Resource and Labor Taxation in Overlapping Generation Economies with an Essential Polluting Resource

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Abstract

This paper uses an overlapping generation framework in which natural resources are introduced in order to prove that an increase in the natural capital share is associated with a decrease in the balanced growth rate. We then characterize combination of labor and resources taxation that allow to decentralize the optimal balanced growth path. Resource-dependant market economies are characterized by a higher rate of resource depletion. Thus, the decentralization of the social balanced growth path requires a lower wage taxation and/or a higher resource taxation. Nevertheless, resource-dependant economies still grow (optimally) slowly. This paper also investigates the impact of a pollution externality on growth. The higher is the detrimental effect of pollution on production, the lower is the negative impact of resource to growth in the long run.

1 Introduction

Since the seminal work of Sachs & Warner (1995), the economic literature focusing on the so-called "resource curse" has developed. The resource curse is an empirical paradox: resource-rich econonomies seems to grow slowly than resource-poor ones. Several channels can lead to a resource curse.[1] The first one is the "Dutch disease" effect: a resource windfall provokes an appreciation of the real exchange rate and deteriorates the terms of trade. Growth enhancing sectors (secondary and tertiary) are thus negatively affected. This channel has been largely documented in the literature (see e.g. Corden (1984), Krugman (1987), Bruno & Sachs (1982), Torvik (2001) or Matsen & Torvik (2005)). A second channel is the crowding-out effect of resources on education (Gylfason (2001), Sachs & Warner (1999)). That results had been mainly criticized by Stijns (2006) which highlights that it is sensitive to the resource abundance indicator choices. He shows that

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[1]See Van der Ploeg (2011) for a great literature review about the resource curse.

indicators that argue for the resource curse are indicators of resource dependance instead of resource abundance. An other very discussed channel relates to the relation between natural resources and institutional quality. Indeed, it seems that resource abundance generates rent-seeking behaviours that could deteriorate the institutionnal quality, through corruption, war and so on. Since institutional quality is a major determinant of growth, natural resources may be growth-reducing. The crowding-out effect of resource on physical capital accumulation has been studied by Sachs & Warner (1995). A resource windfall implies a change in the factor of production's repartition, from secondary and tertiary sectors to the primary sector, which generates less positive externaities and which is charecterized by non-increasing return to scale. Mikesell (1997) argues that the volatility of resources prices discourages investment in resource-abundant economies.

This paper proposes an other explanation of the negative impact of resources on growth. It is based on the literature on non-renewable resources and growth. Since the pessimistic work of the *Club of Rome* (Meadows *et al.* (1972)), a huge literature has developed on the sustainability of growth in a finite world. Thus Dasgupta & Heal (1974), Solow (1974), Stiglitz (1974) have introduced natural resources in a neoclassical framework. They find that the existence of an exogenous technical progress is a necessary condition for the possibility of an infinite growth.

Authors such as Barbier (1999) have introduced non-renewable resources in neoclassical endogenous growth model. Schou (2000) remarks that this framework is very useful to study the impact of polluting resources on growth. He proves that ecological taxation is unnecessary because the flow of emmited pollutants decreases with the stock of resource.

Following Solow (1986) advices, Agnani *et al.* (2005) study the feasibility of positive long-run economic growth in economies where non-renewable resources constitutes an essential input in the production process.

This paper uses their framework to prove that an increase in the natural capital share is associated with a decrease in the balanced growth rate. Moreover, since the social discount rates fully determines the optimal extraction rate, combination of labor and resources taxes that allow to decentralize the optimal balanced growth path are characterized. Resource-dependant market economies describe a higher rate of resource depletion. The decentralization of the social balanced growth thus requires a lower wage taxation and/or a higher resource taxation. Nevertheless, resource-dependant economies still grow (optimally) slowly. Following Schou (2000), the impact of a pollution externality on growth is also investigated. The higher is the detrimental effect of pollution on production, the lower is the negative impact of resource to growth.

Section 2 presents the model. Section 3 characterized the optimal equilibrium. Section 4 derives the balanced growth path. Section 5 analysis the effect of resource dependance and pollution. Section 6 analysis impact of a change in the taxation scheme. Section 7 perform a welfare analysis and discusses the optimal taxation scheme while section 8 concludes.

