Dynamic Panels with MIDAS Covariates: Nonlinearity, Estimation and Fit*

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Abstract

This paper introduces Mixed Data Sampling (MIDAS) to panel regressions suitable for analysis with GMM of the Arellano-Bond form. As MIDAS specification tests are lacking, even in univariate contexts, our proposed methods statistically embed specification checks. We proceed via confidence set estimation for the MIDAS parameter for which empty outcomes signal lack of fit. Conformably, simultaneous and partialled-out confidence sets for the regression coefficient are proposed that address the unidentified boundary parameter (Davies' problem). Inverted statistics are asymptotically pivotal. A simulation study illustrates good size and power and, more broadly, sets a promising template for dealing with shrinkage parameters.

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1 Introduction

In general, a time dependent regression requires that all the variables have the same time interval (for example, annual). When a regressor is sampled at a higher frequency (monthly or weekly), the common practice is to adopt a pre-specified weighting scheme, for example equal weights or mid-point observation, resulting in a single frequency for all variables. However, such assumptions can potentially lead to a loss of information in the high frequency data or specification biases. To correct for such problems, Ghysels, Santa-Clara, and Valkanov (2004) introduce the Mixed Data Sampling (MIDAS) framework which by now is well established in econometrics. For recent extensions and applications of MIDAS, we draw the reader's attention to a Journal of Econometrics special edition (August 2016), specifically in terms of multivariate models see Ghysels (2016), Götz, Hecq, and Smeekes (2016), Zadrozny (2016), and Qian (2016), and more generally to Clements and Galvão (2008), Kuzin, Marcellino, and Schumacher (2011), Bai, Ghysels, and Wright (2013), and Guérin and Marcellino (2013), and Foroni and Marcellino (2013) for a comprehensive survey of recent advances.

One option, the unrestricted MIDAS (UMIDAS) estimation by Foroni, Marcellino, and Schumacher (2013), is to use all the high frequency data points as regressors. This requires sufficently large degrees of freedom particularly as high frequency data become more prevalent. As an alternative to shrinkage techniques, such as LASSO and Bayesian shrinkage, and principle components, the Mixed Data Sampling (MIDAS) technique, of Ghysels et al. (2004), provides a means of extracting more high frequency information in a lower frequency regression. This is accomplished by assuming unequal weights for each of the high frequency observations, and a polynomial aggregation structure that maximizes the retained information while limiting the degrees of freedom losses.

The Almon distributed lag polynomials allow for a variety of weighting schemes, such as proportionally more weight on recent or older observations, equally weighted observations, and hump-shaped weights. These weight profiles are obtained at a minimum cost of a twodimensional parameters, which we define for further references as the MIDAS parameter (θ). To set focus, consider a simple panel regression with one high-frequency covariate which nonlinearly embeds θ and where the associated regression coefficient, denoted as β, is the parameter of interest. Non-linearties in parameters complicate identification. In particular, θ is weakly identified in the near-zero β sub-space. This is the well known Davies (1977, 1987) problem which remains to be addressed in the MIDAS literature, even in the univariate case. This motivates our work.

Furthermore, with the exception of a few recent applications to the Vector Autoregression models, Ghysels (2012) and Binder and Krause (2014), the bulk of this literature remains univariate. In particular, mixed frequency methods dealing with panel data are lacking. We introduce MIDAS to dynamic panel regression models suitable for analysis with GMM methods of the Anderson and Hsiao (1982) and Arellano and Bond (1991) form. This is the first formal extension of MIDAS methods to the panel context. Time series procedures are not guaranteed to extend to dynamic panels, as consideration must be taken to account for the dual indexing of observations resulting in an incidential parameter bias problem. In particular lags may cause various problems, including spurious seasonality as emphasized by Clements and Galvão (2008, 2009). In GMM panel framework, the instruments are lags of the endogenous variables and regressors. The lag structure will thus interfere by construction. In addition, with Almon polynomials specification checks are important because the Almon form introduces nonlinear nuisance parameters. Formal specification checks are rare for MIDAS, even in the univariate case. Lag and nuisance parameter specification issues provide further motivation for our work. This paper contributes to this literature in various ways.

First, we construct a confidence set estimate for the MIDAS parameter, θ, for which empty outcomes signal lack of fit. Although θ is unidentified over a relavant parameter space subset, a fixed θ is compatible with a Sargan test in the framework of Arellano and Bond (1991). For a fixed θ the MIDAS regressor becomes an observable aggregation of the high frequency series. We thus propose to invert the Arellano-Bond Sargan statistic that fixes θ

to a known value, say θ_0. The confidence set for θ is defined as the set of θ_0 values that are not rejected by the modified-Sargan test at the desired level of significance. When $\theta = \theta_0$ the Sargan test is asymptotically pivotal under standard regularity conditions, regardless of the considered data frequency.

Next, we propose conformable confidence sets for the panel regression coefficient, denoted β, controlling for the nonlinearity that results from treating θ as a free nuisance parameter. We put forth three inference methods: (a) joint simultaneous confidence region for θ and β via a modified-Sargan statistic, (b) partialled-out test and confidence set for β obtained by inverting a supremum p-value based on GMM t-statistics, and (c) test and confidence set for β obtained by combining inversion and union-intersection methods. These methods address the nuisance parameter problem resulting from the MIDAS specification including a form of the Davies (1977, 1987) problem, specifically that θ cannot be identified under certain values of the other parameters.

Furthermore, we allow for empty outcomes which statistically embeds specification checks. We thus jointly address specification and the underlying nonlinearity, as has recently been emphasized in the weak-IV literature; see Bun and Windmeijer (2010) for a panel data application, and more generally Stock and Wright (2000), Dufour (1997), Kleibergen (2005), and Ghysels and Wright (2010).

We conduct a simulation study to illustrate promising size and power properties for our procedures. The simulations focuses on exponential Almon lag polynomials.The simulation study employs a special case where the covariance structure of the high frequency observations is nonzero at the low frequency. At first glance this may seem restrictive, however, this unique special case allows for a decomposition of moment condition violations under the alternative, which would not be possible under a more general (and more common to applied analysis) covariance structure. The special case predicts that certain combinations of θ and θ_0 will produce exact zeros in one of the two moments. The strength of individual sets of moments can be then assessed. We find that if the data is generated under unequal weights and the null imposes equal weights then the rejection frequency is large. For inference on β the simultaneous and partialled-out tests are comparable power wise, and controls size whether θ is identified or not.

Overall, our results show concrete promise for panel-MIDAS and, more broadly, for semi-parametric shrinkage methods in which several lags or data frequency make unrestricted estimation costly. The problem of many lags or frequencies is wide-spread and semi-parametric methods could prove more useful in a time-series setting than sparse methods. We provide a formal inference-based template for dealing with shrinkage parameters, by inverting specification tests. With panel models, the information content of Arellano-Bond type methods on specification are well understood. Our method can be extended to non-dynamic panels or univariate time-series models by inverting for example, tests for neglected dynamics.

We present our implementation of MIDAS regressors into the dynamic panel context in Section 2, and introduce our modified-Sargan statistic and limiting distribution. Section 3 presents the moment conditions that are specific to the MIDAS parameters. A special case for the data generating process is outlined in Section 4 and violation of the moment conditions are examined under a variety of data structures. Section 5 describes three methods of obtaining inference on β that are robust to the model's nonlinearity and Davies problem. Section 6 outlines a simulation study to examine the rejection frequency properties of our method. Section 7 provides final comments and outlines the next steps in mixed-frequency dynamic panels.

2 Dynamic Panel with MIDAS regressors

Our proposed method introduces MIDAS regressors into the dynamic panel framework, and the estimation method of Arellano and Bond (1991). To set focus, we consider a prototypical single-regressor dynamic panel model of the form:

$$y_{i,t} = \delta y_{i,t-1} + \beta x_{i,t}(\theta) + u_{i,t}, \tag{1}$$

where the MIDAS aggregate of the high frequency observations is $x_{i,t}(\theta)$. The dependent series is a function of θ due to the recursion, $y_{i,t} := y_{i,t}(\theta)$, but we use the former in the main body of the text for ease of notation. For each MIDAS aggregate there are m high frequency observations, denoted as $x_{i,t,j}$, where $j = 1, 2, \ldots, m$. The MIDAS aggregation is given by

$$x_{i,t}(\theta) = \sum_{j=1}^{m} x_{i,t,j} w_j(\theta), \tag{2}$$

where $w_j(\theta)$ represents the high frequency MIDAS weights. We follow Ghysels, Santa-Clara, and Valkanov (2004), defining the weights as exponential Almon lag polynomial of length m, which can be less than or equal to the high frequency periodicity.

The exponential Almon distributed lag structure [Almon (1965)] is a very simple functional form of $w_j(\theta)$. The general form of the exponential Almon lag uses h parameters, where $\theta = \{\theta_1, \theta_2, \ldots, \theta_h\}$, so the weighing function is defined as:

$$w_j(\theta) = \frac{e^{\theta_1 j + \cdots + \theta_h j^h}}{\sum_{k=1}^{m} e^{\theta_1 k + \cdots + \theta_h k^h}} \tag{3}$$

The functional form ensures that the weights sum to one and the shape of the weights is controlled by the values of θ. The equal weight assumption is recovered when $\theta_i = 0$ for all $i = 1, 2, \ldots, q$,

$$w_j(\mathbf{0}) = \frac{e^{0j + \cdots + 0j^h}}{\sum_{k=1}^{m} e^{0k + \cdots + 0k^h}} = \frac{1}{m}. \tag{4}$$

The exponential Almon lag can describe a variety of patterns to the weights with only two parameters, which we exploit in our simulation design below.

The error term in equation (1), $u_{i,t}$, is in line with the standard dynamic panel framework of the form,

$$u_{i,t} = \mu_i + \nu_{i,t}, \tag{5}$$

where $\mu_i \sim iid(0, \sigma_\mu^2)$ and $\nu_{i,t} \sim iid(0, \sigma_\nu^2)$, and these two components are independent.

