[11] In symmetric equilibrium, rent-seeking efforts are unproductive. This does not prevent conflicts from being harmful, because these rent-seeking efforts will undermine output, especially since a^j_i is high.

[12] The standard "power" form of the contest success function $\alpha^j_i(\mathbf{x}_{t,i}) = x^{d^j}_{t,i,n} / \sum_{s=1}^n x^{d^j}_{t,i,s}$ belongs to the class function $\mathscr{F}$. Indeed, conditions (i) (provided that $d^j < 1$), (ii) and (iii) are trivially satisfied. Condition (iv) immediately follows by fixing $a^j_i = d^j(M_i - 1)/M_i$.

[13] For example, if the challenger is elected, he may benefit from a "honeymoon effect", since citizens are less induced to fight if they face a virgin politician (see, e.g. Mueller, 1973; Kernell, 1978). Following the defeat of the incumbent, the in-place power networks are destroyed, the office staff is renewed, old barriers to entry fall, etc, in such a way that the degree of contestability of internal conflict $(1/a^j)$ increases, hence $a^O \leq a^I$.

2.3. External Conflict

In the second period, citizens can provide an additional fighting effort $y_{2,i,n}$ aiming at escaping from debt repayment. The mechanism underlying external conflicts differs from internal rent-seeking, because the expected gain retrieved from external conflict depends on the collective action of district i citizens and government, and involves a kind of coordination inside the district. Indeed, the government can be induced to organize the collective external fighting effort (e.g., by allowing or instigating growing public protests against the burden of the federal debt) to consolidate his popularity. As we will show below, coordinating the struggle against a common foreign enemy is a mean for an opportunistic government to divert citizens from internal social conflicts to his benefit.[14]

Let us define the collective effort of district i by $\hat{y}_{2,i} := \sum_{n=1}^{M_i} y_{2,i,n}$, and the government's coordination effort by e_i. After the election, citizens n belonging to district i must repay a debt burden $\psi_{i,n}^j(e_i, \hat{\mathbf{y}}_2)(1+r)D_{1,i}$, where $\hat{\mathbf{y}}_2 := (\hat{y}_{2,1}, \cdots, \hat{y}_{2,N})$ is the vector of efforts. Similarly to internal conflict, we assume that $\psi_{i,n}^j$ is a member of the family of functions $\mathscr{G}$, defined as follows.

Definition 2. The class function $\psi_{i,n}^j \in \mathscr{G}$ if $\psi_{i,n}^j$ respects

(i) $\psi_{i,n}^j$ is a twice differentiable mapping from $\mathbb{R}^+ \times [0,N]^N \mapsto [0,N]$.
(ii) $\psi_{i,n}^j$ is decreasing convex in e_i and $y_{2,i,n}$.
(iii) $\psi_{i,n}^j = 1$ in symmetric equilibrium.
(iv) $\varepsilon_i^j := E(\psi_{i,n}^j(\cdot, \mathbf{y}_2))$, and $b_i^j := E(\psi_{i,n}^j(e_i, \cdot))$ in symmetric equilibrium.

Assumption (i) states that the rent-seeking technology is smooth, and assumption (ii) means that citizens and government expect reducing the debt burden by increasing effort, with decreasing returns. However, these efforts will be ineffective in symmetric equilibrium (assumption iii).

Similar to internal conflicts, from assumption (iv), the rent-seeking technology has a constant elasticity $b_i^j \geq 0$ relative to citizens' external fighting effort (in symmetric equilibrium). This elasticity describes the *decisiveness* of the external contest. However, b_i^j differs from a_i^j, since the contest between districts within the federal union is subject to different rules that the contest between individuals within a particular district. Besides, the rent-seeking technology is also a function of government's effort in symmetric equilibrium. This elasticity $\varepsilon_i^j \geq 0$ measures the efficiency of coordinating individual fighting behaviours.

[14]This finding echoes the traditional diversionary hypothesis (see Wright, 1965; Mansfield and Snyder, 2004), suggesting that leaders might trig conflict with another groups to deflect attention from problems at home. Wright stated that *"foreign war as a remedy for internal tension, revolution, or insurrection has been an accepted principle of government"* (Wright, 1965, p.140).

In symmetric equilibrium, external conflicts will be unproductive, but detrimental to output and welfare (as internal conflicts), because each district will repay the same amount $(1+r)D_1/N$ (namely, $\psi_i^j = 1$) of debt. This inefficiency is the higher, the higher elasticities b_i^j and ε_i^j. Similarly to the internal decisiveness parameter (a_i^j), b_i^j and ε_i^j depend on who holds power. Indeed, elected politicians may differ on their abilities to coordinate individual fighting behaviour (hence, $\varepsilon_i^I \neq \varepsilon_i^O$), or to obtain subsides from the federal budget or put the renegotiation of public debt in the federal agenda (hence, $b_i^I \neq b_i^O$).[15]

2.4. Citizens' utility

Taking into account rent-seeking activities, citizen's utility (2) becomes

$$U_i = \alpha_{i,n}^I(\mathbf{x}_{1,i})\left[\tau_i Y_{1,i} + D_{1,i}\right] + \beta\alpha_{i,n}^j(\mathbf{x}_{2,i})\left[\tau_i Y_{2,i} - \psi_{i,n}^j(e_i, \hat{\mathbf{y}}_2)(1+r)D_{1,i}\right]. \quad (6)$$

Beyond engaging into conflicts, citizens provide productive efforts $h_{t,i,n}$ that yield output $Y_{t,i} := \sum_{n=1}^{M_i} h_{t,i,n}$. Thus, their budget constraint is, in the first period[16]

$$x_{1,i,n} + h_{1,i,n} = 1,$$

and, in the second period

$$x_{2,i,n} + y_{2,i,n} + h_{2,i,n} = 1.$$

Hence,

$$Y_{1,i} = \sum_{n=1}^{M_i}(1 - x_{1,i,n}), \text{ and } Y_{2,i} = \sum_{n=1}^{M_i}(1 - x_{2,i,n} - y_{2,i,n}). \quad (7)$$

Therefore, by diverting fighting effort from productive destinations, rent-seeking activities are costly. Finally, by (6) and (7), if politician j is elected, citizen n's utility is

$$U_i^j(x_{1,i,n}, x_{2,i,n}, y_{2,i,n}) = \alpha_{i,n}^j(\mathbf{x}_{1,i})\left[\tau_i \sum_{n=1}^{M_i}(1 - x_{1,i,n}) + D_{1,i}\right]$$
$$+ \beta\alpha_{i,n}^j(\mathbf{x}_{2,i})\left[\tau_i \sum_{n=1}^{M_i}(1 - x_{2,i,n} - y_{2,i,n}) - \psi_{i,n}^j(e_i, \hat{\mathbf{y}}_2)(1+r)D_{1,i}\right], \quad (8)$$

[15] For example, it may be uncomfortable for the incumbent to repudiate a debt that he has himself subscribed. Thus, the challenged (if elected) will be able to coordinate more efficiently the external fight, reducing the degree of contestability; hence $b^O \geq b^I$. As an extreme illustration, the challenger, if elected, can denounce the public debt incurred by his predecessor, using the argument that this debt is "odious" (Kremer and Jayachandran, 2002).

[16] For simplicity, citizens simultaneously and independently decide how to allocate their resources to productive and unproductive activities.

under the non-negativity constraint $g_{2,i} \geq 0$.

From (8), the level of rent in the second period negatively depends on the efficiency of external fighting effort ($\psi^j_{i,n}$), which therefore reduces the inducement to internal fighting (trough the term $\alpha^j_{i,n}$). In equilibrium, this will produce a negative relationship between both types of conflict.

3. Political Competition

Let us now describe the electoral side of the model. Citizens have preferences over ideology and other characteristics of politicians. Thus, in district i, citizen n feels an additional expected utility ($\theta_{i,n}+\xi$) if the politician I takes power. To avoid generating a deterministic election outcome, this term includes two random components: $\theta_{i,n}$ is idiosyncratic and ξ is common to all voters. Following the probabilistic voting models of Lindbeck and Weibull (1987) and Persson and Tabellini (2000), $\theta_{i,n}$ are independent random variables, constant over time, and uniformly distributed on $[-1/2s_i, 1/2s_i]$, with density $s_i > 0$,[17] and ξ reflects the (relative) general popularity of politician I, which is uniformly distributed on $[1/2h, 1/2h]$, with density $h > 0$. Then, if politician j, is elected, citizen n's expected utility becomes

$$\mathbb{E}U^j_{i,n}(x_{1,i,n}, x_{2,i,n}, y_{2,i,n}) := \begin{cases} U^I_{i,n}(x_{1,i,n}, x_{2,i,n}, y_{2,i,n}) + \theta_{i,n} + \xi & \text{if } j = I, \\ U^O_{i,n}(x_{1,i,n}, x_{2,i,n}, y_{2,i,n}) & \text{if } j = O. \end{cases} \tag{9}$$

3.1. Politician's objective

The exercise of power generates psychological gains and losses for the office-holder. First, there is an (exogenous) ego-rent $R > 0$ perceived at each period. Second, after the election, there are additional (endogenous) net gains, because the elected politician beneficiates from extra ego-rents, on the form of "popularity" gains, if the district succeeds in reducing the debt burden. His second period rent then increases with the gain $1 - \psi^j_i(\cdot)$. These extra rents motivate the office-holder to engage costly efforts to wage war on debt burden repayment.[18] Hence, the total psychological rents R_t that the the office-holder perceives are, in the first period[19]

$$R_1 = R,$$

[17] A positive (resp. negative) value of $\theta_{i,n}$ implies that citizen n has a bias in favor of politician I (resp. politician O), whereas citizens with $\theta_{i,n} = 0$ are ideologically neutral. Besides, s_i is a measure the citizens' responsiveness to rent-seeking activities. As s_i increases, citizens care more about rent-captation than ideology, see Eq. (9).

[18] Many empirical works support this specification. For example, the well-known *"Rally 'round the flag syndrome"* suggests that external conflicts often boost popularity of leaders (Grieco et al., 2014, p.199), such that US Presidents who enjoyed extra short-run popular support during the outbreak of international crisis or wars (as, e.g. Lian, 1993; Hetherington and Nelson, 2003, among others).