2 The Model

The model developed in this paper is the OLG model of Diamond (1965) where non-renewable resources are introduced *à la* Agnani *et al.* (2005). Two generations cohabit in this economy. Each new generation is constituted of N_t agents who are alive for two periods. For the sake of simplicity,a no demographical growth assumption is used and the size of a generation is normalized to one.

2.1 The Non-Renewable Resource

Following Agnani *et al.* (2005) the economy is initially endowed with a quantity M_{-1} of a necessary exhaustible resource held by the first generation of aged agents. At each date t, elderly agents sell their resource share to the young generation and a quantity X_t of the resource is used in the production process, and generates an environmental externality. The resource stock in t is thus denoted by $M_t = M_{t-1} - X_t$ and it belongs to the generation t. The rate of exhaustion of the natural asset is $\tau_t = X_t/M_{t-1}$. In per worker terms, it gives

$$\tau_t = \frac{x_t}{m_{t-1}}. \tag{1}$$

The dynamics of the per worker resource stock is thus [2]

$$m_t = (1 - \tau_t)m_{t-1}. \tag{2}$$

It leads, associated with the non renewability of the ressource, to the exhaustibility condition

$$1 \geq \sum_{t=0}^{\infty} \tau_t \prod_{j=1}^{t}(1 - \tau_{j-1})$$

2.2 Consumers

In this economy agents are alive for two periods and maximize the following utility function

$$u_t(c_t; d_{t+1}) = ln(c_t) + \frac{1}{1+\rho} ln(d_{t+1})$$

where c represents the consumption while young, d the consumption while old and ρ the individual rate of time preference.

In the first period of life, the representative agent works to earn a wage w_t, which may be consumed, saved as physical capital s_t or used to buy

[2]It may also be interpreted as a resource market clearing as in Agnani *et al.* (2005)

rights on the resource stock m_t at a price p_t in terms of the representative good. This wage may be taxed by the governement at a rate q_w. His first period budget constraint is

$$(1 - q_w)w_t = c_t + s_t + p_t m_t. \tag{3}$$

While old, he gets his savings increased by the interest rate, and he sells his resource rights at a price p_{t+1}. His second period budget constraint is

$$d_{t+1} = (1 + r_{t+1})s_t + p_{t+1}m_t. \tag{4}$$

Combining 3 and 4, we obtain the following intertemporal budget constraint (IBC herafter)

$$(1 - q_w)w_t = c_t + \frac{d_{t+1}}{1 + r_{t+1}} - \frac{p_{t+1}m_t}{1 + r_{t+1}} + p_t m_t$$

The maximization of utility with respect to m_t, c_t, d_{t+1} subject to the IBC leads to the following first order condition:

$$\frac{d_{t+1}}{c_t} = \frac{1 + r_{t+1}}{1 + \rho} \tag{5}$$

$$\frac{p_{t+1}}{p_t} = 1 + r_{t+1} \tag{6}$$

(5) is the standard Euler equation while (6) is an arbitrage condition between the two assets in this economy, capital and resource.

2.3 Firms

Firms produce the representative good Y_t using a Cobb-Douglas technology. They use capital K_t, labor N_t, and resources X_t, with perfect substitutability and constant returns to scale for a given level of techology A_t which grows at a rate a such that

$$A_{t+1} = (1 + a)A_t \tag{7}$$

The use of the resource in the production process generates a flow of pollution E_t such that $E_t = \phi X_T$, which may be writted in per worker terms as

$$e_t = \phi x_t \tag{8}$$

Pollution resulting from the use of the resource in the production process generates a productivity loss. θ captures the detrimental impact of pollution on the level of production. It is assumed that the governement may tax the use of the ressource in order to reduce the environmental externality. This modelization is consistent with the idea of the carbon tax. The production function is thus:

$$Y_t = A_t K_t^\alpha N_t^\beta X_t^v E_t^{-\theta}$$

with $\alpha + \beta + v = 1$. In per worker terms

$$y_t = A_t k_t^\alpha x_t^v e_t^{-\theta} \tag{9}$$

Capital is remunerated at the interest rate r_t and depreciates at a rate $0 < \delta < 1$. Firms pays a wage w_t to their workers and buys the natural input at its price p_t increased by the taxe rate $1 + q_x$. We focus on the case $\theta < v$. That is, for an identical amount of resource and emissions, we assume that the positive impact of resources on income outweight its negative one. A similar assumption may be found in Schou (2000), where pollution is also seen as a flow. The standard profits maximization leads to the following first order condition:

$$r_t = \alpha A_t k_t^{\alpha-1} x_t^v e_t^{-\theta} - \delta \tag{10}$$

$$w_t = \beta A_t k_t^\alpha x_t^v e_t^{-\theta} \tag{11}$$

$$p_t(1 + q_x) = v A_t k_t^\alpha x_t^{v-1} e_t^{-\theta} \tag{12}$$

Each factor is thus paid at its marginal productivity.[3]