The Arellano-Bond method transforms equation (1) by taking the first difference in t, leading to

$$\Delta y_{i,t} = \delta \Delta y_{i,t-1} + \beta \Delta x_{i,t}(\theta) + \Delta \nu_{i,t}, \tag{6}$$

in which unobserved heterogeneity, μ_i, is eliminated. The MIDAS weights on the high frequency observations are nonlinear except when the weights are equal, but the aggregation structure is linear in variables allowing for the following representations

$$\Delta x_{i,t}(\theta) = \Delta \left(\sum_{j=1}^{m} x_{i,t,j} w_j(\theta) \right) \tag{7}$$

$$= \sum_{j=1}^{m} \left(x_{i,t,j} - x_{i,t-1,j} \right) w_j(\theta). \tag{8}$$

The transformation in (6) results in nonzero correlation between $\Delta y_{i,t-1}$ and $\Delta \nu_{i,t}$. This endogeneity is controlled via a 2-step GMM framework with appropriate lags of $y_{i,t}$ as valid instruments, in addition to available *strictly* or predetermined exogenous regressors.

The model is formulated so that the high frequency observations, $x_{i,t,j}$, intervene only through the MIDAS aggregation. In addition to the advantages pointed out in Ghysels, Santa-Clara, and Valkanov (2006), this framework in panel has two important consequences. First, the resulting lags of the MIDAS regressors are valid instruments in the Arellano-Bond sense. Second, equations (1) to (6) can fit within the Arellano-Bond regularity framework conditional on θ, if θ is fixed. Our approach builds on these two properties. For later reference, let W represent the instrument matrix, and W_i represent the submatrix for the i^{th} cross-section.

Assume that the Arellano-Bond regularity conditions hold for the special case of equation

(6) that sets θ to a known value say θ_0. This assumption forms the basis of our procedure. The MIDAS aggregates are functions of θ_0 as defined by

$$x_{i,t}(\theta_0) = \sum_{j=1}^{m} x_{i,t,j} w_j(\theta_0). \tag{9}$$

With a known θ, the MIDAS regressor at the same frequency of $y_{i,t}$ does not require estimation and can be treated as a usual observable. The errors from equation (6) are functions of θ_0,

$$\Delta\nu_{i,t}(\theta_0) = \Delta y_{i,t} - \delta\Delta y_{i,t-1} - \beta\Delta x_{i,t}(\theta_0) - \beta\left(\Delta x_{i,t}(\theta) - \Delta x_{i,t}(\theta_0)\right), \tag{10}$$

so the estimated 2-step GMM errors will be functions of θ_0 by construction. Under the null, $H_0 : \theta = \theta_0$, equation (10) simplifies to

$$\Delta\nu_{i,t}(\theta_0) = \Delta y_{i,t} - \delta\Delta y_{i,t-1} - \beta\Delta x_{i,t}(\theta_0). \tag{11}$$

The modified-Sargan statistic, which we denote $\mathcal{J}(\theta_0)$, will be applicable, since the value of θ is fixed, and will follow a standard χ^2 distribution (under the Arellano-Bond standard asymptotics), specifically

$$\mathcal{J}(\theta_0) = (\Delta\tilde{\nu}(\theta_0))' W [\tilde{V}_N(\theta_0)]^{-1} W' (\Delta\tilde{\nu}(\theta_0)) \sim \chi^2(\tau - K - 1), \tag{12}$$

where the 'tilde' indicates an estimate from the GMM second step, τ is the number of instruments in W,[1] K is the number of exogenous regressors, and the weighting matrix is defined as

$$\tilde{V}_N(\theta_0) = \sum_{i=1}^{N} W_i'(\Delta\tilde{\nu}_i(\theta_0))(\Delta\tilde{\nu}_i(\theta_0))' W_i. \tag{13}$$

We thus propose to invert the test based on $\mathcal{J}(\theta_0)$. Inverting a test means assembling

[1] See Appendix for the construction and definition of W with MIDAS regressors.

the set of parameter values that are not rejected by this test.[2] In traditional estimation methodology a point estimate is found first and confidence intervals are then constructed. In contrast, first we build a confidence set, then we search for a point estimate [as explained below] within this set. The confidence region, at level $(1-\alpha)$, say 95%, is obtained by inverting $\mathcal{J}(\theta_0)$, which is specifically designed so that, despite the MIDAS structure, its significance level remains at α (in this case, 5%). A point estimate can be obtained by selecting those parameter values that produce the largest test p-value from within the above defined non-rejected set. In particular, if the generated confidence region is empty, the inclusion of MIDAS regressors can be considered rejected at the considered test level. The intuition is that no value of θ - within the considered family - will aggregate the higher frequency data into a model that passes the Arellano-Bond specification test at the considered level. This provides a built-in specification check for the fit of the MIDAS structure within the considered dynamic panel set-up.

The test inversion must be conducted numerically. In the MIDAS literature, the dimension of θ is low (one or two parameters typically). Thus, one can sweep, in turn, the choices for θ_0, and for each choice considered, compute $\mathcal{J}(\theta_0)$, and its associated p-value. The parameter vectors for which the p-values are greater than the level α collected together constitute our proposed confidence region with level $1-\alpha$. Moving from the joint confidence region to individual confidence interval for the components of θ is achieved by projecting this region, i.e. by computing, in turn, the smallest and largest values for each parameter included in this region. Conceptually, a grid search, particle swarm, or simulated annealing could be implemented over a meaningful set of values for θ. A point estimate can also be obtained from the joint confidence set. This corresponds to the model that is most compatible with the data, or, alternatively, that is least-rejected, and is given by the vector of parameter values

[2]To be fully identification-robust which includes weak-instrument and near-unit root issues, the objective function would need to be inverted; see e.g. Stock and Wright (2000), and Kleibergen (2005). These methods have yet to be formally extended to the Panel context; a notable exception is the Panel AR(1) work of Bun and Kleibergen (2016) which to the best of our knowledge excludes regressors.

with the highest p-value in the set.

Projection-based confidence sets are obtained numerically as follows. By definition, a projection-based confidence set can be obtained for any function $g(\theta)$ by minimizing and maximizing the function $g(\theta)$ over the θ values included in the joint confidence region. We thus define each component of θ as a linear combination of θ, of the form $g(\theta) = a'\theta$, where a is a conformable selection vector (consisting of zeros and ones); for example, $\omega_f = (1, 0, \ldots, 0)\,\theta$. We then obtain the projection set by numerical optimization of the associated $a'\theta$ function over θ such that $\mathcal{J}(\theta)$ is less than or equal to its χ^2 cut-off. The latter treats θ as fixed and thus depends on the number of instruments, which in this framework includes lags of the aggregated regressor fixing θ. Let us denote this cut-off by χ^2_α. This implies that the cut-off point for the $\mathcal{J}(\theta_0)$ statistic is the same for any value θ_0 under test. Since it is the case that

$$\min_{\theta_0} \mathcal{J}(\theta_0) \geq \chi^2_\alpha \Leftrightarrow \mathcal{J}(\theta_0) \geq \chi^2_\alpha, \quad \forall \theta_0. \tag{14}$$

Then referring the latter statistic to the χ^2_α cut-off point provides our proposed specification test.

3 Identification of the MIDAS parameter

The previous section considered the model only under the null that $\theta = \theta_0$, in this section we also allow for the model under the alternative. We define the first difference errors from equation (10) as,

$$\Delta\omega_{i,t}(\theta, \theta_0) = \Delta v_{i,t} + (\Delta x_{i,t}(\theta) - \Delta x_{i,t}(\theta_0))\,\beta. \tag{15}$$

These are used to construct the moment conditions of the Arellano-Bond estimator, which are defined from,[3]

$$E\left[W(\theta,\theta_0)'\Delta\omega(\theta,\theta_0)\right] = 0, \tag{16}$$

and lead to the following two sets of moments,

$$A_{t,r} \;:=\; E\left[\sum_{i=1}^{N} y_{i,t-r}(\theta)(\omega_{i,t}(\theta,\theta_0)-\omega_{i,t-1}(\theta,\theta_0))\right], \text{ and} \tag{17}$$

$$B_{t,q} \;:=\; E\left[\sum_{i=1}^{N} x_{i,t-q}(\theta_0)(\omega_{i,t}(\theta,\theta_0)-\omega_{i,t-1}(\theta,\theta_0))\right], \tag{18}$$

where $r = 2,\ldots,t-1$ and $q = 0,\ldots,t-1$, and under the null the following moment conditions must be satisfied $A_{t,r} = 0$ and $B_{t,q} = 0$.

The covariance of the high frequency exogenous observations, is defined as,

$$\Gamma(q,p) = E\left[\sum_{i=1}^{N} x_{i,t,j}x_{i,t-q,j-p}\right], \tag{19}$$

where q and p are the difference between the low-frequency and high-frequency time indexes, respectively.[4] Under the regularity assumptions of the Arellano-Bond framework, the following component of $A_{t,r}$ is

$$E\left[\sum_{i=1}^{N} y_{i,t-r}(\theta)\Delta v_{i,t}\right] = 0, \tag{20}$$

and for $B_{t,q}$ the strict exogeneity assumption will ensure that

$$E\left[\sum_{i=1}^{N} x_{i,t-q}(\theta_0)\Delta v_{i,t}\right] = 0, \tag{21}$$

under both the null and alternative for θ and θ_0. Applying these moment conditions and our definition of the covariance of the MIDAS regressors, equation (19), lead to the following

[3]Refer to equation (71) in the Appendix for definition of $W(\theta,\theta_0)$.

[4]The variance at the high and low frequency are given by $\Gamma(q,p) = \Gamma(0,0)$ for covariance stationary series.

lemma.[5]

Lemma 1 (MIDAS Moments) *Under the Arellano-Bond framework, a dynamic panel model with MIDAS covariates gives the following two sets of moment conditions from the covariance of the MIDAS regressors with lags of dependent series,*

$$\begin{aligned} A_{t,r} &= \left(\sum_{j=1}^{m} \sum_{l=1}^{m} \Gamma(r-1, j-l) \left(w_j(\theta) w_l(\theta) - w_j(\theta) w_l(\theta_0) \right) \right) \beta^2 \\ &+ (1-\delta) \left(\sum_{k=0}^{t-r-1} \delta^k \sum_{j=1}^{m} \sum_{l=1}^{m} \Gamma(r+k, j-l) \left(w_j(\theta) w_l(\theta) - w_j(\theta) w_l(\theta_0) \right) \right) \beta^2, \end{aligned} \quad (22)$$

and the exogenous regressors,

$$B_{t,q} = \left(\sum_{j=1}^{m} \sum_{l=1}^{m} \left(\Gamma(q, j-l) - \Gamma(q+1, j-l) \right) \left(w_j(\theta_0) w_l(\theta) - w_j(\theta_0) w_l(\theta_0) \right) \right) \beta, \quad (23)$$

where $r \geq 2$, $q \geq 0$, θ is the true MIDAS parameter value, and θ_0 is an imposed MIDAS parameter value.