[19] The power-rents are assumed to be purely psychological. This avoids introducing an unnecessary additional conflict between government and district i citizens.

and in the second period,

$$R_2 = R + \lambda(1 - \psi_{i,n}^{j}(\cdot))(1+r)D_{1,i} - c(e_i),$$

where $c(\cdot)$ is the cost of coordination, which is assumed to be an increasing, non-negative, and convex function. $\lambda \geq 0$ reflects how citizens reward the office-holder when the district succeeds in external fighting ($\psi_{i,n}^{j} < 1$). In the opposite case ($\psi_{i,n}^{j} > 1$), the government suffers from a popularity loss. A politician in search for popularity will then undertake efforts to coordinate the external fight, even if, in symmetric equilibrium, this fight will be unproductive ($\psi_{i,n}^{j} = 1$).

The incumbent's objective is to maximize the inter-temporal flow of expected power-rents, namely

$$\mathbb{E}[V] = R_1 + \beta\mu R_2, \tag{10}$$

where $\mathbb{E}$ denotes the expectation operator (with expectations taken over the election outcome) and μ is the (endogenous) reelection probability.

The office-holder has an unique strategic variable in each period. In the first period, the incumbent sets the amount of public debt $D_{1,i}$. In the second period, the elected politician determines his coordination effort e_i, conditionally to the amount of debt initially chosen. Hence, at the time the incumbent decides $D_{1,i}$, he must take into account the effect of this action on the optimal coordination effort he will have to undertake, if reelected. The following subsection details the timing of the model.

3.2. Timing of the game

The timing of events is as follows.

1. **Period 1.** Politician I initially in power in the district i chooses the amount of debt $D_{1,i}$. At this stage, all agents know distributions of $\theta_{i,n}$ and ξ, but not their realized values.
2. Citizen n provides a rent-seeking effort $x_{1,i,n}$; the actual value of ξ is realized, all uncertainty is resolved, and election is held.
3. **Period 2.** The politician j, $j \in \{I, O\}$, who wins the election takes power.
4. The newly elected politician implements the coordination effort e_i. Citizen n makes rent-seeking efforts $x_{2,i,n}$, $y_{2,i,n}$, and the game ends.

As usual, we look for the pure Subgame Perfect Equilibrium, and we solve the model by backwards induction. The two stages (after and before the election) are respectively depicted in the following sections.

4. Second period equilibrium

In the second period, the office-holder computes the optimal coordination effort, and citizens choose their optimal fighting efforts, conditionally to public debt.

4.1. The government's optimal coordination effort

The newly elected politician $j \in \{I, O\}$ in district i, chooses the coordination effort e_i that maximizes his second-period power-rent, namely, using a linear cost function $c(e_i) = e_i$.

$$R_2 = R + \lambda(1 - \psi^j_{i,n}(\cdot))(1 + r)D_{1,i} - e_i. \tag{11}$$

Hence the following first-order condition

$$-\lambda(1 + r)D_{1,i}\partial_1\psi^j_i(e_i, \hat{\mathbf{y}}_2) = 1. \tag{12}$$

The optimal effort is such that the marginal cost just equals the marginal gain. By producing one additional unit of effort, the office-holder takes benefit from an extra psychological gain related to the marginal increase of the collective surplus (the LHS of Eq. (12), which is positive because $\partial_1\psi^j_{i,n} \leq 0$), but suffers one unit marginal cost, through the simple linear cost function.

Focusing on symmetric equilibrium (i.e. $\hat{y}_{2,i} =: \hat{y}_2$, for any i), from Definition 2, the optimal coordination effort is[20]

$$e^{*j}_i(D_{1,i}) = \lambda(1 + r)\varepsilon^j_i D_{1,i}. \tag{13}$$

The optimal effort positively depends on public debt $D_{1,i}$, with a sensitivity related to the elasticity ε^j_i. Indeed, the higher the public debt, the higher the potential gain retrieved from external fighting and the marginal gain of coordination efforts. Besides, as the efficiency of politician's effort (ε^j_i) increases, district i is more likely to escape debt repayment, increasing government's inducement to coordinate the fight.

From (11), the second-period power-rent becomes in symmetric equilibrium ($\psi^j_{i,n} = 1$),

$$R^*_2 = R - e^{*j}_i(D_{1,i}) = R - \lambda(1 + r)\varepsilon^j_i D_{1,i} =: R^{*j}_2(D_{1,i}).$$

In equilibrium, the office-holder's payoff negatively depends on public debt, given unproductive coordination efforts he undertakes to avoid the debt burden repayment.

4.2. Equilibrium internal and external rent-seeking activities

Given the government's effort e^{*j}_i, citizens optimally determine their rent-seeking activities $x_{2,i,n}$ and $y_{2,i,n}$. If politician j holds office, citizen n's programme is then, by

[20] The second order condition is trivially verified, since $\psi^j_{i,n}(\cdot, \hat{\mathbf{y}}_2)$ is a convex mapping.

(2),

$$\max_{(x_{2,i,n},y_{2,i,n})\in\mathscr{C}} \mathbb{E}U^j_{i,n}(x_{1,i,n},x_{2,i,n},y_{2,i,n}),$$
$$\text{s.t. } \tau_i\sum_{s=1}^{M_i}(1-x_{2,i,s}-y_{2,i,s}) \geq \psi^j_{i,n}(e_i^{*j},\hat{\mathbf{y}}_2)(1+r)D_{1,i}, \tag{14}$$

where $\mathscr{C} := \{(x,y)\in[0,1]^2;\ x+y\leq 1\}$.

Using (8), the first-order condition on $x_{2,i,n}$ is (the strict complementarity slackness condition holds at equilibrium, see Appendix A)

$$\frac{\partial\alpha^j_{i,n}(\mathbf{x}_{2,i})}{\partial x_{2,i,n}}\left[\tau_i\sum_{s=1}^{M_i}(1-x_{2,i,s}-y_{2,i,s})-\psi^j_{i,n}(e_i^{*j},\hat{\mathbf{y}}_2)(1+r)D_{1,i}\right]=\alpha^j_{i,n}(\mathbf{x}_{2,i})\tau_i, \tag{15}$$

The LHS of (15) is the marginal gain to catch an additional share of public good by exerting one more unit of intra-district effort ($g_{2,i}\partial\alpha^j_{i,n}(\cdot)/\partial x_{2,i,n}$). This marginal gain negatively depends on $D_{1,i}$, because the debt burden reduces the amount of feasible public good ($g_{2,i}$). The RHS is the marginal cost, which is the opportunity cost of rent-seeking activities in term of fiscal resources ($\alpha^j_{i,n}\tau_i$).

Using (8), the first order condition on $y_{2,i,n}$ is

$$\tau_i=-\partial_2\psi^j_{i,n}(e_i,\hat{\mathbf{y}}_2)\frac{\partial\hat{y}_{2,i}}{\partial y_{2,i,n}}(1+r)D_{1,i.} \tag{16}$$

The LHS of (16) is the marginal cost of external rent-seeking behavior, which, as previously, corresponds to the loss of fiscal resource (τ_i). The RHS of (16) represents the marginal gain of fighting for citizens. As from the government's perspective, this gain positively depends on $D_{1,i}$, which is the stake of external conflict.

In symmetric equilibrium, citizens simultaneously and independently solve the same maximization problem (14), and the optimal rent-seeking efforts are characterized by $x_{t,i,n} =: x_{t,i}$ and $y_{2,i,n} =: y_{2,i}$. The first order conditions (15) and (16) then lead to

$$\frac{a^j_i}{x_{2,i}}=\frac{\tau_i}{\tau_iM_i(1-x_{2,i}-y_{2,i})-(1+r)D_{1,i}}, \tag{17}$$
$$\tau_i=\frac{b^j_iD_{1,i}(1+r)}{y_{2,i}M_i}. \tag{18}$$

The following Proposition establishes the unique couple of rent-seeking efforts $(x^{*j}_{2,i},y^{*j}_{2,i})$.

Proposition 1. *Let $\kappa_i := (1+r)/\tau_iM_i$. The unique optimal set of rent-seeking efforts*

$(x^{*j}_{2,i}, y^{*j}_{2,i}) \in \mathscr{C}$ *is*

$$x^{*j}_{2,i} = A^j_i \left[1 - \kappa_i B^j_i D_{1,i}\right] =: x^{*j}_{2,i}(D_{1,i}), \quad (19)$$
$$y^{*j}_{2,i} = \kappa_i (B^j_i - 1) D_{1,i} =: y^{*j}_{2,i}(D_{1,i}), \ \forall D_{1,i} \in [0, \overline{D}_{1,i}], \quad (20)$$

where $A^j_i := a^j_i M_i/(1 + a^j_i M_i) < 1$, $B^j_i := b^j_i + 1$, *and* $\bar{D}_{1,i} := \{D_{1,i} | x^{*j}_{2,i} = 0; j = I, O\}$ *corresponds to the highest public debt level consistent with positive effort* $x^{*j}_{2,i} \geq 0$.

Proof: See Appendix A.

Optimal fighting efforts $(x^{*j}_{2,i}, y^{*j}_{2,i})$ depend on the type of politician (j) in office, through parameters A^j_i and B^j_i (which are equivalent to a^j_i and b^j_i, respectively). Indeed, from (19) and (20), rent-seeking activities are positively related to decisiveness parameters (i.e. $\partial x^{*j}_{2,i}/\partial a^j_i > 0$ and $\partial y^{*j}_{2,i}/\partial b^j_i > 0$), which increase the expected efficiency of effort and the marginal gain of fighting.

In addition, $x^{*j}_{2,i}$ negatively depends on b^j_i. Effectively, the higher b^j_i, the sharper external fight. This reduces production and the available public good, which is the base of internal conflict. Hence lesser inducements to fight inside district i. There is no corresponding inverse relation between a^j_i and $y^{*j}_{2,i}$ because the base of the external conflict (the public debt burden) does not depend on the roughness of internal fight.