2.4 The Government

The governement can choose to fix positive tax rates on wages and resources. In each period, its revenue is thus

$$g_t = p_t m_t q_x + q_w w_t$$

Since we are not interested in how these revenues are used, but solely in where they come from, it may be assumed that they are used to reimburse the external debt or to do some improductive expenditures.

3 The Intertemporal Equilibrium

The economy produces a representative good which may be consumed or saved as physical capital. Following Diamond (1965), the good market clearing condition is

$$s_t = k_{t+1} \tag{13}$$

Definition 1. *An intertemporal competitive equilibrium is a solution of the system* (1)-(13)*. It is thus charactrized by the following properties:*

[3]This paper concentrates on interior solutions.

1. Agents maximize their utility according to their intertemporal budget constraint.

2. Firms maximize their profits.

3. The resource stock evolves according its law.

4. Markets clear.

4 The Balanced Growth Path

This paper focuses on balanced growth path because they constitute the only case where long-run positive growth is possible, as noted in Agnani *et al.* (2005).

Definition 2. *An intertemporal equilibrium where all variables grow at a constant rate is defined as a balanced growth path (BGP hereafter).*

Let μ_h be the BGP notation of the ratio h_{t+1}/h_t. According to definition 2, μ_m should be constant. Thus (2) implies a constant rate of extraction along the BGP, i.e. $\tau_t = \tau_{t+1} = \tau$.

Proposition 1. *This overlapping economy is characterized by the following growth rates:*

$$\begin{aligned}
\mu_k &= \mu_y = \mu_s = \mu_w = \mu_c = \mu \\
\mu_x &= \mu_e = \mu_m = 1 - \tau \\
\mu_p &= \mu/\mu_x \\
\mu_a &= 1 + a \\
\mu_r &= 1 \\
\mu \ &= (1+a)^{\frac{1}{1-\alpha}}(1-\tau)^{\frac{v-\theta}{1-\alpha}}
\end{aligned}$$

Proof. Proof is reported in appendix A. □

From Proposition 1, it may be estabished that a necessary condition to a long run positive growth is $\tau < 1 - (1+a)^{\frac{1}{\theta - v}}$. This threshold will be refered as a positive growth threshold (*PGT* hereafter). To analyse how the balanced growth path is affected by a change in the taxation scheme, it is necessary to characterize the constant extraction rate. The market clearing condition (13) may be written, using (3), (4), (5), (11), (12), as

$$k_{t+1} = \left[\frac{(1-q_w)\beta}{2+\rho} - \frac{v(1-\tau)}{\tau(1+q_x)} \right] A_t k_t^{\alpha} x_t^{v} e_t^{-\theta}$$

Evaluating this equation at the BGP, we can obtain:

$$\mu = \left[\frac{(1-q_w)\beta}{2+\rho} - \frac{v(1-\tau)}{\tau(1+q_x)}\right] A_t k_t^{\alpha-1} x_t^v e_t^{-\theta}$$

which may be rewritten as[4]

$$\frac{\mu\alpha(2+\rho)\tau(1+q_x)}{\beta(1-q_w)\tau(1+q_x) - v(1-\tau)(2+\rho)} = \frac{\mu}{1+\tau} - (1-\delta)$$

Substituting μ by its expression leads to

$$\frac{(1+a)^{\frac{1}{1-\alpha}}(1-\tau)^{\frac{v-\theta}{1-\alpha}}\alpha(2+\rho)\tau(1+q_x)}{\beta(1-q_w)\tau(1+q_x) - v(1-\tau)(2+\rho)} + (1-\delta) = \frac{(1+a)^{\frac{1}{1-\alpha}}(1-\tau)^{\frac{v-\theta}{1-\alpha}}}{1-\tau} \tag{14}$$