Under the null of $\theta = \theta_0$, we find that the moment conditions are satisfied, $A_{t,r} = 0$ and $B_{t,q} = 0$, since $w_l(\theta) = w_l(\theta_0)$ and $w_j(\theta_0) = w_j(\theta_0)$.

4 A Special Case under the Alternative

Under the alternative, the MIDAS moments of the general case, equations (22) and (23), can both be nonzero but the magnitude of each of the moments could vary significantly for different values of θ_0. To illustrate this effect, we introduce a special case for the data generation process of the MIDAS regressor. This allows us to decompose the influence of each MIDAS moments under a variety of alternative hypotheses. Our special case is defined as:

[5]See Appendix for derivation of Lemma 1.

1. $\Gamma(q,p) = 0$ for all $p \neq 0$, and

2. $\Gamma(q,0) \neq 0$ for all $q \geq 0$,

which means that the covariance between high frequency observations are zero except when the pair are a low frequency match (e.g., $x_{i,t,j}$ and $x_{i,t-q,j}$). This special case is used in the simulation study reported below.

The second and greater lag of the dependent series are suitable instruments in the Arellano-Bond method, which leads to the following moment condition,

$$\begin{aligned} A_{t,r} &= E\left[\sum_{i=1}^{N} y_{i,t-r}(\theta)(x_{i,t}(\theta) - x_{i,t}(\theta_0))\right]\beta \\ &\quad -E\left[\sum_{i=1}^{N} y_{i,t-r}(\theta)(x_{i,t-1}(\theta) - x_{i,t-1}(\theta_0))\right]\beta, \end{aligned} \tag{24}$$

and Lemma 2.

Lemma 2 (MIDAS Moment from Dependent Series) *Under the Arellano-Bond framework, a dynamic panel model with MIDAS covariates gives the following set of moment conditions from the covariance of the MIDAS regressors with lags of dependent series,*

$$\begin{aligned} A_{t,r} &= \left(\Gamma(r-1,0)\sum_{j=1}^{m}\left(w_j(\theta)^2 - w_j(\theta)w_j(\theta_0)\right)\right)\beta^2 \\ &+ (1-\delta)\left(\sum_{k=0}^{t-r-1}\delta^k\Gamma(r+k,0)\sum_{j=1}^{m}\left(w_j(\theta)^2 - w_j(\theta)w_j(\theta_0)\right)\right)\beta^2, \end{aligned} \tag{25}$$

where $r \geq 2$, θ is the true MIDAS parameter value, and θ_0 is an imposed MIDAS parameter value.

In the context of a predetermined MIDAS exogenous regressor, all the lags and contemporaneous observations are valid instruments leading to the second set of moment conditions,

$$\begin{aligned} B_{t,q} &= E\left[\sum_{i=1}^{N} x_{i,t-q}(\theta_0)(x_{i,t}(\theta) - x_{i,t}(\theta_0))\right]\beta \\ &\quad -E\left[\sum_{i=1}^{N} x_{i,t-q}(\theta_0)(x_{i,t-1}(\theta) - x_{i,t-1}(\theta_0))\right]\beta, \end{aligned} \tag{26}$$

and gives Lemma 3.

Lemma 3 (MIDAS Moment from Exogenous Series) *Under the Arellano-Bond framework, a dynamic panel model with MIDAS covariates gives the following set of moment conditions from the covariance of the MIDAS regressors with constructed exogenous series,*

$$B_{t,q} = \left((\Gamma(q,0) - \Gamma(q+1,0)) \sum_{j=1}^{m} \left(w_j(\theta_0) w_j(\theta) - w_j(\theta_0)^2 \right) \right) \beta, \tag{27}$$

where $q \geq 0$, θ is the true MIDAS parameter value, and θ_0 is an imposed MIDAS parameter value.

The influence of the MIDAS parameters on the modified-Sargan statistic can range from none to relatively strong through the moment conditions defined in Lemma (2) and (3). The following theorems address special cases, and we choose not to focus on the trivial case where $w_j(\theta) = w_j(\theta_0)$, for all j, since this results in $A_{t,r} = 0$ and $B_{t,q} = 0$. In the context of the theorems presented below, θ represents the MIDAS parameters from the data generating process, while θ_0 are the MIDAS parameter values imposed under the null.

Theorem 1 (Equal MIDAS Weights) *In this case, θ generates **equal** MIDAS weights on each high frequency observation ($w_j(\theta) = 1/m$), where the null hypothesis imposes **unequal** weights, so*

$$H_0 : w_j(\theta_0) \neq \frac{1}{m} \text{ for any } j. \tag{28}$$

The MIDAS moment conditions are

$$\begin{aligned} A_{t,r} &= 0 \text{ and} && (29) \\ B_{t,q} &= (\Gamma(q,0) - \Gamma(q+1,0)) \left(\frac{1}{m} - \sum_{j=1}^{m} w_j(\theta_0)^2 \right) \beta. && (30) \end{aligned}$$

By theorem (1), the first moment condition is zero by construction thereby limiting the moment violations relative to the first difference of the low frequency correlation of the MIDAS regressors.

Theorem 2 (Null of Equal MIDAS Weights) *In this case, θ generates* ***unequal*** *MIDAS weights on each high frequency observation ($w_j(\theta) \neq 1/m$), where the null hypothesis imposes* ***equal*** *weights, so*

$$H_0 : w_j(\theta_0) = \frac{1}{m} \text{ for all } j. \quad (31)$$

The MIDAS moment conditions are

$$\begin{aligned} A_{t,r} &= \left(\Gamma(r-1,0) + (1-\delta) \sum_{k=0}^{t-r-1} \delta^k \Gamma(r+k,0) \right) \left(-\frac{1}{m} + \sum_{j=1}^{m} w_j(\theta)^2 \right) \beta^2 \text{ and} && (32) \\ B_{t,q} &= 0. && (33) \end{aligned}$$

By theorem (2), the second moment condition is zero by construction, and the first moment condition compounds the low frequency covariance and is weighted as a function of the lagged dependent parameter. The effect of the unequal weights on the MIDAS moments in theorems (1) and (2) lead to

$$0 \geq \frac{1}{m} - \sum_{j=1}^{m} w_j(\theta_0)^2 \geq \left(\frac{1}{m} - 1 \right) \text{ and } 0 \leq -\frac{1}{m} + \sum_{j=1}^{m} w_j(\theta)^2 \leq \left(1 - \frac{1}{m} \right). \quad (34)$$

Theorem 3 (Extreme Opposite MIDAS Weights) *In this case, both θ and θ_0 generate* ***unequal*** *MIDAS weights on each high frequency observation. However, the weighting schemes are extreme such that, $w_j(\theta)w_j(\theta_0) = 0$ for all j. The MIDAS moment conditions are*

$$A_{t,r} = \left(\Gamma(r-1,0) + (1-\delta)\sum_{k=0}^{t-r-1} \delta^j \Gamma(r+k,0)\right)\left(\sum_{j=1}^{m} w_j(\theta)^2\right)\beta^2 \text{ and} \tag{35}$$

$$B_{t,q} = -\left(\Gamma(q,0) - \Gamma(q+1,0)\right)\left(\sum_{j=1}^{m} w_j(\theta_0)^2\right)\beta. \tag{36}$$

Theorem 4 (Local Alternative MIDAS Weights) *In this case, θ and θ_0 generate* ***unequal*** *MIDAS weights on each high frequency observation. The weighting schemes are similar, $w_j(\theta)w_j(\theta_0) \neq 0$ for all j, but distinct such that $w_j(\theta)^2 - w_j(\theta)w_j(\theta_0) \neq 0$ and $w_j(\theta_0)w_j(\theta) - w_j(\theta_0)^2 \neq 0$. The MIDAS moment conditions are*

$$A_{t,r} \neq 0 \quad \text{and} \quad B_{t,q} \neq 0, \tag{37}$$

under the above assumptions and equal to formula presented in Lemma (2) and (3).

Theorems (3) and (4) are at the extremes of the weighting schemes under the alternative. Interestingly, $B_{t,q}$ could be either positive or negative under the alternative, and if this is the case then as the weights shift to the extreme opposite it could be zero. As the local weighting schemes tend towards equivalence in weighting schemes then the MIDAS moment conditions will tend to zero.

5 Inference on β

The previous sections focus on inference regarding the MIDAS parameter where the other parameters are estimated with the Arellano and Bond estimator. We now draw our attention to inference on the other parameters of the model. We show above that for a given value of θ, the other parameter estimates will have the same finite-sample properties and limiting distribution determined under the Arellano and Bond framework.[6] In this section, we exploit this fact to define adequate confidence sets for β, acknowledging θ formally as a nuisance parameter.

To begin, we apply a simple transformation to equation (1) that formalizes the joint and nonlinear inference problem at hand,

$$\begin{aligned} z_{i,t}(\beta_0, \theta_0) &= y_{i,t} - \beta_0 x_{i,t}(\theta_0) && (38) \\ &= \delta y_{i,t-1} + \beta x_{i,t}(\theta) - \beta_0 x_{i,t}(\theta_0) + u_{i,t} && (39) \\ &= \delta y_{i,t-1} + (\beta - \beta_0) x_{i,t}(\theta_0) + \beta(x_{i,t}(\theta) - x_{i,t}(\theta_0)) + u_{i,t}, && (40) \end{aligned}$$

where β_0 and θ_0 are known values of the parameters. The Arellano and Bond estimator is applied to this transformed model,

$$\begin{aligned} z_{i,t}(\beta_0, \theta_0) &= \delta y_{i,t-1} + (\beta - \beta_0) x_{i,t}(\theta_0) + \omega_{i,t} \text{ where} && (41) \\ \omega_{i,t} &= \beta(x_{i,t}(\theta) - x_{i,t}(\theta_0)) + u_{i,t}. && (42) \end{aligned}$$

We consider two methods with regards to inference on β and θ: (a) a union-intersection method using both the Sargan and GMM t-statistic on β, and (b) a bounds test using the latter.