Proposition 1 highlights an outstanding feature. By issuing debt in the first period, the incumbent can generate a tradeoff between both types of conflicts. Indeed, public debt reduces the base of intra-district conflict through the decline of the feasible public good, while broadening the base of external conflict through the fight against the debt repayment.

Figure 1 describes this tradeoff. Since $(x^{*j}_{2,i})'(\cdot) < 0$, and $(y^{*j}_{2,i})'(\cdot) > 0$, as public debt increases, the equilibrium moves from point E_1 to point E_3 along the parametric curve with respect to $D_{1,i}$.

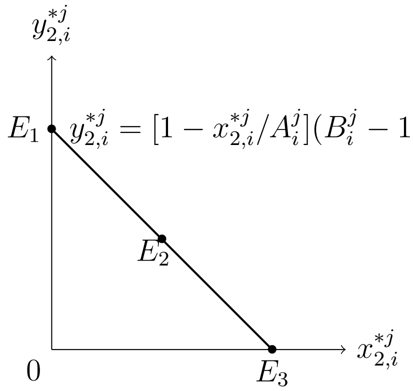

Figure 1: The tradeoff between internal and external conflicts

This tradeoff is closely related to Sumner (1906) well-known concept of ethnocentrism, whereby *"the relation of comradeship and peace in the we-group and that of hostility and war towards the other-groups are correlative to each other"* (Sumner, 1906, p.12). Our model offers a novel perspective on this mechanism, based on public debt management. Similarly to the diversionary theory of war, our argument is that public debt can be used to create diversion in managing internal and external social conflicts. Through public debt, the incumbent can generate, after the election, either internal conflict only (point E_1 in Figure 1), external conflict only (point E_3), or any combination of internal and external conflicts (like in point E_2).

By directing citizens' fighting behaviour toward the rest of the federation rather toward themselves, public debt is then a political instrument in the hands of the incumbent. Therefore, a Machiavellian incumbent can use public debt to secure social peace during his possible future mandate, for his personal interest.

5. First Period Equilibrium

This section solves the first-period equilibrium by a two-step backwards induction. According to our timing, in the first step, citizens compute their rent-seeking efforts, and in the second step, the incumbent determines the amount of debt.

5.1. First period rent-seeking behaviour

Given optimal levels of second-period effort $(x^{*j}_{2,i}, y^{*j}_{2,i})$, citizens optimally determine their first-period intra-district effort $x_{1,i,n}$. The optimal effort $x^*_{1,i}$ satisfies

$$x^{*I}_{1,i,n} := \underset{x_{1,i,n}\in[0,1]}{\text{argmax}}\ \mathbb{E}U^j_i(x_{1,i,n}, x^{*j}_{2,i}, y^{*j}_{2,i}).$$

From (8), the first-order condition is

$$\frac{\partial \alpha^I_{i,n}(\mathbf{x}_{1,i})}{\partial x_{1,i,n}}\left[\tau_i \sum_{s=1}^{M_i}(1-x_{1,i,s}) + D_{1,i}\right] = \alpha^I_{i,n}(\mathbf{x}_{1,i})\tau_i. \tag{21}$$

As in the previous section, the LHS of (21) represents the marginal gain to grasp an additional amount of rent, while the RHS is the marginal opportunity cost of rent-seeking activities. In symmetric equilibrium ($x_{1,i,n} =: x_{1,i}$), Eq. (21) becomes $a^I_i/x_{1,i} = \tau_i/[\tau_i M_i(1-x_{1,i}) + D_{1,i}]$, and the optimal effort positively depends on public debt, which increases the available public good in the first period, namely[21]

$$x^{*I}_{1,i} := A^I_i\left[1 + \frac{\kappa_i D_{1,i}}{1+r}\right] =: x^{*I}_{1,i}(D_{1,i}). \tag{22}$$

[21]The second-order condition is satisfied since $(\alpha^j_i)'' < 0$, and in symmetric equilibrium: $\partial^2\mathbb{E}U^j_i/\partial x^2_{1,i} = (\alpha^j_i)''(x_{1,i})[\tau_i M_i(1-x_{1,i}) + D_{1,i}] - 2(\alpha^j_i)'(x_{1,i})\tau_i < 0$.

5.2. The voting process

At the end of period 1 the election takes place and citizens vote for the candidate who brings them the highest expected utility. Citizen n supports politician I if and only if

$$\mathbb{E}U^{*I}_{i,n}(D_{1,i}) > \mathbb{E}U^{*O}_{i,n}(D_{1,i}),$$

where $U^{*j}_{i,n}(D_{1,i}) := U^{j}_{i,n}(x^{*I}_{1,i}(D_{1,i}), x^{*j}_{2,i}(D_{1,i}), y^{*j}_{2}(D_{1,i}))$, namely, by (9), iff

$$\theta_{i,n} > \overline{\theta}_i := -\beta\Delta_i(D_{1,i}) - \xi,$$

with $\Delta_i(\cdot) := U^{*I}_{i,n}(\cdot) - U^{*O}_{i,n}(\cdot)$ the differential of welfare.

Citizens with $\theta_{i,n} > \overline{\theta}_i$ prefer politician I. Thus, given our assumptions about the distribution of ideological preferences, politician I's vote share is

$$\pi_i = \sum_{n=1}^{M_i} \mathbb{P}\{\theta_{i,n} > \overline{\theta}_{i,n}\} = \sum_{n=1}^{M_i} \int_{\overline{\theta}_{i,n}}^{1/2s_i} s_i \, dz = \frac{M_i}{2} - M_i s_i \overline{\theta}_i. \tag{23}$$

From both candidates point of view, π_i is a random variable, since it is a transformation of the random shock ξ. The electoral outcome is thus a random event, related to the realization of the shock ξ. Let us consider a majoritarian rule in which the politician having obtained at less 50% of votes wins the election. Under this rule, the reelection probability of politician I is

$$\mu = \mathbb{P}\left\{\pi_i \geq \frac{M_i}{2}\right\} = \mathbb{P}\left\{\xi \geq -\Delta_i(D_{1,i})\right\}. \tag{24}$$

Hence, given our assumptions about the distribution of ξ ,[22]

$$\mu = \frac{1}{2} + h\Delta_i(D_{1,i}) =: \mu(D_{1,i}). \tag{25}$$

Using (8), the differential of welfare is in symmetric equilibrium ($\alpha^j_i = 1/M_i$ and $\psi^j_i = 1$)

$$\Delta_i(D_{1,i}) = \beta\tau_i[x^{*O}_{2,i}(D_{1,i}) + y^{*O}_{2,i}(D_{1,i}) - x^{*I}_{2,i}(D_{1,i}) - y^{*I}_{2,i}(D_{1,i})].$$

For tractability convenience, let us define the differentials of decisiveness parameters $\tilde{A}_i := (A^I_i - A^O_i)/2$ and $\tilde{B}_i := (B^I_i - B^O_i)/2$, and the average $\bar{A}_i := (A^I_i + A^O_i)/2$ and $\bar{B}_i := (B^I_i + B^O_i)/2$. To save notations, we henceforth drop i subscript. Proposition 1

[22] There is an interior solution, provided that h is small enough.

leads to[23]

$$\Delta(D_1) = 2\tau\beta\left[\kappa D_1\bar{B}(\tilde{A} - \sigma\tilde{B}) - \tilde{A}\right], \tag{26}$$

where $\sigma := (1 - \bar{A})/\bar{B} > 0$. Reintroducing in (25), we obtain

$$\mu(D_1) = \frac{1}{2} + 2\beta h\tau\left[\kappa D_1\bar{B}(\tilde{A} - \sigma\tilde{B}) - \tilde{A}\right]. \tag{27}$$

The reelection probability ($\mu(D_1)$) depends on public debt through the interaction between comparative advantages or disadvantages of politicians in managing internal and external conflicts. The tilde variables $\tilde{A}$ and $\tilde{B}$ reflect citizens' comparative incitement to fight (in internal and external conflicts, respectively) if the incumbent, rather than the challenger, is elected. Obviously, any increase in $\tilde{B}$ reduces the reelection probability of the incumbent, because external conflict weakens if his challenger is elected. In contrast, an increase in $\tilde{A}$ exerts two conflicting effects. First, the higher $\tilde{A}$, the higher internal conflicts if the incumbent is elected compared to the challenger (this corresponds to the negative effect in (27)). Second, there is a positive effect in (27) due to the interaction between both types of conflict (more internal conflict leads to less external conflict, as we have already emphasized).

In addition, the reelection probability depends on public debt through two channels ($\tilde{A}$ and $\tilde{B}$). As regards the first channel, if $\tilde{A} > 0$, citizens know that internal conflict will be larger if the incumbent is elected rather than his challenger. Therefore, by this channel, public debt increases the reelection probability by reducing internal conflict and the comparative disadvantage of the incumbent. As regards the second channel, if $\tilde{B} > 0$, there will be comparatively more external conflict if the incumbent is elected. By strengthening external conflicts, public debt reduces the chances of the incumbent. Consequently, from an electoral perspective, the incumbent is induced to issue public debt only if the first effect outweighs the last, namely, if the relative differential in internal conflict is large enough (i.e. $\tilde{A} > \sigma\tilde{B}$). In the opposite case ($\tilde{A} < \sigma\tilde{B}$) , public debt reduces his probability of being renewed.

However, the incumbent's objective is not to maximize the probability of reelection, but the expected value of inter-temporal power-rents. Let us now turn our attention to the determination of the optimal level of public debt.