It may now be established that τ^* is solution to the preceding non linear equation. We denote LHS and RHS the left and right hand side of (14). $RHS(\tau)$ is defined on $[0;1[$ with $RHS(0) = (1+a)^{\frac{1}{1-\alpha}}$. Since $\lim_{\tau\to 1} RHS(\tau) = +\infty$, $RHS(\tau)$ admits a vertical asymptote in $\tau = 1$. Moreover, $\frac{\partial RHS(\tau)}{\partial \tau} > 0$ and $\frac{\partial RHS(\tau)^2}{\partial^2 \tau} > 0$ imply an increasing and convex function. $LHS(\tau)$ is defined on $[0;\hat{\tau}[U]\hat{\tau};1]$ with $\hat{\tau} = \frac{v(2+\rho)}{\beta(1+q_x)(1-q_w)+v(2+\rho)}$. $LHS(\tau) < 0 \,\forall\, \tau < \hat{\tau}$ which do not allow the possibility of an equilibrium extraction rate given that $RHS(\tau) > 0 \,\forall\, \tau \in [0;1[$. Since $lim_{\tau\to\hat{\tau}_{(+)}} = +\infty$ and $lim_{\tau\to 1} = 0$ it exists a unique τ^* such that (14) is satisfied. This situation is represented in Figure 1.

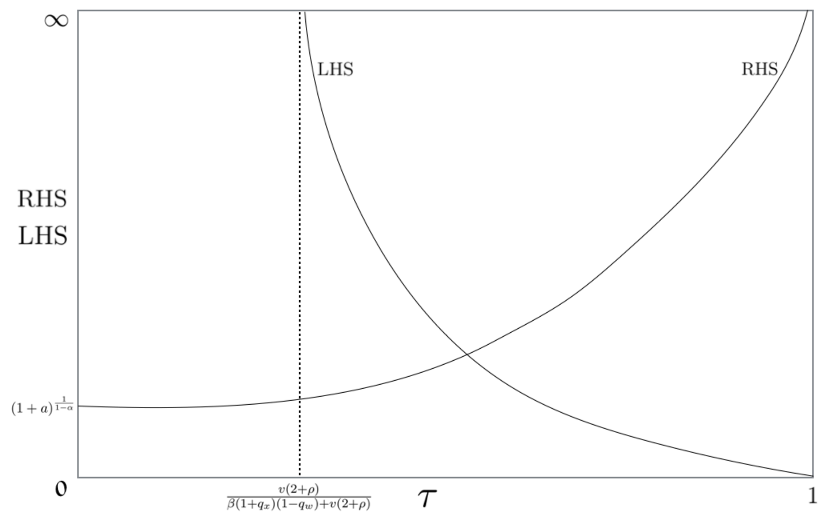

Figure 1: Characterization of the competitive equilibrium extraction rate

[4]see appendix A

The threshold $\hat{\tau}$ is called the growth area threshold (GAT hereafter). Indeed, an increase in the GAT reduces the set of τ such that the economy grows at the BGP, because the economy contracts if $PGT < \hat{\tau}$.

From Proposition 1 it appears that an higher extraction rate is associated with a lower growth while looking at (1) and (9), an increase in τ implies an higher income. It is thus necessary to distinguish between short and long run impacts of a higher extraction rate. In the short run, an increase in τ, *ceteris paribus*, implies an increase of one input in the production process. Current production thus increases. Nevertheless, it will be harder to maintain this level of production, because less natural resources are available to produce. In the long run, the higher is the extraction, the more the economy needs capital to compensates resource depletion. Saving may thus be not sufficient to maintain the growth level if resource extraction increases.

5 Comparative Statics

The resource curse literature seems to show that a higher resource abundance is associated with a lower growth rate. Since the seminal work of Sachs & Warner (1995), there has been an huge literature on the resource curse, using different abundancy indicators. Gylfason & Zoega (2006) among others, uses the share of natural wealth in total wealth. Stijns (2006) argues that this is more a indicator of dependancy. In that cobb-douglas economy, the resource dependance is capturated by v. It constitutes the income share of natural resources. Indeed, if the taxation is set to 0, $w_t/y_t = \beta$, $(r_t - \delta)k_t/y_t = \alpha$ and $p_t x_t/y_t = v$. The model developed here shows some ability in reproducing a resource curse. Indeed,

$$\frac{\partial \mu}{\partial v} = \mu \left[\frac{ln(1-\tau)}{1-\alpha} - \frac{\partial \tau}{\partial v} \frac{v-\theta}{(1-\tau)(1-\alpha)} \right] \tag{15}$$

which is always negative because $\frac{\partial \tau}{\partial v} > 0$.