The first method provides simultaneous inference on both β and θ, which coincides with the joint null,

$$H_{1,0} : \beta = \beta_0 \text{ and } \theta = \theta_0. \quad (43)$$

Since our objective is joint inference, then a goodness of fit measure is an appropriate test

[6] This could include taking into account the correction to the standard errors put forth in Windmeijer (2005).

statistic. The Sargan statistic, $\mathcal{J}(\beta_0, \theta_0)$, is suitable for testing this joint null because it can indicate poor model fit and/or over-identification. This approach differs from Bun and Kleibergen (2016) in that inversion over δ is not considered; asymptotics here retain the Arellano-Bond framework. The confidence interval for both parameters can be constructed in a similar manner to that outlined above, leading to

$$CI(\beta, \theta; \alpha) = \{\beta_0, \theta_0; \mathcal{J}(\beta_0, \theta_0) \geq \chi^2_\alpha(\tau - K - 1)\}, \tag{44}$$

which can be projected for inference on each parameter in turn as explained above.

The second and third methods are primarily concerned with inference on β, so their distinction is on the treatment of the nuisance parameter, θ. The null is

$$H_{2,0} : \beta - \beta_0 = 0, \tag{45}$$

which allows for testing via the t-statistic of $(\beta - \beta_0)$ in equation (41).

The second method combines inversion and union-intersection methods as follows. For a given MIDAS parameter, θ_0, there is an associated t-statistic, $\mathtt{t}((\beta - \beta_0); \theta_0)$. For a given β_0, we search for smallest t-statistic over the confidence set for θ, specifically

$$\mathtt{t}_C(\beta - \beta_0) = \inf_{\theta \in CI(\theta; \alpha_1)} \{ \ |\mathtt{t}((\beta - \beta_0); \theta)| \ \}, \tag{46}$$

where $CI(\theta; \alpha_1)$ is the confidence set for θ described in the previous section at given level of significance (α_1).[7] The confidence set for β is the collection of all β_0 where $\mathtt{t}_C(\beta - \beta_0)$ do not exceed the t-statistic critical value at significance level α_2,

$$CI(\beta; \alpha) = \{\beta_0; \mathtt{t}_C(\beta - \beta_0) < \mathtt{t}_{crit}(\alpha_2)\}, \tag{47}$$

[7] Subscript ‘C’ indicates the use of a confidence set for θ.

where the combined significance is approximately $\alpha = \alpha_1 + \alpha_2$. In this case, the significance level takes into account that θ is an estimated parameter.

The third method ignores sample-based information on θ and thus minimizes the t-statistic over the the entire MIDAS parameter space, denoted as Θ. The t-statistic of β is used to construct the confidence set in a similar manner as the second method, specifically[8]

$$\mathtt{t}_A(\beta - \beta_0) = \inf_{\theta \in \Theta} \{ \, |\mathtt{t}((\beta - \beta_0); \theta)| \, \} \text{ and} \quad (48)$$

$$CI(\beta; \alpha) = \{\beta_0; \mathtt{t}_A(\beta - \beta_0) < \mathtt{t}_{crit}(\alpha)\}. \quad (49)$$

The model, defined by equation 1, is nonlinear in the sense that β is multiplied to the MIDAS weights. The weights cannot be zero (must sum to one) but β can equal zero, which gives rise to a Davies (1977, 1987) problem. The Davies problem can be succinctly stated as, the case where nuisance parameters (θ) are not present under the null ($\beta = 0$) but are present under the alternative ($\beta \neq 0$). When $\beta = 0$, equation (1) is reduced to a dynamic panel model without regressors,

$$y_{i,t} = \delta y_{i,t-1} + u_{i,t}. \quad (50)$$

So including a MIDAS regressor in the Arellano and Bond framework would, by construction, result in over-identification.[9] This leads to Theorem 5.

Theorem 5 (MIDAS regressor not present) *Under the null that the dependent series does not depend on the MIDAS aggregated exogenous series,*

$$H_0 : \beta = 0, \quad (51)$$

[8] Subscript 'A' indicates the use of all possible values of θ.

[9] The addition of MIDAS regressors would be considered redundant, in the context of regressors and instruments.

then the modified-Sargan statistic is

$$\mathcal{J}(\theta_0) = \mathcal{J} \text{ for all } \theta_0 \in \Theta. \tag{52}$$

Under the null, $\beta = 0$, all values of θ satisfy the model, so identification is no longer possible since the moments $A_{t,r}$ and $B_{t,q}$ from Lemma (1) no longer enter into the model, and invalidates standard asymptotic theory and, in contrast, justifies the above simultaneous or partialled-out alternatives. Our proposed approach is in the spirit of the Davies solution, where we use our modified-Sargan test, $\mathcal{J}(\theta)$, and screen over the range of θ to construct a confidence set.

Furthermore, inference under the null of $\beta = 0$ remains valid: the transformed model in equation (41), the t-statistic for $(\beta - \beta_0)$, the inversion of the modified-Sargan test, $\mathcal{J}(\beta_0, \theta_0)$. This is true whether or not the confidence set of θ is the entire parameter space or a subspace constructed from the inversion of $\mathcal{J}(\theta_0)$.

6 Simulation Study

The simulations are drawn based on equation (1), and we implement the special case, described in the previous section, of the exogenous regressors for the construction of $x_{i,t}(\theta)$ given by:

$$x_{i,t,j} = \eta_i + \rho x_{i,t-1,j} + \epsilon_{i,t,j}. \tag{53}$$

It is apparent from the equation above that the high frequency autoregressive parameter is preserved for MIDAS aggregation.[10] This special case gives a clear demonstration of the theorem, and the effect of weighting schemes over the parameter space.

[10]Further discussion on the DGP of $x_{i,t}(\theta)$ is given in appendix A.

With the exception of θ, the null model parameters are taken from Arellano and Bond, which provides a degree of comparability. So the null model parameters[11] are:

$$\delta = 0.5, \qquad \beta = 1.0,$$
$$\eta = 0, \qquad \rho = 0.8.$$

The case of $\eta = 0$ assumes that there are no fixed effects in the MIDAS regressor.[12] The simulations draw from the standard normal distribution for each of the error terms, so $\mu_i \sim N(0, \sigma_\mu^2)$, $\nu_{i,t} \sim N(0, \sigma_\nu^2)$ and $\epsilon_{i,t,j} \sim N(0, \sigma_\epsilon^2)$, with variances set to:

$$\begin{aligned} \sigma_\mu^2 &= 1, \\ \sigma_\nu^2 &= 1, \\ \sigma_\epsilon^2 &= 0.9. \end{aligned}$$

The settings of the error varinances were taken from the Arellano and Bond simulation design. Bun and Windmeijer (2010) show that the variance ratio $(\sigma_\mu^2/\sigma_\nu^2)$ affects the consistency of the GMM estimates, however we remain true to the Arellano and Bond structure to ensure comparability.

This study uses a two-parameter exponential Almon weights, $\theta = [\theta_1, \theta_2]$, with five different assumptions:

- $\theta^A = [0, 0]$ - flat weights or arithmetic average,
- $\theta^B = [0.1, -0.2]$ - rapid decay with more weight on recent observations,
- $\theta^C = [0.03, -0.02]$ - slow decay with relatively more weight on recent observations, and
- $\theta^D = [-0.06, 0.01]$ - slow increase with relatively more weight on older observations.

[11] In their paper, alternative values of δ were examined, 0.2 and 0.8.
[12] This was also a feature of the Arellano and Bond simulation design.

- $\theta^E = [-0.04, 0.02]$ - rapid increase with relatively more weight on older observations.

The grid search parameter space, $\Theta = \{(|\theta_1| \leq 1), (|\theta_2| \leq 1)\}$, encompasses nearly all possible weighing schemes of a two-parameter exponential Almon lag. The number of high frequency observations is comparable to the number of trading days in a month, so $m = 20$.

Results are summarized in Tables 1-4 and more details are provided in the appendix. Table 1 shows that our procedure exhibits size control for all considered designs. At this stage, the results suggest that power is consistent with conventional wisdom as it increases in both N and T. However, the power appears to be more dependent on the second θ parameter such that grid parameter values that are more skewed than the true model exhibit power.[13]

Table 1: Rejection Frequency under the Null

Weight	DGP & Null		$N = 500$			$N = 1000$		
	θ_1	θ_2	$T = 5$	$T = 10$	$T = 15$	$T = 5$	$T = 10$	$T = 15$
older	-0.04	0.02	0.036	0.036	0.039	0.035	0.037	0.042
⇑	-0.06	0.01	0.042	0.034	0.044	0.035	0.046	0.035
equal	0	0	0.041	0.051	0.041	0.052	0.042	0.043
⇓	0.03	-0.02	0.042	0.049	0.040	0.046	0.053	0.050
recent	0.1	-0.2	0.028	0.051	0.049	0.020	0.046	0.042

Table 2: Rejection Frequency with $\theta_0 = (0, 0)'$

Weight	DGP		$N = 500$			$N = 1000$		
	θ_1	θ_2	$T = 5$	$T = 10$	$T = 15$	$T = 5$	$T = 10$	$T = 15$
older	-0.04	0.02	0.060	0.242	0.472	0.275	0.680	0.968
⇑	-0.06	0.01	0.043	0.077	0.091	0.077	0.109	0.225
equal	0	0						
⇓	0.03	-0.02	0.044	0.078	0.082	0.062	0.073	0.127
recent	0.1	-0.2	0.068	0.655	0.866	0.161	0.597	0.944

Table 3: Rejection Frequency with $\theta_0 = (1, 1)'$, $w_{20}(\theta_0) = 1$

Weight	DGP		$N = 500$			$N = 1000$		
	θ_1	θ_2	$T = 5$	$T = 10$	$T = 15$	$T = 5$	$T = 10$	$T = 15$
older	-0.04	0.02	0.046	0.051	0.063	0.03	0.043	0.06
⇑	-0.06	0.01	0.059	0.077	0.101	0.057	0.121	0.168
equal	0	0	0.06	0.058	0.06	0.047	0.038	0.092
⇓	0.03	-0.02	0.049	0.067	0.091	0.067	0.096	0.247
recent	0.1	-0.2	0.065	0.144	0.259	0.122	0.361	0.755

[13] Recall, this is an inversion of the specification test so the objective is to obtain the non-rejected region. The size results show that the true parameter values will be within this set.