6. Public debt and social cohesion

In the first period, the incumbent sets the amount of public debt D_1 that maximizes his expected inter-temporal payoff. He internalizes the consequences of his choice on

[23]We use: $\Delta(D_1) = -2\beta\tau\tilde{A} + \beta\tau\kappa D_1\left\{(\bar{B} + \tilde{B})(\bar{A} + \tilde{A}) - (\bar{B} - \tilde{B})(\bar{A} - \tilde{A}) - 2\tilde{B}\right\} = -2\beta\tau\tilde{A} + \beta\tau\kappa D_1\{\bar{B}\tilde{A} - \tilde{B}(1 - \bar{A})\}$.

citizens' rent-seeking activities and of the level of coordination effort he will have to engage in the second period, if reelected. Thus, using (10), the incumbent maximizes

$$\mathbb{E}[V] = R_1 + \beta\mu(D_1)R_2^*(D_1) = R + \beta\mu(D_1)[R - e^*(D_1)].$$

The solution of this problem leads to the following first-order condition

$$\mu'(\cdot)[R - e^*(\cdot)] = \mu(\cdot)(e^*)'(\cdot). \tag{28}$$

The incumbent chooses public debt such that the marginal gain of being reelected equals the marginal cost of effort, if reelected. The LHS of (28) is the marginal gain to issue public debt, namely the marginal increase in the probability to be elected ($\mu'(\cdot)$), adjusted by the power-rent ($R - e^{*I}(\cdot)$). The RHS of (28) is the marginal cost of debt: high public debt today forces the incumbent (if reelected, with probability μ) to undertake high effort tomorrow.

As we have discussed above, the reelection probability positively ($\mu' > 0$) or negatively ($\mu' < 0$) depends on public debt, according to the sign of the term $\tilde{A} - \sigma\tilde{B}$. As $(e^*)' > 0$, and $\mu > 0$, we obtain an interior maximum only if $\tilde{A} - \sigma\tilde{B} > 0$. In the opposite case, there is a corner solution. The following Proposition fully characterizes the nature of equilibrium according to the value of the couple $(\tilde{A}, \tilde{B})$.

Proposition 2. *(Equilibrium Characterization) For small ε^I, there is a unique subdivision of the $(\tilde{A}, \tilde{B})$-plane by the family of non-empty disjoint set $\{\mathscr{C}_0, \mathscr{C}_m, \mathscr{C}_{int}\}$, such that, the unique equilibrium D_1^* is characterized by the three following cases.*

i. *If $(\tilde{A}, \tilde{B}) \in \mathscr{C}_0$, $D_1^* = 0$.*

ii. *If $(\tilde{A}, \tilde{B}) \in \mathscr{C}_m$, $D_1^* = \bar{D}_1$.*

iii. *If $(\tilde{A}, \tilde{B}) \in \mathscr{C}_{int}$,*

$$D_1^* = \bar{R} + \frac{\tilde{A} - \Psi}{2\kappa\bar{B}(\tilde{A} - \sigma\tilde{B})},$$

where $\Psi := 1/4h\beta\tau$ and $\bar{R} := R/2\lambda(1+r)\varepsilon^I$.

Proof: See Appendix B.

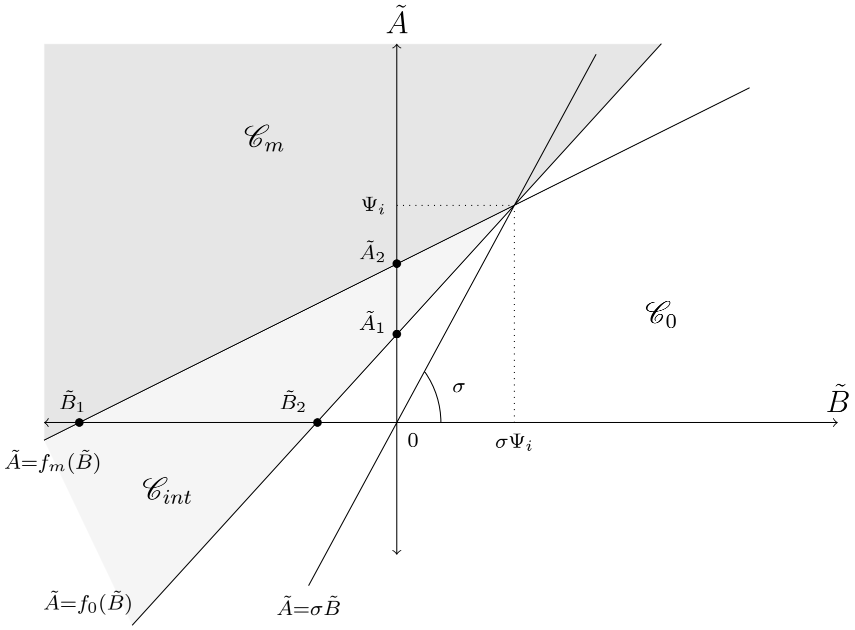

Figure 2: Equilibrium characterization in the $(\tilde{A}, \tilde{B})$-plane

Figure 2 establishes the public debt strategy of the incumbent, as a function of his comparative advantages or disadvantages in managing internal and external conflicts. We can distinguish 3 areas. (i) For high values of $\tilde{A}$ and low values of $\tilde{B}$, public debt is set at its maximum level, (ii) on the contrary, for small values of $\tilde{A}$ and high values of $\tilde{B}$, the incumbent implements a zero public debt. (iii) In the case $(\tilde{A}, \tilde{B}) \in \mathscr{C}_{int}$, there is an interior solution. These features extend the previous discussion on the probability of reelection, by considering the cost to coordinate external conflict.

6.1. Equilibrium public debt and social conflicts

Figure 3 depicts the optimal debt strategy of the incumbent, an the corresponding equilibrium level of internal and external conflicts in period 2, if he is reelected, in function of the decisiveness gaps $\tilde{A}$ and $\tilde{B}$.[24]

[24] Figures 3a and 3b are established as cross-sections of Figure 2, for $\tilde{A} = 0$, and $\tilde{B} = 0$, respectively.

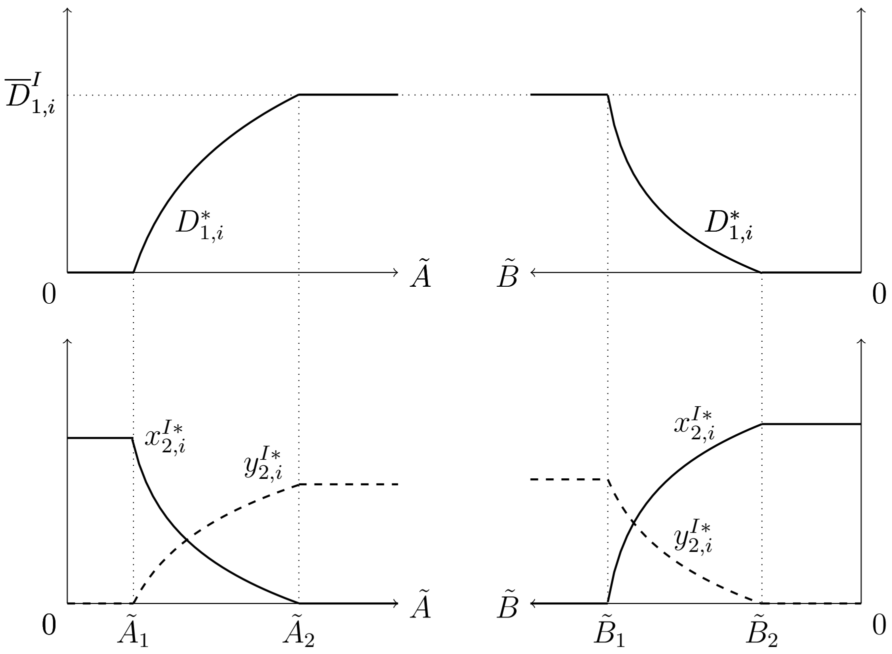

Figure 3a – $\tilde{B} = 0$ – Figure 3b – $\tilde{A} = 0$ –

Figure 3: Optimal debt and conflicts in function of the decisiveness gap

If $\tilde{B} = 0$ and $\tilde{A} > 0$, the incumbent has a comparative disadvantage in internal conflicts relative to his challenger. As $\tilde{A}$ increases, it is in the incumbent's interest to rise public debt to reduce this disadvantage (see Figure 3a). Indeed, by cutting the stake of the internal conflict, public debt allows reducing internal rent-seeking activities during the second period ($x_{2,i}^{I*}$ decreases). If $\tilde{A} = 0$ and $\tilde{B} > 0$, in contrast, the challenger has a comparative advantage in external conflict. As $\tilde{B}$ increases, the incumbent is induced to reduce public debt to destroy this comparative advantage by limiting external rent-seeking activities during the second period ($y_{2,i}^{I*}$ decreases, see Figure 3b).

For $\tilde{A}_1 < \tilde{A} < \tilde{A}_2$ and $\tilde{B}_1 < \tilde{B} < \tilde{B}_2$, respectively, the optimal debt strategy gives rise to an interior solution $0 < D_{1,i}^{*} < \overline{D}_{1,i}^{I}$,[25] but for other configurations of parameters, we obtain corner solutions, without change in interpretation. Noteworthy, without any comparative advantage or disadvantage ($\tilde{A} = \tilde{B} = 0$), the incumbent never issue public debt ($D_1^{*} = 0$). Indeed, the office-holder is induced to issue public debt only if his comparative advantages are high enough to offset the coordination effort he will have to undertake, if reelected.

[25]Such an interior solution is likely to occur if we consider, for example, $a_i^I > a_i^0$ and $b^I < b^O$, as discussed in footnotes 10 and 12.

Figure 3 highlights the strategic use of public debt as a diversionary device. This diversionary tactic is maximal if the comparative advantage of the opponent in internal (resp. external) conflict resolution is high (resp. low), i.e. if $\tilde{A} > A_2$ (resp. if $\tilde{B} < B_1$). In contrast, the incentive to use public debt for diversionary motives disappears if $\tilde{A} < A_1$ and/or $\tilde{B} > B_2$.

Finally, to establish a benchmark, the following subsection determines the optimal public debt that maximizes social welfare in district i.