Proposition 2. *This overlaping economy may be affected by a resource curse. It means that an increase in resource dependance is associated with a lower growth rate.*

Proof. Proof is reported in appendix B. □

This results is quite intuitive. Indeed, because of the exhaustibility of the resource stock, a given growth rate implies to save more and more on the resource. It is not surprising that when the importance of the resource increases growth is negatively affected because it necessites more saving to be sustained. However, the constant return to scale assumption has been losen.

To keep the constant return to scale assumtpion, an increase in v necessarily implies a decrease in either β or α by the same order (or both obviously). Since $\frac{\partial \mu}{\partial \beta} > 0$, an increase in v compensated by a decrease in β implies the same result. If the labor share in the production diminishes, households will earn lower wages so they cannot save enough to achieve the same growth. The effect of a decrease of α on growth is, for now, uncertain so we cannot conclude on the effect of a change in v compensated by a change in α. Nevertheless, a numerical simulation reported in appendix D tends to show that an increase in v compensated by a decrease in α is associated with a lower balanced growth for reasonable parameter values.

The effect of an increase in θ is:

$$\frac{\partial \mu}{\partial \theta} = \mu \left[-\frac{ln(1-\tau)}{1-\alpha} - \frac{\partial \tau}{\partial \theta} \frac{v-\theta}{(1-\tau)(1-\alpha)} \right] \tag{16}$$

which is always positive because $\frac{\partial \tau}{\partial \theta} < 0$.

Proposition 3. *When pollution hurts severely productivity, growth is higher.*

Proof. Proof is reported in appendix C. □

This effect may seems puzzling, it is nevertheless quite intuitive. θ diminishes the net resource's contribution to GDP. The BGP extraction rate is thus lower and that implies that less savings are needed to reach a given growth rate.

6 Implication of a change in the taxation scheme

The environmental taxation affects the extraction rate which in turn affects growth following:

$$\frac{\partial \mu}{\partial q_x} = \frac{-\mu(v-\theta)}{(1-\alpha)(1-\tau)} \frac{\partial \tau}{\partial q_x} \tag{17}$$

which is always positive for $\frac{\partial \tau}{\partial q_x} < 0$.

Proposition 4. *An increase in the environmental fiscality implies a larger growth rate.*

Proof. Proof is reported in appendix E. □

An increase in resource taxation means that resources are less attractive to produce and should be replaced by labor or man-made capital. It thus reduces the extraction rate on the balanced growth path. Since growth depends negatively on the extraction, the environmental taxation increases growth. Nevertheless, in the short run, an increase in environmental taxation impacts negatively the level of production. As in Schou (2000), the pollution decreases towards 0 on the balanced growth path because the quantity of resources used as an input decreases. Schou (2000) argues that this means that the environmental taxation is unnecessary. This paper challenges that view: in an OLG model, the extraction scheme (which describes the intertemporal allocation of resources and income) is affected by a resource tax, because agents are not infinitely lived and do not internalize the effect of their behaviour on future generation. Resource taxation, which is traditionnaly seen as having no effect in the infinitely lived agents framework, impacts the economy if its intergenerational dimension is considered. That will be discussed more precisely in the next section.

The labor taxation affects the extraction rate which in turn affects growth following:

$$\frac{\partial \mu}{\partial q_w} = \frac{-\mu(v-\theta)}{(1-\alpha)(1-\tau)} \frac{\partial \tau}{\partial q_w} \tag{18}$$

which is always negative for $\frac{\partial \tau}{\partial q_w} > 0$.

Proposition 5. *A decrease in labor taxes implies a larger growth rate.*

Proof. Proof is reported in appendix F. □

A decrease in wages taxation implies that a larger income is available for households to save. Since saving is the engine of capital accumulation, that put the economy on a larger BGP.

Combining proposition 4 and 5, we can imagine a change in the taxation scheme which allow to reach a higher growth. Since $\mu_p \times \mu_x = \mu_w$, this possible change in the taxation scheme does not modify the sustainability of the government budget because $\mu_g = \mu$ which means that on the BGP, the government expenditures to GDP ratio is constant. Nevertheless, since a change in the taxation scheme has also instantaneous effects, it can modify the level of that ratio.