Table 4: Rejection Frequency with $\theta_0 = (-1, -1)'$, $w_1(\theta_0) = 1$

Weight	DGP		$N = 500$			$N = 1000$		
	θ_1	θ_2	$T = 5$	$T = 10$	$T = 15$	$T = 5$	$T = 10$	$T = 15$
older	-0.04	0.02	0.062	0.192	0.738	0.031	0.51	0.987
$\Uparrow$	-0.06	0.01	0.07	0.06	0.088	0.038	0.117	0.235
equal	0	0	0.054	0.055	0.048	0.034	0.054	0.086
$\Downarrow$	0.03	-0.02	0.055	0.049	0.045	0.05	0.083	0.126
recent	0.1	-0.2	0.061	0.038	0.028	0.051	0.049	0.06

We examine the simulations under the alternative with respect to the theorems in the previous section. Theorem 1 is when the data generating process is equal weights, the power is close to size as shown by the middle rows of tables 3 and 4, and by the full results presented in table 7. Table 2 illustrates the rejection frequency when the simulations represent theorem 2, which shows that the rejection frequency rises as the true θ differs from the equal weights imposed under the null.

The results based on the theorems of extreme and local weighting schemes are presented in tables 3, 4, and tables 8 to 11. The rejection frequency under the alternative rises as the weights approach the extreme opposites (Theorem 3), so local alternatives (Theorem 4) have much lower rejection frequencies.

We now turn our attention to inference on β and results reported in Tables **??** and **??**. In Table **??**, data is drawn with $\beta = 0$ [that is from equation (50)]; numbers reported the rejection frequencies of the joint null at representative combinations of β and θ, for a nominal level of 5% and using the joint Sargan test based on equation (41) for various values of β_0 and θ_0. For each choice of θ_0, the tested value for β_0 is reported as a column heading, in which case zero values of β_0 pertain to size and non-zero to power. This setting is used to examine the effect of the Davies problem on inference regarding θ and β.

Table **??** reports rejections of the GMM t-statistic based on equation (41), where the level has been adjusted to 2.5%, to provide a fair assessment of the union-intersection method. As in Table **??**, data is drawn with $\beta = 0$ [that is from equation (50)]; the test is applied varying the tested value of β_0 while fixing θ to the value used to draw the data, which is reported in the left panel of the Table. On the one hand, this is done so that Tables **??** and **??** can be compared informatively. More to the point here, a design with oracle choices for θ, which is common in the literature on nuisance parameter dependent methods, will best allow us to justify the simultaneous or union-intersection methodologies as proposed. Indeed, such a design will reveal the severity of the nuisance parameter problem if rejection probabilities vary importantly across nuisance parameter choices. This is particularly important in our design since data is generated with $\beta = 0$ which evacuates θ completely from the DGP. Significance tests are particularly relevant empirically, and in view of the Davies problem, we purposely stress-test our proposed methods via this scenario.

Results of Tables **??** confirm that the Sargan statistic exhibits level control under the null, $\beta = 0$, for any combination of θ, as expected since all θ satisfy the model. Rejection frequency increases as β deviates from the zero null, and as θ deviates from equal weight aggregation. The rejection frequency for a bounds test on β based on the joint statistic over all relevant θ values is best indicated by the minimum rejection frequency in each column, which should be interpreted recalling that the true β is zero in the DGP regardless of the tested value of θ and β.

Results of Table **??** suggest that our proposed simultaneous and partialled-out procedures are comparable power-wise, and that both tests hold power when the model is identified, which in view of our theoretical analysis corresponds to unequal weights and a non-zero β. Since projections of multi-parameter regions are conservative, we find that the level-adjusted t-test fares equally well. The bounds t-test which as in Table **??** is illustrated via the minimum rejection frequency in each column, conveys the same information as the bounds Sargan test, which is important to verify given that β is zero for all simulated DGPs in this table as well. Overall, results confirm the usefulness of testable information on the equal weights and zero β null hypothesis, which would concretely produce unbounded confidence regions. The latter, in addition to empty outcomes reveal misspecification that should be flagged empirically.

Table 5: Rejection Frequency for $\beta = 0^\dagger$

Weight	θ_0		$\mathtt{t}(\beta - \beta_0)$ at 2.5% level				
	$\theta_{1,0}$	$\theta_{2,0}$	$\beta_0 = 0$	$\beta_0 = 0.5$	$\beta_0 = 1$	$\beta_0 = 1.5$	$\beta_0 = 2$
older	-0.04	0.02	0.004	0.530	0.904	0.959	0.984
⇑	-0.06	0.01	0.007	0.191	0.630	0.861	0.933
equal	0	0	0.010	0.070	0.339	0.633	0.801
⇓	0.03	-0.02	0.004	0.183	0.615	0.840	0.911
recent	0.1	-0.2	0.006	0.577	0.897	0.954	0.977
			Sargan Statistic at 5% level				
older	-0.04	0.02	0.024	0.024	0.024	0.024	0.024
⇑	-0.06	0.01	0.020	0.020	0.020	0.020	0.020
equal	0	0	0.015	0.015	0.015	0.015	0.015
⇓	0.03	-0.02	0.019	0.019	0.019	0.019	0.019
recent	0.1	-0.2	0.029	0.029	0.029	0.029	0.029
			$\inf \mathtt{t}(\beta - \beta_0)$ and Sargan at 2.5% level each				
	$[-1, 1]$	$[-1, 1]$	0.000	0.001	0.056	0.217	0.416

$\dagger T = 5$ and $N = 500$.

Table 6: Rejection Frequency for $\theta = (-0.04, 0.02)'$ and $\beta = 2^\dagger$

Weight	θ_0		$\mathtt{t}(\beta - \beta_0)$ at 2.5% level				
	$\theta_{1,0}$	$\theta_{2,0}$	$\beta_0 = 0$	$\beta_0 = 0.5$	$\beta_0 = 1$	$\beta_0 = 1.5$	$\beta_0 = 2$
older	-0.04	0.02	0.959	0.907	0.751	0.301	0.006
⇑	-0.06	0.01	0.898	0.843	0.755	0.614	0.400
equal	0	0	0.205	0.132	0.071	0.042	0.048
⇓	0.03	-0.02	0.026	0.054	0.176	0.369	0.576
recent	0.1	-0.2	0.024	0.165	0.562	0.808	0.920
			Sargan Statistic at 5% level				
older	-0.04	0.02	0.024	0.024	0.024	0.024	0.024
⇑	-0.06	0.01	0.094	0.094	0.094	0.094	0.094
equal	0	0	0.414	0.414	0.414	0.414	0.414
⇓	0.03	-0.02	0.420	0.420	0.420	0.420	0.420
recent	0.1	-0.2	0.418	0.418	0.418	0.418	0.418
			$\inf \mathtt{t}(\beta - \beta_0)$ and Sargan at 2.5% level each				
	$[-1, 1]$	$[-1, 1]$	0.007	0.004	0.001	0.000	0.000

$\dagger T = 5$ and $N = 500$.

7 Concluding Remarks

Our proposed method introduces MIDAS regressors into the context of the Arellano-Bond dynamic panel framework. We show that a modified-Sargan test can be used as a specification check on the MIDAS parameters. This modified-Sargan test can be inverted to obtain the confidence set for the MIDAS parameters. An empty confidence set indicates a lack of fit of the imposed model. Our method is examined in a simulation study, which demonstrates level control, local power for alternatives with similar weighing schemes, and power against an equal weight assumption.

Since available works on MIDAS specification tests are scarce, our results provide useful guidelines for further work. Our method is based on the asymptotic Sargan test as the specification test, and it may be possible to alter the framework to the minimum Anderson-Rubin test studied by Bun and Kleibergen (2016). These authors suggest that asymptotics in this context can be tricky, but worthy since their methodology is robust to unit root panels.

More broadly, this paper sets a promising template for dealing with shrinkage parameters, when unrestricted estimation is not desirable nor even possible. Specification checks in general hold concrete information on shrinkage parameters, that can be harvested to formally identify intervening parameters.

References

Shirley Almon. The distributed lag between capital appropriations and expenditures. *Econometrica: Journal of the Econometric Society*, pages 178–196, 1965.

Theodore Wilbur Anderson and Cheng Hsiao. Formulation and estimation of dynamic models using panel data. *Journal of econometrics*, 18(1):47–82, 1982.

Manuel Arellano and Stephen Bond. Some tests of specification for panel data: Monte carlo evidence and an application to employment equations. *The review of economic studies*, 58 (2):277–297, 1991.

Jennie Bai, Eric Ghysels, and Jonathan H Wright. State space models and MIDAS regressions. *Econometric Reviews*, 32(7):779–813, 2013.

Michael Binder and Melanie Krause. When is inequality detrimental to growth? - perspectives from a novel cross-country approach. *Manuscript*, 2014.

Maurice Bun and Frank Kleibergen. Identification and inference in moments based analysis of linear dynamic panel data models. Technical report, Working Paper, 2016.

Maurice JG Bun and Frank Windmeijer. The weak instrument problem of the system GMM estimator in dynamic panel data models. *The Econometrics Journal*, 13(1):95–126, 2010.

Michael P Clements and Ana Beatriz Galvão. Macroeconomic forecasting with mixed-frequency data: Forecasting output growth in the united states. *Journal of Business & Economic Statistics*, 26(4):546–554, 2008.

Michael P Clements and Ana Beatriz Galvão. Forecasting us output growth using leading indicators: An appraisal using MIDAS models. *Journal of Applied Econometrics*, 24(7): 1187–1206, 2009.

Robert B Davies. Hypothesis testing when a nuisance parameter is present only under the alternative. *Biometrika*, 64(2):247–254, 1977.

Robert B Davies. Hypothesis testing when a nuisance parameter is present only under the alternative. *Biometrika*, 74(1):33–43, 1987.

Jean-Marie Dufour. Some impossibility theorems in econometrics with applications to structural and dynamic models. *Econometrica: Journal of the Econometric Society*, pages 1365–1387, 1997.

Claudia Foroni and Massimiliano Giuseppe Marcellino. A survey of econometric methods for mixed-frequency data. *Available at SSRN 2268912*, 2013.

Claudia Foroni, Massimiliano Marcellino, and Christian Schumacher. Unrestricted mixed data sampling (MIDAS): MIDAS regressions with unrestricted lag polynomials. *Journal of the Royal Statistical Society: Series A (Statistics in Society)*, 2013.