6.2. Social Welfare

The inter-temporal social welfare of district i (W_i) is the sum of all citizens' payoff, and depends on the type of politician who takes office in the second period. Therefore, we can compute expected social welfare as

$$\mathbb{E}[W_i] = \sum_{n=1}^{M_i} W_{1,i,n}^I + \beta\mu^I \sum_{n=1}^{M_i} W_{2,i,n}^I + \beta(1-\mu^I) \sum_{n=1}^{M_i} W_{2,i,n}^O, \tag{29}$$

where $\mathbb{E}$ is the expectation operator over the election outcome, and $W_{t,i,n}$ the payoff of citizen n, $n \in \{1, \cdots, M_i\}$. Using the inter-temporal utility (8), we have

$$W_{1,i,n}^I = \alpha_{i,n}^j(\mathbf{x}_{1,i}) \left[\tau_i \sum_{n=1}^{M_i} (1 - x_{1,i,n}) + D_{1,i} \right],$$
$$W_{2,i,n}^j = \alpha_{i,n}^j(\mathbf{x}_{2,i}) \left[\tau_i \sum_{n=1}^{M_i} (1 - x_{2,i,n} - y_{2,i,n}) - \psi_i^j(\mathbf{y}_2)(1+r)D_{1,i} \right].$$

In symmetric equilibrium ($W_{t,i,n} =: W_{t,i}$, for any n), expected social welfare (29) becomes (by dropping subscript i)

$$\mathbb{E}[W] = MW_1^I + \beta M\mu^I[W_2^I - W_2^O] + \beta MW_2^O, \tag{30}$$

hence, using optimal effort levels (19), (20) and (22)

$$\mathbb{E}[W](D_1) = M\Delta(D_1)\left[\frac{1}{2} + h\Delta(D_1)\right] + \beta\tau M(1 - x_2^{*O} - y_2^{*O}) + [1 - \beta(1+r)]D_1 + \tau M(1 - x_1^{*I}). \tag{31}$$

The following Proposition determines the socially-optimal level of public debt (D_1^{SW}) in function of decisiveness parameters ($\tilde{A}$ and $\tilde{B}$).

Proposition 3. *Let $\beta = 1/(1+r)$. If $\bar{D}_1 > \sigma/(1-\sigma)$, there are two levels $K_1(\tilde{A})$ and*

$K_2(\tilde{A})$, *where* $K_1 < 0 < K_2$, *such that*

$$D_1^{SW} = \begin{cases} 0 & \textit{if } K_1(\tilde{A}) \leq \tilde{B} \leq K_2(\tilde{A}), \\ \bar{D}_1 & \textit{else.} \end{cases}$$

Proof: By inspecting Eq. (31), we note that $D_1 \mapsto \mathbb{E}[W]$ is a quadratic function, namely

$$\mathbb{E}[W] = \epsilon_1 D_1^2 + \epsilon_2 D_1 + \epsilon_3,$$

where, by (26), $\epsilon_1 := Mh[2\tau\beta\kappa(1+r)\bar{B}(\tilde{A} - \sigma\tilde{B})]^2 > 0$. Thus, $D_1 \mapsto \mathbb{E}[W]$ describes an U-shaped curve, and the social planner of district i maximizes the expected social welfare (31) by issuing the amount of public debt $D_1^{SW} \in \{0, \overline{D}_1\}$. Appendix C characterizes this critical level in function of the decisiveness parameters. □

As welfare is an U-shaped function of public debt in Eq. (31), an interior solution cannot appear and the optimum is a bang-bang equilibrium: the social planner chooses either a zero or a maximal ($\bar{D}_1$) public debt. Indeed, the social planner internalizes second-period citizens' payoff for both election outcomes (i.e. W_2^I and W_2^O). Specifically, in Eq. (31), the expected welfare is quadratic in the gap of payoffs $\Delta = W_2^I - W_2^O$, thereby the society enjoys extreme values of Δ.

Effectively, it is important, for citizens welfare, that the politician who brings the highest utility is very likely to be elected. This is the case if Δ is strongly positive (the incumbent brings high gains relative to the challenger, and has a high probability of reelection) or strongly negative (the challenger brings high gains relative to the incumbent, and is more likely to be elected). Proposition 3 shows that such extreme values of Δ arise if $D_1 = \bar{D}_1$ or $D_1 = 0$, depending on the gaps of decisiveness in internal and external conflicts.

Interestingly, the equilibrium under an opportunistic incumbent corresponds to the social optimum if the incumbent has a strong comparative advantage in external conflict. As shows Figure 2, when $\tilde{B}$ is highly negative the couple $(\tilde{A}, \tilde{B})$ belongs to the area $\mathscr{C}_m$, and $D_1^* = D_1^{SW} = \bar{D}_1$. Indeed, in this context, citizens highly benefit from the incumbent reelection (Δ is high) precisely because public debt is maximal, which favors both the collective welfare and the incumbent payoff.

This benchmark is attractive in the light of the diversionary theory of conflicts. Opportunistic political behaviour leads to a social optimum in the case of a maximal diversionary effect. Indeed, the level of debt $\bar{D}_1$ gives rise to the lowest internal rent-seeking effort and the highest external fighting effort. Along these lines, the diversionary motivation is not only an opportunistic way to ensure incumbent's private interests but also a channel to reach a greater social welfare.

7. Conclusion

There is a substantial consensus within social science disciplines about the positive relationship between intergroup conflicts and group cohesion (see Tajfel, 1982, for a review). In a relevant paper, Stein (1976) concluded that "*there is a clear convergence in the literature in both the specific studies and in the various disciplines that suggests external conflict does increase internal cohesion*" (Stein, 1976, p.165).

In this paper, we have developed such an idea in the context of a macroeconomic model where a strategic incumbent manages public debt to monitor internal and external conflicts in function of his comparative advantages relative to the challenger. Specifically, by increasing public debt, an incumbent whose challenger beneficiates from a comparative advantage in managing internal conflicts can lessen this advantage by reducing expected rent-seeking behaviour, if reelected. By so doing, he ties his own hand in order to credibly commit to escape citizens claims during his term. Thus, public debt does make internal social cohesion.

Additionally, by developing external conflicts, public debt can be even more profitable for the incumbent, who will beneficiate from less external conflicts than his challenger, if reelected. This double dividend of public debt remains, even if, in symmetric equilibrium, the incumbent does not retrieve any popularity advantage from the fight against other districts.

Our paper is built on the long-lasting idea that competition over public expenditures, like over any resource whose property rights are imperfectly defined, gives rise to social conflicts or rent seeking activities. A number of examples show that windfall budget surpluses exacerbate claims from various social groups (as highlights the famous episode of the "fiscal jackpot" offered by the resumption of french economic growth in 2000).[26] Other prominent occurrences emphasize that public debt could serve to generate anger against international institutions or foreign powers and the citizens thereof. For instance, in the current decade, hampered by the burden of a huge debt, the Greek government oriented social claims against the foreign creditors, namely the well-known *"Troïka"*.[27] The high level of public debt then served as a political instrument both to continue international negotiations, and to force citizens to accept austerity measures (see Ardagna and Caselli, 2014 and Katiskas, 2012). Moreover, from an electoral perspective, international negotiations had been used by the governments to be perceived as combative and active in the eyes of the Greek electorate, improving their chances to be reelected (Katiskas, 2012).

[26] During the same year, the United States also benefited from such a windfall fiscal gain, with the same effect on citizens' claims, due to the strong april tax revenues (the so-called *April Surprise*), allowing a reevaluation of the primary budget surplus close to 50%.

[27] For example, the prime minister A. Tsipras was claimed in 2016 that creditors make Greek crisis worse. See, e.g., https://www.theguardian.com/world/2016/sep/11/alexis-tsipras-greece-criticises-creditors-thessaloniki

Our setup may lead to interesting prospects for future research. First, it would be particularly interesting to study the interplay between the electoral process and citizens' welfare in an inter-temporal framework, to examine the interaction between reputational strengths and the diversionary mechanism. Second, formalizing a probabilistic environment would allow describing how the propagation of idiosyncratic shocks in the Federation is affected by the diversionary channel. Finally, a very fruitful research avenue would be to use time-series or cross-national data sets to empirically test the diversionary mechanism of the public debt and examine how incumbent politicians may increase public debt to pacify the society.

References

Aghion, P., Bolton, P.P., 1990. Government Domestic Debt and the Risk of Default: A Political-Economic Model of the Strategic Role of Government Debt, in: Public Debt Management: Theory, History (Eds.), Dornbusch, R. and Draghi, M.. Cambridge University Press, NY, pp. 315–344.

Alesina, A., Tabellini, G., 1990. A Positive Theory of Fiscal Deficits and Government Debt. Review of Economic Studies 57, 403–414.

Bertola, G., Drazen, A., 1993. Trigger Points and Budget Cuts: Explaining the Effects of Fiscal Austerity. The American Economic Review 83, 11–26.

Brender, A., Drazen, A., 2008. How do budget deficits and economic growth affect reelection prospects? Evidence from a large panel of countries. The American Economic Review 98, 2203–2220.

Dahrendorf, R., 1964. Homo Sociologicus. VS Verlag für Sozialwissenschaften, Wiesbaden. pp. 13–96. URL: https://doi.org/10.1007/978-3-322-85721-7_1, doi:10.1007/978-3-322-85721-7{_}1.

Garmann, S., 2017. Political budget cycles and fiscally conservative voters. Economics Letters 155, 72–75.

Grieco, J., Ikenberry, G.J., Mastanduno, M., 2014. Introduction to International Relations. Palgrave.

Grossman, S.J., Hart, O.D., 1982. Corporate financial structure and managerial incentives, in: The economics of information and uncertainty. University of Chicago Press, Chicago, pp. 107–140.

Hetherington, M.J., Nelson, M., 2003. Anatomy of a rally effect: George W. Bush and the war on terrorism. PS: Political Science Politics 36, 37–42.

Hirshleifer, J., 1989. Conflict and rent-seeking success functions: Ratio vs. difference models of relative success. Public Choice 63, 101–112.

Hirshleifer, J., 1991. The paradox of power. Economics and Politics 3, 177–200.

Hirshleifer, J., 2008. Anarchy and its breakdown, in: Garfinkel, M.R., Skaperdas, I.S. (Eds.), The Political Economy of Conflict and Appropriation. Cambridge University Press, Cambridge. chapter 2.

Hodler, R., 2011. Elections and the strategic use of budget deficits. Public Choice 148, 149–161.

Jensen, M.C., 1986. Agency costs of free cash flow, corporate finance, and takeovers. The American Economic Review 76, 323–329.