7 Welfare analysis

The analysis of a Ramsey central planner problem[5] allow to study the optimality of the BGP. A benevolent social planner is assumed to solve the following problem:

$$\max_{\{c_t;d_t;m_t;k_t;e_t\}_{t=0}^{\infty}} = \frac{1}{1+\rho} ln(d_0) + \sum_{t=0}^{\infty} \frac{1}{(1+R)^{t+1}} \left[ln(c_t) + \frac{1}{1+\rho} ln(d_{t+1}) \right]$$

subject to

[5]See Ramsey (1928).

$$y_t = A_t k_t^{\alpha} x_t^{v} e_t^{-\theta} \tag{19}$$
$$y_t = c_t + d_t + k_{t+1} - (1-\delta)k_t \tag{20}$$
$$A_{t+1} = (1+a)A_t \tag{21}$$
$$e_t = \phi x_t \tag{22}$$
$$m_t = (1-\tau_t)m_{t-1} \tag{23}$$
$$x_t = \tau_t m_{t-1} \tag{24}$$
$$m_{-1} = \sum_{t=0}^{\infty} \tau_t m_{t-1} \tag{25}$$
$$k_0,\, m_{-1},\, e_{-1},\, A_0 > 0\,\text{given}, \tag{26}$$

where R is the social discount rate. (19) represents the production function. (20) established that the economy consumes or invets exactly its net production in each period. (21) represents the exogenous technological progress. (22) is the emissions implied by the resource use while (23) and (24) represents the dynamics of the resource. (25) is a total exaustibility condition for the resource while (26) represents initial endowments.

The FOC of the previous problems may be reduced to:

$$\frac{1+R}{1+\rho} = \frac{d_t}{c_t} \tag{27}$$
$$(1+\rho)\frac{d_{t+1}}{c_t} = \alpha A_{t+1} k_{t+1}^{\alpha-1} x_{t+1}^{v} e_{t+1}^{-\theta} + 1 - \delta \tag{28}$$
$$\frac{A_{t+1} k_{t+1}^{\alpha} x_{t+1}^{v} e_{t+1}^{-\theta}(v x_{t+1}^{-1} + \phi\theta e_{t+1}^{-1})}{A_t k_t^{\alpha} x_t^{v} e_t^{-\theta}(v x_t^{-1} + \phi\theta e_t^{-1})} = A_{t+1}\alpha k_{t+1}^{\alpha-1} x_{t+1}^{v} e_{t+1}^{-\theta} + 1 - \delta \tag{29}$$
$$\lim_{t\to\infty} \left(\frac{1}{1+R}\right)^t \frac{k_{t+1}}{c_t} = 0 \tag{30}$$

(27) is an intergenerational optimality condition, (28) is an intragenerational optimality condition, while (29) characterize the optimal intertemporal resource allocation. (30) is the transversality condition associated with the planner problem.

Proposition 6. *The optimal balanced growth path is definded by:*

$$\tilde{\mu}_k = \tilde{\mu}_y = \tilde{\mu}_c = \tilde{\mu}$$
$$\tilde{\mu}_x = \tilde{\mu}_e = \tilde{\mu}_m = 1 - \tilde{\tau}$$
$$\tilde{\mu}_a = 1 + a$$
$$\tilde{\mu} = (1+a)^{\frac{1}{1-\alpha}} (1-\tilde{\tau})^{\frac{v-\theta}{1-\alpha}}$$

Proof. Proof is reported in appendix G. □

Proposition 6 implies that the market equilibrium BGP is optimal if $\tau = \tilde{\tau}$. Using (27), (28), (29), we have $(1+a)\tilde{\mu}_k^{\alpha}\tilde{\mu}_x^{v}\tilde{\mu}_e^{-\theta}\tilde{\mu}_x^{-1} = (1+R)\tilde{\mu}_c$. Since $\tilde{\mu}_c = \tilde{\mu}_k$, it can be established that $\tilde{\tau} = \frac{R}{1+R}$.

We have seen that both labor and resource taxation influence the market balanced growth through the extraction rate. It is thus possible to decentralize the optimal BGP using a resource tax and/or a labor tax. From Propositions 4 and 5, we can conlude that for a given social discount rate, resource rich economies are characterized by an higher resource taxation rate and/or a lower labor taxation one.

Since $\frac{\partial \tilde{\mu}}{\partial v} < 0$, resource-dependent economies are always characterized by a lower optimal balanced growth rate.