Eric Ghysels. Macroeconomics and the reality of mixed frequency data. *Available at SSRN 2069998*, 2012.

Eric Ghysels. Macroeconomics and the reality of mixed frequency data. *Journal of Econometrics*, 2016.

Eric Ghysels and Jonathan H Wright. Midas instruments. Technical report, Working Paper, 2010.

Eric Ghysels, Pedro Santa-Clara, and Rossen Valkanov. The MIDAS touch: Mixed data sampling regression models. *Finance*, 2004.

Eric Ghysels, Pedro Santa-Clara, and Rossen Valkanov. Predicting volatility: getting the most out of return data sampled at different frequencies. *Journal of Econometrics*, 131(1): 59–95, 2006.

Thomas B Götz, Alain Hecq, and Stephan Smeekes. Testing for granger causality in large mixed-frequency vars. *Journal of Econometrics*, 2016.

Pierre Guérin and Massimiliano Marcellino. Markov-switching MIDAS models. *Journal of Business & Economic Statistics*, 31(1):45–56, 2013.

Frank Kleibergen. Testing parameters in GMM without assuming that they are identified. *Econometrica*, 73(4):1103–1123, 2005.

Vladimir Kuzin, Massimiliano Marcellino, and Christian Schumacher. MIDAS vs. mixed-frequency VAR: Nowcasting GDP in the euro area. *International Journal of Forecasting*, 27(2):529–542, 2011.

Hang Qian. A computationally efficient method for vector autoregression with mixed frequency data. *Journal of Econometrics*, 2016.

James H Stock and Jonathan H Wright. GMM with weak identification. *Econometrica*, 68 (5):1055–1096, 2000.

Frank Windmeijer. A finite sample correction for the variance of linear efficient two-step GMM estimators. *Journal of econometrics*, 126(1):25–51, 2005.

Peter A Zadrozny. Extended yule–walker identification of varma models with single-or mixed-frequency data. *Journal of Econometrics*, 2016.

A MIDAS Data Generating Process

Taking the first difference of a MIDAS regressor implies a specific structural form of the underlying data generation process. The first formulation of the high frequency observations defines a relationship between the j^{th} observation in period t and the j^{th} observation in period $t-1$, given by:

$$x_{i,t,j} = \alpha_i + \rho x_{i,(t-1),j} + \epsilon_{i,t,j}. \tag{54}$$

The MIDAS regressor generated from equation (54) gives:

$$\begin{aligned} x_{i,t}(\theta) &= \sum_{j=1}^{m} w_j(\theta) x_{i,t,j} & (55) \\ &= \sum_{j=1}^{j^{max}} w_j(\theta)(\alpha_i + \rho x_{i,t-1,j} + \epsilon_{i,t,j}) & (56) \\ &= \alpha_i + \rho \sum_{j=1}^{m} w_j(\theta) x_{i,t-1,j} + \sum_{j=1}^{m} w_j(\theta) \epsilon_{i,t,j} & (57) \end{aligned}$$

The first difference (in t) of this MIDAS regressor gives:

$$\Delta x_{i,t}(\theta) = \rho \sum_{j=1}^{j^{max}} w_j \Delta x_{i,t-1,j}(\theta) + \sum_{j=1}^{j^{max}} w_j(\theta) \Delta \epsilon_{i,t,j} \tag{58}$$

The second formulation is a predetermined relationship between the j and $j-1$ observations in period t, given by:

$$x_{i,t,j} = \alpha_i + \gamma x_{i,t,j+1} + \epsilon_{i,t,j} \tag{59}$$

To obtain the first difference in t of this formulation of the high frequency observations, we begin with recursion of the series to obtain,

$$x_{i,t,j} = \alpha_i \sum_{k=0}^{m-1} \gamma^k + \gamma^m x_{i,t-1,j} + \left[\sum_{k=0}^{m-j} \gamma^k \epsilon_{i,t,j+k} + \sum_{k=1}^{j} \gamma^{m-j+k} \epsilon_{i,t,k} \right], \tag{60}$$

where the bracketed part is a moving average error term, denoted $e_{i,t,j}(\gamma)$.

The MIDAS aggregation from equation 60 gives:

$$
\begin{aligned}
x_{i,t}(\theta) &= \sum_{j=1}^{m} w_j(\theta) x_{i,t,j} && (61)\\
&= \sum_{j=1}^{m} w_j(\theta) \left(\alpha_i \sum_{k=0}^{m-1} \gamma^k + \gamma^m x_{i,t-1,j} + e_{i,t,j}(\gamma) \right) && (62)\\
&= \alpha_i \sum_{k=0}^{m-1} \gamma^k + \gamma^m \sum_{j=1}^{m} w_j(\theta) x_{i,t-1,j} + \sum_{j=1}^{m} w_j(\theta) e_{i,t,j}(\gamma) && (63)
\end{aligned}
$$

Taking the first difference in t of this second MIDAS formulation results in:

$$
\Delta x_{i,t}(\theta) = \gamma^m \sum_{j=1}^{m} w_j(\theta) \Delta x_{i,t-1,j} + \sum_{j=1}^{m} w_j(\theta) \Delta e_{i,t,j}(\gamma). \tag{64}
$$

Equations (58) and (64) both statify the condition that the MIDAS aggregate is uncorrelated with $\nu_{i,s}$ from equation (5) for all s in t. It is apparent that the moving average component in equation (64) leads to correlation at the high frequency observations.

For the simulation study, the original Arellano and Bond assumption that $\rho = 0.8$ at the low frequency is maintained. It is clear that equation (58) is more managable, since the value can be set. For equation (64), the relationship between ρ and γ can be derived from the recursion of the series, $\gamma = \rho^{1/m}$, so for a stationary low frequency series the value of γ approaches unity as m increases, and is further complicated by the moving average component of the errors. To avoid complications of the moving average terms and the approach of the high frequency observations to unity, our simulation study employs the first representation of the high frequency observations which leads to correlation at the low frequency only.

B Derivation of Lemmas

We assume that $E[y_{i,0}x_{i,t,j}] = 0$ for all $j = 1, \ldots, m$, where the MIDAS aggregation of the high frequency observations can be defined as,

$$x_{i,t}(\theta) = \sum_{j=1}^{m} x_{i,t,j} w_j(\theta). \tag{65}$$

Under the assumption of covariance stationary high-frequency observations, then

$$E\left[\sum_{i=1}^{N} x_{i,t,j} x_{i,,t-q,j-p}\right] = \Gamma(q,p), \tag{66}$$

and if $p = 0$ then $\Gamma(q, p; p = 0) = \Gamma(q, 0)$.

The dynamic panel model with MIDAS covariates can be written as,

$$y_{i,t}(\theta) = \delta y_{i,t-1}(\theta) + x_{i,t}(\theta)\beta + v_{i,t} + \mu_i, \tag{67}$$

where the dependent series is a function of θ, based on the Wold decomposition.

For any given θ_0, the model can be written as,

$$y_{i,t}(\theta, \theta_0) = \delta y_{i,t-1}(\theta) + x_{i,t}(\theta_0)\beta + \left[v_{i,t} + \mu_i + \left(x_{i,t}(\theta) - x_{i,t}(\theta_0)\right)\beta\right], \tag{68}$$

leading to the error term as a function of θ and θ_0,

$$\begin{aligned} \omega_{i,t}(\theta, \theta_0) &= v_{i,t} + \mu_i + \left(x_{i,t}(\theta) - x_{i,t}(\theta_0)\right)\beta \\ &= y_{i,t}(\theta) - \delta y_{i,t-1}(\theta) - x_{i,t}(\theta_0)\beta. \end{aligned} \tag{69}$$

The instrument matrix is constructed as a block diagonal matrix with each block is indexed by $s = 3, \dots, T$, and defined by

$$z_{i,s} := z_{i,s}(\theta, \theta_0) = (y_{i,1}(\theta), \dots, y_{i,s-2}(\theta), x_{i,1}(\theta_0), x_{i,2}(\theta_0), \dots, x_{i,s}(\theta_0)), \tag{70}$$

where the exogenous regressors are predetermined by assumption. The instruments in each block are functions of θ and θ_0, so we denote the instrument matrix as $W(\theta, \theta_0)$. The matrix of instruments for each i is denoted $W_i(\theta, \theta_0)$ takes the following form,

$$W_i := W_i(\theta, \theta_0) = \begin{bmatrix} z_{i,3} & 0 & 0 & 0 \\ 0 & z_{i,4} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & z_{i,T} \end{bmatrix}. \tag{71}$$

W_i is a $(T-2) \times \tau$ matrix, where τ is a function of T and the number of exogenous regressors (K). The full matrix of instruments, for all i, is $W := W(\theta, \theta_0) = [W_1'(\theta, \theta_0), \dots, W_N'(\theta, \theta_0)]'$ with dimensions $N(T-2) \times \tau$.

The moment condition of the Arellano-Bond estimator is defined as

$$E\left[W(\theta, \theta_0)' \Delta\omega(\theta, \theta_0)\right] = 0, \tag{72}$$

leading to the following two moment conditions

$$E\left[\sum_{i=1}^{N} y_{i,t-r}(\theta)(\omega_{i,t}(\theta, \theta_0) - \omega_{i,t-1}(\theta, \theta_0))\right] = 0, \tag{73}$$

and

$$E\left[\sum_{i=1}^{N} x_{i,t-q}(\theta_0)(\omega_{i,t}(\theta, \theta_0) - \omega_{i,t-1}(\theta, \theta_0))\right] = 0, \tag{74}$$

where $r = 2, \dots, t-1$ and $q = 0, \dots, t-1$.