Katz, E., Nitzan, S., Rosenberg, J., 1990. Rent Seeking for Pure Public Goods. Public Choice 65, 49–60.

Kernell, S., 1978. Explaining presidential popularity: How ad hoc theorizing, misplaced emphasis, and insufficient care in measuring one's variables refuted common sense and led conventional wisdom down the path of anomalies. American Political Science Review 72, 506–522.

Kremer, M., Jayachandran, S., 2002. Odious debt. NBER Working Papers w8953 .

Levine, R.A., Campbell, D.T., 1972. Ethnocentrism: Theories of conflict, ethnic attitudes and group conflict. Wiley, NY.

Levy, J., 1989. Conflict resolution in elementary and secondary education. Conflict Resolution Quarterly 7, 73–87.

Lian, B., .O.J.R., 1993. Presidents, the use of military force, and public opinion. Journal of Conflict Resolution 37, 277–300.

Lindbeck, A., Weibull, J., 1987. Balanced-Budget Redistribution as the Outcome of Political Competition. Public Choice 52, 273–297.

Mansfield, E.D., Snyder, J., 2004. Electing to fight. MIT Press, Cambridge.

Mueller, J.E., 1973. War, presidents, and public opinion. John Wiley and Sons.

Münster, J., Staal, K., 2011. War with outsiders makes peace inside. Conflict Management and Peace Science 28, 91–110.

Nordhaus, W.D., 1975. The Political Business Cycle. The Review of Economic Studies 42, 169–190.

Peltzman, S., 1992. Voters as Fiscal Conservatives. The Quarterly Journal of Economics 107, 327–361.

Persson, T., Svensson, L., 1989. Why a Stubborn Conservative Would Run a Deficit: Policy with Time-Inconsistent Preferences. Quarterly Journal of Economics 104, 325–346.

Persson, T., Tabellini, G., 2000. Political Economics: Explaining Economic Policy. MIT Press, MA.

Rogoff, K., Sibert, A., 1988. Elections and Macroeconomic Policy Cycles. Review of Economic Studies 55, 1–16.

Stein, A.A., 1976. Conflict and cohesion: A review of the literature. Journal of conflict resolution 20, 143–172.

Stulz, R., 1990. Managerial discretion and optimal financing policies. Journal of financial Economics 26, 3–27.

Sumner, W.G., 1906. Folkways, a study of the sociological importance of usages, manners, customs, mores, and morals. Ginn.

Tajfel, H., 1982. Social psychology of intergroup relations. Annual review of psychology 33, 1–39.

Wright, Q., 1965. A Study of War. University of Chicago Press, Chicago.

Appendix A. Proof of Proposition 1

Let $i \in \{1, \cdots, N\}$. The vectors of efforts are denoted by $\mathbf{x}_{2,i} := (x_{2,i,1}, , x_{2,i,M_i})$ and $\hat{\mathbf{y}}_2 := (\hat{y}_{2,1}, \cdots, \hat{y}_{2,N})$, where $\mathrm{y}_{2,i} := f(e_i) \sum_{s=1}^{M_i} y_{2,i,s}$. The Lagrange function $\mathcal{L}$ related to the maximization problem (14) is

$$\begin{aligned} \mathcal{L}(x_{2,i,n}, y_{2,i,n}, \lambda_{i,n}) = \alpha_{i,n}^j(\mathbf{x}_{1,i}) &\left[\tau_i \sum_{s=1}^{M_i} (1 - x_{1,i,s}) + D_{1,i} \right] \\ &+ \beta \alpha_{i,n}^j(\mathbf{x}_{2,i}) \left[\tau_i \sum_{s=1}^{M_i} (1 - x_{2,i,s} - y_{2,i,s}) - \psi_i^j(\hat{\mathbf{y}}_2)(1+r) D_{1,i} \right] \\ &+ \lambda_{i,n} \left[\tau_i \sum_{s=1}^{M_i} (1 - x_{2,i,s} - y_{2,i,s}) - \psi_i^j(\hat{\mathbf{y}}_2)(1+r) D_{1,i} \right], \end{aligned} \tag{A.1}$$

where $\lambda_{i,n} \geq 0$ is the Lagrange multiplier of the constraint $g_{2,i} \geq 0$. The following subsection computes the first-order conditions.

Appendix A.1. First-Order Conditions

By (A.1), the Karush-Kuhn-Tucker (KKT) conditions are, since $\partial \hat{y}_{2,i} / \partial y_{2,i,n} = f(e_i)$,

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial x_{2,i,n}} = \beta \frac{\partial \alpha_{i,n}^j(\mathbf{x}_{2,i})}{\partial x_{2,i,n}} &\left[\tau_i \sum_{s=1}^{M_i} (1 - x_{2,i,s} - y_{2,i,s}) - \psi_i^j(\hat{\mathbf{y}}_2)(1+r) D_{1,i} \right] \\ &- \beta \tau_i \alpha_{i,n}^j(\mathbf{x}_{2,i}) - \lambda_{i,n} \tau_i = 0, \end{aligned} \tag{A.2}$$

$$\frac{\partial \mathcal{L}}{\partial y_{2,i,n}} = - \left[\tau_i + f(e_i) \frac{\partial \psi_i^j(\hat{\mathbf{y}}_2)}{\partial y_{2,i,n}} (1+r) D_{1,i} \right] [\lambda_{i,n} + \beta \alpha_i^j(x_{2,i,n})] = 0, \tag{A.3}$$

$$\lambda_{i,n} \left[\tau_i \sum_{s=1}^{M_i} (1 - x_{2,i,s} - y_{2,i,s}) - (1+r) \psi_i^j(\hat{\mathbf{y}}_2) D_{1,i} \right] = 0. \tag{A.4}$$

The symmetric equilibrium is characterized by $x_{2,i,n} =: x_{2,i}$, $y_{2,i,n} =: y_{2,i}$, $\lambda_{i,n} =: \lambda_i$, $\forall n \in \{1, \cdots, M_i\}$, and $\hat{y}_{2,i} = \hat{y}_2$, $\forall i \in \{1, \cdots, N\}$. Using Definitions 1 and 2, the symmetric equilibrium leads to

$$\frac{\partial \alpha_{i,n}^j(\mathbf{x}_{2,i})}{\partial x_{2,i,n}} =: (\alpha_i^j)'(x_{2,i}) = \left[\frac{(\alpha_i^j)'(x_{2,i}) x_{2,i}}{\alpha_i^j(x_{2,i})} \right] \frac{\alpha_i^j(x_{2,i})}{x_{2,i}} = \frac{a_i^j}{M_i x_{2,i}},$$

$$\frac{\partial \psi_i^j(\hat{\mathbf{y}}_2)}{\partial y_{2,i,n}} =: (\psi^j)'(\hat{y}_2) = \left[\frac{(\psi^j)'(\hat{y}_2) \hat{y}_2}{\psi^j(\hat{y}_2)} \right] \frac{\psi^j(\hat{y}_2)}{\hat{y}_2} = - \frac{b^j}{\hat{y}_2}.$$

Since $\psi^j = 1$ and $\alpha_i^j = 1/M_i$, the KKT conditions (A.2)-(A.3)-(A.4) become

$$\frac{\partial \mathcal{L}}{\partial x_{2,i}} = \frac{\beta a_i^j}{x_{2,i}}[\tau_i M_i(1 - x_{2,i} - y_{2,i}) - (1+r)D_{1,i}] - \frac{\beta \tau_i}{M_i} - \lambda_i \tau_i = 0, \tag{A.5}$$

$$\frac{\partial \mathcal{L}}{\partial y_{2,i}} = -\left[\tau_i - \frac{b^j(1+r)D_{1,i}}{y_{2,i}M_i}\right]\left[\lambda_i + \frac{\beta}{M_i}\right] = 0, \tag{A.6}$$

$$\lambda_i \left[\tau_i M_i(1 - x_{2,i} - y_{2,i}) - (1+r)D_{1,i}\right] = 0. \tag{A.7}$$

Now, we look at the complementary slackness condition, considering two cases.

Case 1. $\lambda_i > 0$. Using (A.7), $\tau_i M_i(1 - x_{2,i} - y_{2,i}) = (1+r)D_{1,i}$. By substituting in (A.5), we have $\lambda_i = -\beta/M_i < 0$, which establishes a contradiction.

Case 2. $\lambda_i = 0$. Using (A.7), $\tau_i M_i(1 - x_{2,i} - y_{2,i}) > (1+r)D_{1,i}$, and (A.5)-(A.6) become

$$\frac{\partial \mathcal{L}}{\partial x_{2,i}} = \frac{\beta a_i^j}{x_{2,i}}[\tau_i M_i(1 - x_{2,i} - y_{2,i}) - (1+r)D_{1,i}] - \frac{\beta \tau_i}{M_i} = 0,$$

$$\frac{\partial \mathcal{L}}{\partial y_{2,i}} = \frac{\beta}{M_i}\left[\frac{b^j(1+r)D_{1,i}}{y_{2,i}M_i} - \tau_i\right] = 0,$$

hence; the unique critical-point $(\check{x}_{2,i}, \check{y}_{2,i})$ is

$$\check{x}_{2,i} = \frac{a_i^j M_i}{1 + a^j M_i}\left[1 - \left(1 + b^j\right)\frac{(1+r)D_{1,i}}{\tau_i M_i}\right], \tag{A.8}$$

$$\check{y}_{2,i} = \frac{b^j(1+r)D_{1,i}}{\tau_i M_i}. \tag{A.9}$$

By denoting $A_i^j := a_i^j/(1 + a_i^j M_i)$, and $B^j := 1 + b^j$, Eqs. (19) and (20) in the main tex immediately follow.

Finally, the couple of efforts $(\check{x}_{2,i}, \check{y}_{2,i})$ is the unique equilibrium if and only if it is the unique global maximum on the compact-space $\mathscr{C} := \{(s,t) \in [0,1]^2;\ s + t < 1\}$, as shows the following subsection.