Considering the first moment condition, equation (73), we substitute in equations (67) and (69) for any θ_0 to obtain the following,

$$
\begin{aligned}
0 &= E\left[\sum_{i=1}^{N} \left(\delta y_{i,t-1-r}(\theta) + x_{i,t-r}(\theta)\beta + v_{i,t-r} + \mu_i\right)\left(v_{i,t} - v_{i,t-1}\right)\right] \\
&+ E\left[\sum_{i=1}^{N} \left(v_{i,t-r} + \mu_i\right)\left(x_{i,t}(\theta) - x_{i,t}(\theta_0)\right)\beta\right] \\
&- E\left[\sum_{i=1}^{N} \left(v_{i,t-r} + \mu_i\right)\left(x_{i,t-1}(\theta) - x_{i,t-1}(\theta_0)\right)\beta\right] \\
&+ E\left[\sum_{i=1}^{N} \left(\delta y_{i,t-1-r}(\theta)\right)\left(x_{i,t}(\theta) - x_{i,t}(\theta_0)\right)\beta\right] \\
&- E\left[\sum_{i=1}^{N} \left(\delta y_{i,t-1-r}(\theta)\right)\left(x_{i,t-1}(\theta) - x_{i,t-1}(\theta_0)\right)\beta\right] \\
&+ E\left[\sum_{i=1}^{N} \left(x_{i,t-r}(\theta)\beta\right)\left(x_{i,t}(\theta) - x_{i,t}(\theta_0)\right)\beta\right] \\
&- E\left[\sum_{i=1}^{N} \left(x_{i,t-r}(\theta)\beta\right)\left(x_{i,t-1}(\theta) - x_{i,t-1}(\theta_0)\right)\beta\right],
\end{aligned} \tag{75}
$$

where the first three expectation terms are zero under the usual Arellano-Bond framework and strict exogeneity. The continuous recursion of the dependent series leads to

$$
\begin{aligned}
0 &= E\left[\sum_{i=1}^{N} \left(\delta^t y_{i,0}\right)\left(x_{i,t}(\theta) - x_{i,t}(\theta_0)\right)\right]\beta \\
&- E\left[\sum_{i=1}^{N} \left(\delta^t y_{i,0}\right)\left(x_{i,t-1}(\theta) - x_{i,t-1}(\theta_0)\right)\right]\beta \\
&+ E\left[\sum_{i=1}^{N} \left(\sum_{k=0}^{t-r-1} \delta^k x_{i,t-r-k}(\theta)\right)\left(x_{i,t}(\theta) - x_{i,t}(\theta_0)\right)\right]\beta^2 \\
&- E\left[\sum_{i=1}^{N} \left(\sum_{k=0}^{t-r-1} \delta^k x_{i,t-r-k}(\theta)\right)\left(x_{i,t-1}(\theta) - x_{i,t-1}(\theta_0)\right)\right]\beta^2,
\end{aligned} \tag{76}
$$

and under the assumption $E_i\left[y_{i,0}x_{i,t}(\theta)\right] = 0$ for all t, then the first two moments in the equation are zero. This leads to the following representation of the first moment condition,

$$
\begin{aligned}
0 &= \left(\sum_{k=0}^{t-r-1} \delta^k \sum_{j=1}^{m} \sum_{l=1}^{m} \Gamma(r+k, j-l) \left(w_j(\theta) w_l(\theta) - w_j(\theta) w_l(\theta_0) \right) \right) \beta^2 \\
&- \left(\sum_{k=0}^{t-r-1} \delta^k \sum_{j=1}^{m} \sum_{l=1}^{m} \Gamma(r+k-1, j-l) \left(w_j(\theta) w_l(\theta) - w_j(\theta) w_l(\theta_0) \right) \right) \beta^2. \qquad (77)
\end{aligned}
$$

which simplifies and we define as,

$$
\begin{aligned}
\bar{A}_{t,r} &= \left(\sum_{j=1}^{m} \sum_{l=1}^{m} \Gamma(r-1, j-l) \left(w_j(\theta) w_l(\theta) - w_j(\theta) w_l(\theta_0) \right) \right) \beta^2 \\
&+ (1-\delta) \left(\sum_{k=0}^{t-r-1} \delta^k \sum_{j=1}^{m} \sum_{l=1}^{m} \Gamma(r+k, j-l) \left(w_j(\theta) w_l(\theta) - w_j(\theta) w_l(\theta_0) \right) \right) \beta^2. \qquad (78)
\end{aligned}
$$

For the second moment condition, a similar substitution and recursion are applied to equation (74) to obtain,

$$
\begin{aligned}
0 &= \left(\sum_{j=1}^{m} \sum_{l=1}^{m} \Gamma(q, j-l) \left(w_j(\theta_0) w_l(\theta) - w_j(\theta_0) w_l(\theta_0) \right) \right) \beta \\
&- \left(\sum_{j=1}^{m} \sum_{l=1}^{m} \Gamma(q+1, j-l) \left(w_j(\theta_0) w_l(\theta) - w_j(\theta_0) w_l(\theta_0) \right) \right) \beta, \qquad (79)
\end{aligned}
$$

which simplifies and we define as,

$$
\bar{B}_{t,q} = \left(\sum_{j=1}^{m} \sum_{l=1}^{m} \left(\Gamma(q, j-l) - \Gamma(q+1, j-l) \right) \left(w_j(\theta_0) w_l(\theta) - w_j(\theta_0) w_l(\theta_0) \right) \right) \beta. \qquad (80)
$$

Lemma (2) and (3) restrict focus on the case where $\Gamma(q,p) = 0$ for all $p \neq 0$, and $\Gamma(q,0) \neq 0$, which allows for correlation of the exogenous regressors over the low frequency time periods but not between the high frequency observations. Under these assumptions, Lemma (2) is derived from equation (78) as

$$\begin{aligned} 0 &= \left(\Gamma(r-1,0)\sum_{j=1}^{m}\big(w_j(\theta)^2 - w_j(\theta)w_j(\theta_0)\big)\right)\beta^2 \\ &+ (1-\delta)\left(\sum_{k=0}^{t-r-1}\delta^k\Gamma(r+k,0)\sum_{j=1}^{m}\big(w_j(\theta)^2 - w_j(\theta)w_j(\theta_0)\big)\right)\beta^2 \end{aligned} \tag{81}$$

and equation (80) leads to Lemma (3) given by,

$$0 = \left((\Gamma(q,0) - \Gamma(q+1,0))\sum_{j=1}^{m}\big(w_j(\theta_0)w_j(\theta) - w_j(\theta_0)^2\big)\right)\beta. \tag{82}$$

C Tables

Table 7: Sargan Test: Empirical Size and Power for MIDAS using θ^A at $\alpha = 0.05$

		$N = 100$			$N = 500$			$N = 1000$		
θ_1^A	θ_2^A	$T = 5$	$T = 10$	$T = 15$	$T = 5$	$T = 10$	$T = 15$	$T = 5$	$T = 10$	$T = 15$
Size										
0	0	0.047	0.024	0.025	0.041	0.051	0.041	0.052	0.042	0.043
Power										
-1	-1	0.064	0.029	0.019	0.054	0.055	0.048	0.034	0.054	0.086
	-0.5	0.055	0.038	0.024	0.054	0.049	0.051	0.048	0.061	0.077
	0	0.046	0.041	0.014	0.046	0.042	0.041	0.057	0.071	0.063
	0.5	0.042	0.034	0.018	0.053	0.05	0.053	0.042	0.055	0.091
	1	0.037	0.034	0.021	0.051	0.048	0.058	0.051	0.059	0.099
-0.5	-1	0.063	0.029	0.021	0.059	0.045	0.054	0.035	0.067	0.08
	-0.5	0.056	0.034	0.02	0.053	0.041	0.056	0.042	0.063	0.066
	0	0.059	0.042	0.014	0.055	0.042	0.06	0.041	0.05	0.06
	0.5	0.043	0.033	0.011	0.042	0.05	0.064	0.049	0.054	0.087
	1	0.045	0.031	0.02	0.054	0.042	0.057	0.06	0.065	0.086
0	-1	0.051	0.034	0.013	0.056	0.04	0.035	0.047	0.063	0.077
	-0.5	0.06	0.035	0.014	0.05	0.047	0.063	0.05	0.053	0.077
	0	-	-	-	-	-	-	-	-	-
	0.5	0.048	0.034	0.016	0.052	0.051	0.057	0.049	0.052	0.086
	1	0.028	0.033	0.019	0.052	0.047	0.051	0.045	0.064	0.079
0.5	-1	0.054	0.037	0.02	0.045	0.05	0.053	0.052	0.055	0.077
	-0.5	0.05	0.039	0.013	0.062	0.042	0.053	0.05	0.061	0.067
	0	0.041	0.029	0.019	0.051	0.045	0.06	0.039	0.057	0.068
	0.5	0.057	0.026	0.017	0.05	0.058	0.052	0.054	0.055	0.112
	1	0.052	0.022	0.013	0.038	0.063	0.058	0.053	0.038	0.083
1	-1	0.053	0.038	0.013	0.046	0.046	0.059	0.047	0.054	0.081
	-0.5	0.057	0.025	0.018	0.04	0.036	0.056	0.04	0.055	0.073
	0	0.043	0.026	0.017	0.034	0.036	0.075	0.064	0.062	0.08
	0.5	0.047	0.024	0.019	0.046	0.046	0.046	0.038	0.055	0.081
	1	0.05	0.036	0.02	0.06	0.058	0.06	0.047	0.038	0.092

Table 8: Sargan Test: Empirical Size and Power for MIDAS using θ^B at $\alpha = 0.05$

		$N = 100$			$N = 500$			$N = 1000$		
θ_1^B	θ_2^B	$T = 5$	$T = 10$	$T = 15$	$T = 5$	$T = 10$	$T = 15$	$T = 5$	$T = 10$	$T = 15$
Size										
0.1	-0.2	0.035	0.043	0.043	0.028	0.051	0.049	0.02	0.046	0.042
Power										
-1	-1	0.057	0.158	0.073	0.062	0.192	0.738	0.031	0.51	0.987
	-0.5	0.057	0.13	0.071	0.055	0.134	0.541	0.037	0.379	0.909
	0	0.034	0.041	0.056	0.051	0.039	0.07	0.018	0.041	0.072
	0.5	0.044	0.056	0.043	0.038	0.047	0.046	0.014	0.043	0.066
	1	0.04	0.056	0.047	0.046	0.06	0.055	0.023	0.03	0.066
-0.5	-1	0.055	0.131	0.088	0.064	0.153	0.7	0.041	0.512	0.979
	-0.5	0.061	0.094	0.061	0.046	0.104	0.414	0.037	0.279	0.813
	0	0.041	0.055	0.043	0.038	0.047	0.059	0.017	0.061	0.064
	0.5	0.046	0.048	0.043	0.042	0.049	0.047	0.023	0.045	0.063
	1	0.061	0.056	0.053	0.031	0.058	0.061	0.021	0.034	0.059
0	-1	0.05	0.123	0.089	0.056	0.147	0.613	0.05	0.415	0.965
	-0.5	0.048	0.072	0.067	0.042	0.078	0.218	0.035	0.157	0.507
	0	0.039	0.068	0.161	0.032	0.655	0.597	0.026	0.866	0.944
	0.5	0.04	0.063	0.044	0.04	0.045	0.04	0.021	0.049	0.062
	1	0.033	0.055	0.049	0.05	0.042	0.054	0.02	0.037	0.058
0.5	-1	0.05	0.112	0.086	0.054	0.138	0.564	0.025	0.365	0.925
	-0.5	0.037	0.054	0.047	0.041	0.046	0.101	0.02	0.068	0.117
	0	0.048	0.038	0.064	0.05	0.09	0.042	0.029	0.042	0.026
	0.5	0.049	0.056	0.055	0.036	0.062	0.047	0.033	0.041	0.067
	1	0.042	0.046	0.047	0.04	0.056	0.052	0.019	0.044	0.061
1	-1	0.052	0.084	0.082	0.048	0.101	0.435	0.024	0.308	0.819
	-0.5	0.036	0.04	0.049	0.03	0.041	0.048	0.016	0.053	0.046
	0	0.05	0.034	0.066	0.055	0.054	0.035	0.029	0.034	0.046
	0.5	0.048	0.043	0.036	0.039	0.044	0.045	0.018	0.027	0.055
	1	0.045	0.052	0.053	0.046	0.051	0.063	0.03	0.043	0.06

Table 9: Sargan Test: Empirical Size and Power for MIDAS using θ^C at $\alpha = 0.05$

		$N = 100$			$N = 500$			$N = 1000$		
θ_1^C	θ_2^C	$T = 5$	$T = 10$	$T = 15$	$T = 5$	$T = 10$	$T = 15$	$T = 5$	$T = 10$	$T = 15$
Size										
0.03	-0.02	0.038	0.038	0.018	0.042	0.049	0.04	0.046	0.053	0.05
Power										
-1	-1	0.066	0.038	0.026	0.07	0.06	0.088	0.038	0.117	0.235
	-0.5	0.065	0.05	0.027	0.058	0.051	0.074	0.042	0.112	0.211
	0	0.043	0.047	0.021	0.052	0.043	0.061	0.055	0.109	0.158
	0.5	0.044	0.034	0.015	0.07	0.058	0.109	0.047	0.102	0.162
	1	0.041	0.036	0.015	0.068	0.078	0.097	0.061	0.112	0.163
-0.5	-1	0.065	0.047	0.022	0.064	0.044	0.11	0.045	0.108	0.223
	-0.5	0.065	0.035	0.021	0.065	0.048	0.094	0.043	0.125	0.202
	0	0.037	0.041	0.016	0.057	0.046	0.061	0.036	0.051	0.083
	0.5	0.043	0.032	0.011	0.054	0.072	0.098	0.061	0.11	0.159
	1	0.05	0.027	0.02	0.058	0.068	0.101	0.061	0.096	0.165
0	-1	0.053	0.041	0.02	0.058	0.036	0.068	0.045	0.115	0.21
	-0.5	0.064	0.038	0.026	0.061	0.046	0.101	0.058	0.113	0.203
	0	0.05	0.025	0.021	0.044	0.078	0.082	0.062	0.073	0.127
	0.5	0.051	0.033	0.017	0.066	0.067	0.11	0.049	0.102	0.16
	1	0.03	0.027	0.018	0.062	0.07	0.09	0.052	0.112	0.152
0.5	-1	0.055	0.044	0.025	0.066	0.052	0.108	0.051	0.109	0.218
	-0.5	0.046	0.049	0.02	0.056	0.048	0.082	0.051	0.109	0.175
	0	0.04	0.032	0.017	0.055	0.059	0.1	0.041	0.094	0.136
	0.5	0.052	0.021	0.022	0.07	0.076	0.104	0.067	0.123	0.183
	1	0.04	0.025	0.013	0.058	0.071	0.095	0.054	0.119	0.156
1	-1	0.056	0.052	0.013	0.044	0.05	0.107	0.06	0.121	0.225
	-0.5	0.05	0.037	0.016	0.041	0.048	0.074	0.045	0.114	0.165
	0	0.044	0.028	0.016	0.043	0.049	0.115	0.058	0.11	0.168
	0.5	0.043	0.032	0.013	0.063	0.06	0.1	0.04	0.088	0.157
	1	0.045	0.033	0.026	0.059	0.077	0.101	0.057	0.121	0.168

Table 10: Sargan Test: Empirical Size and Power for MIDAS using θ^D at $\alpha = 0.05$

θ_1^D	θ_2^D	$N=100$			$N=500$			$N=1000$		
		$T=5$	$T=10$	$T=15$	$T=5$	$T=10$	$T=15$	$T=5$	$T=10$	$T=15$
Size										
-0.06	0.01	0.048	0.032	0.016	0.042	0.034	0.044	0.035	0.046	0.035
Power										
-1	-1	0.063	0.036	0.015	0.055	0.049	0.045	0.05	0.083	0.126
	-0.5	0.054	0.037	0.011	0.048	0.044	0.052	0.062	0.078	0.114
	0	0.051	0.044	0.019	0.05	0.034	0.049	0.068	0.093	0.101
	0.5	0.037	0.032	0.013	0.047	0.07	0.092	0.053	0.096	0.229
	1	0.032	0.033	0.024	0.042	0.067	0.084	0.062	0.094	0.238
-0.5	-1	0.067	0.036	0.015	0.056	0.05	0.052	0.058	0.085	0.119
	-0.5	0.054	0.038	0.016	0.04	0.042	0.05	0.056	0.084	0.1
	0	0.055	0.043	0.014	0.052	0.045	0.063	0.053	0.083	0.093
	0.5	0.046	0.034	0.013	0.036	0.062	0.097	0.066	0.099	0.249
	1	0.043	0.029	0.016	0.056	0.06	0.101	0.074	0.096	0.235
0	-1	0.059	0.032	0.015	0.05	0.044	0.038	0.062	0.081	0.112
	-0.5	0.065	0.033	0.019	0.041	0.047	0.059	0.058	0.075	0.117
	0	0.05	0.025	0.021	0.043	0.077	0.091	0.077	0.109	0.225
	0.5	0.045	0.032	0.018	0.051	0.057	0.099	0.052	0.096	0.225
	1	0.033	0.031	0.022	0.046	0.056	0.07	0.061	0.094	0.231
0.5	-1	0.06	0.035	0.02	0.052	0.046	0.042	0.065	0.082	0.112
	-0.5	0.045	0.037	0.02	0.062	0.039	0.063	0.053	0.08	0.102
	0	0.039	0.023	0.022	0.058	0.043	0.046	0.043	0.046	0.038
	0.5	0.05	0.028	0.013	0.05	0.061	0.093	0.066	0.096	0.23
	1	0.04	0.029	0.013	0.033	0.073	0.103	0.062	0.094	0.22
1	-1	0.058	0.037	0.014	0.045	0.05	0.061	0.064	0.08	0.113
	-0.5	0.061	0.031	0.02	0.045	0.033	0.055	0.052	0.079	0.108
	0	0.036	0.024	0.01	0.035	0.048	0.087	0.062	0.066	0.094
	0.5	0.048	0.036	0.02	0.043	0.068	0.084	0.055	0.099	0.204
	1	0.044	0.032	0.019	0.049	0.067	0.091	0.067	0.096	0.247

Table 11: Sargan Test: Empirical Size and Power for MIDAS using θ^E at $\alpha = 0.05$

		$N=100$			$N=500$			$N=1000$		
θ_1^E	θ_2^E	$T=5$	$T=10$	$T=15$	$T=5$	$T=10$	$T=15$	$T=5$	$T=10$	$T=15$
Size										
-0.04	0.02	0.044	0.032	0.021	0.036	0.036	0.039	0.035	0.037	0.042
Power										
-1	-1	0.071	0.057	0.022	0.061	0.038	0.028	0.051	0.049	0.06
	-0.5	0.074	0.045	0.015	0.051	0.03	0.024	0.054	0.039	0.04
	0	0.054	0.046	0.01	0.043	0.023	0.024	0.054	0.054	0.036
	0.5	0.039	0.04	0.027	0.077	0.149	0.273	0.109	0.367	0.76
	1	0.04	0.042	0.028	0.067	0.153	0.264	0.103	0.361	0.746
-0.5	-1	0.078	0.04	0.019	0.047	0.03	0.026	0.05	0.051	0.054
	-0.5	0.071	0.045	0.013	0.041	0.026	0.031	0.05	0.058	0.05
	0	0.05	0.039	0.011	0.046	0.032	0.037	0.04	0.055	0.048
	0.5	0.061	0.043	0.017	0.057	0.151	0.269	0.127	0.365	0.738
	1	0.044	0.037	0.021	0.075	0.135	0.27	0.119	0.345	0.751
0	-1	0.08	0.042	0.017	0.051	0.029	0.021	0.055	0.051	0.043
	-0.5	0.075	0.032	0.02	0.039	0.034	0.035	0.052	0.043	0.053
	0	0.039	0.026	0.018	0.06	0.242	0.472	0.275	0.68	0.968
	0.5	0.054	0.039	0.029	0.08	0.145	0.276	0.107	0.329	0.728
	1	0.037	0.034	0.031	0.069	0.145	0.261	0.11	0.367	0.741
0.5	-1	0.061	0.035	0.02	0.046	0.031	0.027	0.05	0.044	0.051
	-0.5	0.057	0.039	0.013	0.058	0.024	0.036	0.044	0.042	0.047
	0	0.038	0.026	0.022	0.049	0.042	0.051	0.05	0.053	0.054
	0.5	0.052	0.038	0.025	0.071	0.155	0.283	0.12	0.367	0.718
	1	0.045	0.036	0.016	0.045	0.147	0.285	0.117	0.361	0.753
1	-1	0.058	0.045	0.02	0.041	0.037	0.023	0.054	0.047	0.053
	-0.5	0.061	0.031	0.018	0.049	0.025	0.025	0.049	0.045	0.047
	0	0.034	0.026	0.009	0.035	0.041	0.053	0.055	0.055	0.043
	0.5	0.046	0.037	0.022	0.072	0.134	0.274	0.116	0.367	0.71
	1	0.052	0.043	0.032	0.065	0.144	0.259	0.122	0.361	0.755