Appendix A.2. Second-Order Conditions

To establish the concavity of the Lagrange-function, we compute the following second-derivatives. By (A.2), and (A.3) we obtain

$$\frac{\partial^2 \mathcal{L}}{\partial (x_{2,i,n})^2} = \beta \frac{\partial^2 \alpha_{i,n}^j(\mathbf{x}_{2,i})}{\partial (x_{2,i,n})^2}\left[\tau_i \sum_{s=1}^{M_i}(1 - x_{2,i,s} - y_{2,i,s}) - \psi_i^j(\hat{\mathbf{y}}_2)(1+r)D_{1,i}\right] - 2\beta\tau_i \frac{\partial \alpha_{i,n}^j(\mathbf{x}_{2,i})}{\partial x_{2,i,n}},$$

$$
\begin{aligned}
\frac{\partial^2 \mathcal{L}}{\partial(x_{2,i,n})\partial(y_{2,i,n})} &= -\beta \frac{\partial \alpha^j_{i,n}(\mathbf{x}_{2,i})}{\partial x_{2,i,n}} \left[\tau_i + f(e_i) \frac{\partial \psi^j_i(\hat{\mathbf{y}}_2)}{\partial \hat{y}_{2,i}} (1+r) D_{1,i} \right], \\
\frac{\partial^2 \mathcal{L}}{\partial(y_{2,i,n})^2} &= -f(e_i)^2 \frac{\partial^2 \psi^j_i(\hat{\mathbf{y}}_2)}{\partial(\hat{y}_{2,i})^2} (1+r) D_1 \left[\lambda_{i,n} + \beta \alpha^j_{i,n}(\mathbf{x}_{2,i}) \right], \\
\frac{\partial^2 \mathcal{L}}{\partial(x_{2,i,n})\partial(\lambda_{i,n})} &= -\tau_i, \\
\frac{\partial^2 \mathcal{L}}{\partial(y_{2,i,n})\partial(\lambda_{i,n})} &= -\left[\tau_i + f(e_i) \frac{\partial \psi^j_i(\hat{\mathbf{y}}_2)}{\partial \hat{y}_{2,i}} (1+r) D_{1,i} \right],
\end{aligned}
$$

and, using Eq. (A.1), we have $\partial^2 \mathcal{L}/\partial(\lambda_{i,n})^2 = 0$.

In symmetric equilibrium, by (A.8) and (A.9), second-derivatives evaluated at the critical-point $(\check{x}_{2,i}, \check{y}_{2,i})$ become

$$
\left. \frac{\partial^2 \mathcal{L}}{\partial x_{2,i} \partial y_{2,i}} \right|_{(\check{x}_{2,i}, \check{y}_{2,i})} = \left. \frac{\partial^2 \mathcal{L}}{\partial y_{2,i} \partial \lambda_i} \right|_{(\check{x}_{2,i}, \check{y}_{2,i})} = 0,
$$

since first-order conditions lead to $\tau_i + (1+r) D_{1,i}\ \psi^{j\prime}(f(e_i) M_i \check{y}_{2,i}) = 0$. In addition, we have

$$
\begin{aligned}
\left. \frac{\partial^2 \mathcal{L}}{\partial(x_{2,i})^2} \right|_{(\check{x}_{2,i}, \check{y}_{2,i})} &= \beta (\alpha^j_i)''(\check{x}_{2,i}) [\tau_i M_i (1 - \check{x}_{2,i} - \check{y}_{2,i}) - (1+r) D_{1,i}] \\
&\quad - 2\beta \tau_i (\alpha^j_i)'(\check{x}_{2,i}) =: E_1, \quad \text{(A.10)}
\end{aligned}
$$

$$
\left. \frac{\partial^2 \mathcal{L}}{\partial(y_{2,i})^2} \right|_{(\check{x}_{2,i}, \check{y}_{2,i})} = -f(e_i)^2 (\psi^j)''(f(e_i) M_i \check{y}_{2,i}) (1+r) D_{1,i} \left[\lambda_i + \frac{\beta}{M_i} \right] =: E_2. \quad \text{(A.11)}
$$

Consequently, the Hessian-matrix, denoted by $\mathbf{H}(\cdot,\cdot)$, related to the Lagrange function evaluated at the critical-point $(\check{x}_{2,i}, \check{y}_{2,i})$ is given by

$$
\mathbf{H}|_{(\hat{x}_{2,i}, \hat{y}_{2,i})} = \begin{pmatrix} E_1 & 0 & -\tau_i \\ 0 & E_2 & 0 \\ -\tau_i & 0 & 0 \end{pmatrix}.
$$

According to Definitions 1 and 2, since $(\alpha^j_i)'' < 0$ and $(\psi^j_i)'' < 0$, we have: $E_1 < 0$, $E_2 < 0$; namely, $\mathrm{tr}(\mathbf{H}) = E_1 + E_2 < 0$ and $\det(\mathbf{H}) = -\tau_i^2 E_2 > 0$. Thus, the unique critical-point $(\hat{x}_{2,i}, \hat{y}_{2,i})$ is the unique global maximum of the Lagrange-function.

In the last step, we show that $(\check{x}_{2,i}, \check{y}_{2,i}) \in \mathscr{C}$, namely $\check{x}_{2,i} \geq 0$, $\check{y}_{2,i} \geq 0$, and $\check{x}_{2,i} + \check{y}_{2,i} \leq 1$. First, by (A.9), we have $\check{y}_{2,i} \geq 0$, $\forall D_{1,i} \geq 0$, and by (A.8), $\check{x}_{2,i} \geq 0$ if and only if $D_{1,i} \in [0, \overline{D}_{1,i}]$, where

$$\overline{D}^j_{1,i} := \frac{\tau_i M_i}{(1+r)(1+b^j)}. \tag{A.12}$$

Second, using Eqs. (A.8)-(A.9), the function $\zeta(D_{1,i}) := \hat{x}_{2,i} + \hat{y}_{2,i}$ is given by

$$\zeta(D_{1,i}) = A^j_i - \frac{(1+r)D_{1,i}}{\tau_i M_i}\left[1 - B^j(1 - A^j_i)\right]. \tag{A.13}$$

From (A.13), as $\zeta(D_{1,i})$ linearly depends on $D_{1,i}$, we can distinguish the two following cases.

(i) If ζ decreases with $D_{1,i}$, we have, by (A.13),

$$\max_{D_{1,i} \in [0, \overline{D}_{1,i}]} \zeta(D_{1,i}) = \zeta(0) = A^j_i < 1.$$

(ii) If ζ increases with $D_{1,i}$, using (A.12), we have

$$\max_{D_{1,i} \in [0, \overline{D}_{1,i}]} \zeta(D_{1,i}) = \zeta(\overline{D}^j_{1,i}) = A^j_i - \frac{1}{B^I} + \frac{B^j}{B^I}(1 - A^j_i).$$

In addition, as $0 < B^j \leq B^I$, $\forall j \in \{I, O\}$, we obtain: $\zeta(\overline{D}^j_{1,i}) \leq 1 - 1/B^I < 1$.

Finally, for any $D_{1,i} \in [0, \overline{D}_{1,i}]$, the couple $(\check{x}_{2,i}, \check{y}_{2,i})$ is the unique global maximum on $\mathscr{C}$, namely the unique equilibrium, and we note $(\check{x}_{2,i}, \check{y}_{2,i}) =: (\check{x}^*_{2,i}, \check{y}^*_{2,i})$.

Appendix B. The incumbent's programme

By (13) and (27), the first-order condition (28) can be written: $2\beta h \tau \varepsilon^I \lambda(1 + r)\phi(D_1) = 0$, where

$$\phi(D_1) := 2\kappa\rho\bar{R} + \tilde{A} - \Psi - 2\kappa\rho D_{1,i}, \tag{B.1}$$

with $\rho := \bar{B}(\tilde{A} - \sigma\tilde{B})$, $\Psi := 1/4\beta h\tau$, and $\bar{R} := R/2\lambda(1+r)\varepsilon^I$.

Clearly, $\phi'(D_1) = -2\kappa\rho$, and the sign of ρ determines the sign of the second-order derivative. If $\rho > 0$, ϕ is strictly decreasing, the objective function $\mathbb{E}[V]$ of the incumbent is strictly concave, and, if there is a critical point in $(0, \bar{D}_1)$, it defines the unique global maximum. If $\rho \leq 0$, in contrast, ϕ is increasing, and there is no interior maximum. Let us characterize the solution of the maximization problem in these two cases, respectively.

Appendix B.1. Strict concavity: $\tilde{A} > \sigma\tilde{B}$.

Eq. (B.1) defines the unique critical point

$$\check{D}_1 = \bar{R} + \frac{\tilde{A} - \Psi}{2\kappa\bar{B}(\tilde{A} - \sigma\tilde{B})}.$$

We must ensure that $\check{D}_{1,i} \in (0, \overline{D}_{1,i})$. First, by (B.1), we have $\phi_i(0) > 0$ if and only if

$$\tilde{A} > f_0(\tilde{B}) := \gamma_1\sigma\tilde{B} + \tilde{A}_1, \tag{B.2}$$

where

$$\gamma_1 := \frac{2\kappa\bar{R}\bar{B}}{2\kappa\bar{R}\bar{B} + 1} \in (0,1), \text{ and } \tilde{A}_1 := \frac{\Psi}{2\kappa\bar{R}\bar{B} + 1} > 0.$$

Second, by (B.1), we have $\phi(\bar{D}_1) < 0$ if and only if

$$\tilde{A} < f_m(\tilde{B}) := \delta_1\sigma\tilde{B} + \tilde{A}_2, \tag{B.3}$$

where,

$$\delta_1 := \frac{2\kappa\bar{B}(\bar{R} - \bar{D}_1)}{2\kappa\bar{B}(\bar{R} - \bar{D}_1) + 1} < \gamma_1, \text{ and } \tilde{A}_2 := \frac{\Psi}{2\kappa\bar{B}(\bar{R} - \bar{D}_1) + 1} > \tilde{A}_1.$$

We assume that $\bar{R} > \bar{D}_1$, namely $R > 2\lambda(1+r)\varepsilon^I\bar{D}_1$ (which is true for small value of ε^I). There is a unique point, denoted by $(\tilde{A}_0, \tilde{B}_0) \in \mathbb{R}^2_+$, such that $\tilde{A}_0 = f_0(\tilde{B}_0) = f_m(\tilde{B}_0) = \sigma\tilde{B}_0$. We compute the coordinates of this point by taking $\rho = 0$ in (B.1); thus: $\tilde{B}_0 = \Psi/\sigma$, and $\tilde{A}_0 = \Psi$. In other words, the curves $\tilde{A} = f_0(\tilde{B})$, $\tilde{A} = f_m(\tilde{B})$, and $\tilde{A} = \sigma\tilde{B}$ intersect once at the point $(\tilde{A}_0, \tilde{B}_0) = (\Psi, \Psi/\sigma)$, as in Figure C.1. Besides, in the $(\tilde{A}, \tilde{B})$-plane, f_m and f_0 cross the x-axis at $\tilde{B}_1 := -\tilde{A}_2/\sigma\delta_1$, and $\tilde{B}_2 := -\tilde{A}_1/\sigma\gamma_1$, respectively, with $\tilde{B}_1 < \tilde{B}_2$.

Consequently, if $\tilde{A} > \sigma\tilde{B}$, the objective function ($\mathbb{E}[V]$) is strictly concave, the first-order derivative is a decreasing continuous function, and the maximum D_1^* in $[0, \bar{D}_1]$ is characterized by the following three cases.

i. If $\tilde{A} \leq f_0(\tilde{B})$, $\phi(0) \leq 0$, and $D_1^* = 0$.
ii. If $\tilde{A} \geq f_m(\tilde{B})$, $\phi(\bar{D}_1) \geq 0$, and $D_1^* = \bar{D}_1$.
iii. If $f_0(\tilde{B}) < \tilde{A} < f_m(\tilde{B})$, $\phi(\bar{D}_1) < 0 < \phi(0)$, and $D_1^* = \check{D}_1 \in (0, \bar{D}_1)$.

In this respect, we can introduce the non-empty subset

$$\mathscr{C}_{int} := \Big\{(\tilde{A}, \tilde{B}) |\ \tilde{A} > \sigma\tilde{B};\ f_0(\tilde{B}) < \tilde{A} < f_m(\tilde{B})\Big\} \subset \mathbb{R}^2,$$

which defines the set of couple $(\tilde{A}, \tilde{B})$ giving rise to the unique interior solution $D_{1,i}^* \in (0, \bar{D}_1)$.

Appendix B.2. Convexity: $\tilde{A} \leq \sigma\tilde{B}$

In this case, the objective function ($\mathbb{E}[V]$) is convex, the first-order derivative (ϕ) is a continuous and increasing function, and the global maximum is reached at the bound, namely $D_1^* \in \{0, \bar{D}_1\}$. Therefore, we can distinguish the three following cases.

i. If $\tilde{A} \geq f_0(\tilde{B})$, $\phi(0) \geq 0$, and $D_1^* = \bar{D}_1$.
ii. If $\tilde{A} \leq f_m(\tilde{B})$, $\phi(\bar{D}_1) \leq 0$, and $D_1^* = 0$.
iii. If $f_m(\tilde{B}) < \tilde{A} < f_0(\tilde{B})$, $\phi(0) < 0 < \phi(\bar{D}_1)$, and there is an unique interior minimum in $(0, \overline{D}_{1,i}^I)$; hence $D_1^* = \text{argmax}\{\mathbb{E}[V](0),\ \mathbb{E}[V](\bar{D}_1)\}$.
   By (27), as $\tilde{A} \leq \sigma\tilde{B}$, we have: $\mu(0) > \mu(\bar{D}_1)$. By (13), as e^* increases in D_1, we have: $e^*(0) < e^*(\bar{D}_1)$. Consequently, it follows that

$$R + \beta\mu(0)[\tilde{R} - e^*(0)] = \mathbb{E}[V](0) > \mathbb{E}[V](\bar{D}_1) = R + \beta\mu(\bar{D}_1)[\tilde{R} - e^*(\bar{D}_1)],$$

   hence, $D_1^* = 0$.

Appendix B.3. Equilibrium Characterization

Summing up, we can characterize the equilibrium in Figure C.1. By overlapping the convex and the concave analysis, we can introduce the two following non-empty subsets

$$\mathscr{C}_0 := \{(\tilde{A}, \tilde{B})|\ \tilde{A} \leq f_0(\tilde{B})\}, \tag{B.4}$$

$$\mathscr{C}_m := \{(\tilde{A}, \tilde{B})|\ \tilde{A} \geq f_m(\tilde{B})\ ; \tilde{A} \geq f_0(\tilde{B})\}. \tag{B.5}$$

Therefore, if $(\tilde{A}, \tilde{B}) \in \mathscr{C}_0$, then $D_1^* = 0$; and if $(\tilde{A}, \tilde{B}) \in \mathscr{C}_m$, then $D_1^* = \bar{D}_1$. Besides, as we have shown above, we have clearly: $\mathscr{C}_0 \cup \mathscr{C}_m \cup \mathscr{C}_{int} = \mathbb{R}^2$, $\mathscr{C}_0 \cap \mathscr{C}_m = \emptyset$, $\mathscr{C}_0 \cap \mathscr{C}_{int} = \emptyset$, and $\mathscr{C}_{int} \cap \mathscr{C}_m = \emptyset$, as described in Figure C.1.

Appendix C. Social Welfare

Let $\rho := \bar{B}(\tilde{A} - \sigma\tilde{B})$. Using Eq. (31), the objective of the social planner $D_1 \mapsto \mathbb{E}[W](D_1)$ is a quadratic function, and describes an U-shaped curve. The solution of the maximization problem is then

$$\max_{D_1 \in [0, \bar{D}_1]} \mathbb{E}[W] = \max\left\{\mathbb{E}[W](0)\ ;\ \mathbb{E}[W](\bar{D}_1)\right\}.$$

From Eq. (31), we obtain, using (19),(20), (22) and (26)

$$\mathbb{E}[W](0) = -2\tau M\beta\tilde{A}\left[\frac{1}{2} - 2h\tau\beta\tilde{A}\right] + \beta\tau M(1 - A^O) + \tau M(1 - A^I), \tag{C.1}$$

and

$$\mathbb{E}[W](\bar{D}_1) = 2\tau M\beta[\kappa\rho\bar{D}_1 - \tilde{A}]\left[\frac{1}{2} + 2h\tau\beta(\kappa\rho\bar{D}_1 - \tilde{A})\right] + \beta\tau M[1 - \kappa(B^O - 1)\bar{D}_1)] + \tau M[1 - \beta(1+r)]\bar{D}_1 + \tau M\left[1 - A^I - \frac{\kappa A^I\bar{D}_1}{1+r}\right]. \quad \text{(C.2)}$$

Let us introduce $\Phi(\rho) := \mathbb{E}[W](\bar{D}_1) - \mathbb{E}[W](0)$. From (C.1), $\mathbb{E}[W](0)$ is independent of ρ. From (C.2), $\mathbb{E}[W](\bar{D}_1)$ is quadratic in ρ; hence, there is a critical value $\overline{\rho} \in \mathbb{R}$ such that $\mathbb{E}[W](\bar{D}_1)$ increases in ρ if and only if $\rho \geq \overline{\rho}$, and $\lim_{\rho\to\pm\infty}\mathbb{E}[W](\bar{D}_1) = +\infty$. In this way, Φ is a quadratic continuous function in ρ, where $\Phi'(\rho) \geq 0 \Leftrightarrow \rho \geq \overline{\rho}$, and $\lim_{\rho\to\pm\infty}\Phi(\rho) = +\infty$.

Let $\beta = 1/(1+r)$. By (C.1) and (C.2), we have

$$\Phi(0) = \beta\left\{\tau MA^O - \kappa\tau M(B^O - 1)\bar{D}_1 - \kappa A^I\bar{D}_1\right\}.$$

Hence, $\Phi(0) < 0 \Leftrightarrow A^O - \kappa(B^O - 1)\bar{D}_1 - \kappa A^I\bar{D}_1 < 0 \Leftrightarrow (\bar{A} - \tilde{A}) - \kappa(\bar{B} - \tilde{B} - 1)\bar{D}_1 - \kappa(\bar{A} + \tilde{A})\bar{D}_1 < 0$. As $\rho := \bar{B}(\tilde{A} - \sigma\tilde{B}) = 0$, we have $\tilde{A} = \sigma\tilde{B}$, and

$$\Phi(0) < 0 \Leftrightarrow (\bar{A} - \sigma\tilde{B}) - \kappa(\bar{B} - \tilde{B} - 1)\bar{D}_1 - \kappa(\bar{A} + \sigma\tilde{B})\bar{D}_1 := \varphi(\tilde{B}) < 0$$

We can compute $\varphi'(\tilde{B}) = -\sigma + \bar{D}_1(1-\sigma)$, which is assumed to be positive. Since $\sigma\tilde{B} \leq \bar{A}$ (because $A^0 \geq 0$), it follows that: $\max\varphi(\tilde{B}) = \varphi(\bar{A}/\sigma) = -\kappa(\bar{B} - \bar{A}/\sigma - 1)\bar{D}_1 - 2\kappa\bar{A}\bar{D} < 0$, for any $(\tilde{A}, \tilde{B})$; hence, $\Phi(0) < 0$.

Consequently, according to the Intermediate Value Theorem, there are two critical values: $Z_1(\tilde{A}) < 0$ and $Z_2(\tilde{A}) > 0$, such that $\Psi(\rho) < 0$, namely $\mathbb{E}[W](\bar{D}_1) < \mathbb{E}[W](0)$ iff $Z_1(\tilde{A}) < \rho < Z_2(\tilde{A})$, for any $\tilde{A} \in \mathbb{R}$. In other words, $D_1^{SW} = 0$ iff

$$K_1(\tilde{A}) := \frac{1}{\sigma}\left(\tilde{A} - \frac{Z_2(\tilde{A})}{\bar{B}}\right) < \tilde{B} < K_2(\tilde{A}) := \frac{1}{\sigma}\left(\tilde{A} - \frac{Z_1(\tilde{A})}{\bar{B}}\right)$$