8 Conclusion

In an overlapping economy, an increase in resource dependance can cause a resource curse due to the need to save more and more on the resource. A negative environmental externality, in the form of a productivity loss, helps the economy to reach a higher BGP. Nevertheless, the intergenerational dimension whithout altruism considered here imply a need for a regulatory policy. Indeed, the decentralization of the optimal balanced growth path may be achieved through labor and/or resource taxation. Resource-dependant economies should set an higher resource taxation and a lower labor one. However, the resource curse may be seen as an optimal phenomenon.

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Appendix

A Proof of Proposition 1

The proof uses continuously definitions 1 and 2.

- (2) at the BGP leads to $\mu_m = 1 - \tau$
- (1) at the BGP leads to $\mu_m = \mu_x$
- (8) at the BGP leads to $\mu_e = \mu_x$.
- From (13) at the BGP $\mu_s = \mu_k$.
- (12) at the BGP leads to $\mu_p = (1+a)\mu_k^{\alpha}\mu_x^{v-1}\mu_e^{-\theta}$
- (11) at the BGP leads to $\mu_w = (1+a)\mu_k^{\alpha}\mu_x^{v}\mu_e^{-\theta}$
- (9) at the BGP leads to $\mu_y = (1+a)\mu_k^{\alpha}\mu_x^{v}\mu_e^{-\theta}$
- (7) at the BGP leads to $\mu_a = 1 + a$
- (6) may be writted at the BGP as $\mu_p = 1 + r_{t+1} \Rightarrow \mu_r = 1$
- (5) at the BGP leads to $\mu_c = \mu_d$
- (10) at the BGP leads to $(1+a)\mu_k^{\alpha-1}\mu_x^{v}\mu_e^{-\theta} = 1 \Rightarrow \mu_k = (1+a)^{\frac{1}{1-\alpha}}(1-\tau)^{\frac{v-\theta}{1-\alpha}}$
- (5) in the CBI implies $w_t = c_t(2+\theta) \Rightarrow \mu_w = \mu_c$

B Proof of Proposition 2

Proposition 2 establish that the economy may be affected by a resource curse. This is always true if $\frac{\partial \tau}{\partial v} > 0$.

$$\frac{\partial \tau}{\partial v} = -\frac{\frac{\partial LHS}{\partial v} - \frac{\partial RHS}{\partial v}}{\frac{\partial LHS}{\partial \tau} - \frac{\partial RHS}{\partial \tau}}$$

Since $\frac{\partial LHS}{\partial \tau} - \frac{\partial RHS}{\partial \tau} < 0$, the sign of $\frac{\partial \tau}{\partial v}$ is the sign of its numerator. A sufficient condition that ensures $\frac{\partial LHS}{\partial v} - \frac{\partial RHS}{\partial v} \geq 0$ is

$$-\frac{ln(1-\tau)}{(1-\alpha)(1-\tau)}+\frac{\alpha(2+\rho)\tau ln(1-\tau)(1+q_x)}{(1-\alpha)\left[\beta\tau(1-q_w)(1+q_x)-(2+\rho)(1-\tau)v\right]}\geq 0$$

which may be reduced to a simpler form as

$$\beta\tau(1-q_w)(1+q_x)\geq v(2+\rho)(1-\tau)$$

which is always true for all $\tau\in]\hat{\tau},1[$.

C Proof of Proposition 3

Proposition 3 establish that the higher is the productivity loss due to pollution, the higer is growth. This is always true if $\frac{\partial\tau}{\partial\theta}>0$.

$$\frac{\partial\tau}{\partial\theta}=-\frac{\frac{\partial LHS}{\partial\theta}-\frac{\partial RHS}{\partial\theta}}{\frac{\partial LHS}{\partial\tau}-\frac{\partial RHS}{\partial\tau}}$$

Since $\frac{\partial LHS}{\partial\tau}-\frac{\partial RHS}{\partial\tau}<0$, the sign of $\frac{\partial\tau}{\partial\theta}$ is the sign of its numerator. A sufficient condition that ensures $\frac{\partial LHS}{\partial\theta}-\frac{\partial RHS}{\partial\theta}\geq 0$ is

$$\frac{ln(1-\tau)}{(1-\alpha)(1-\tau)}-\frac{\alpha(2+\rho)\tau ln(1-\tau)(1+q_x)}{(1-\alpha)\left[\beta\tau(1-q_w)(1+q_x)-(2+\rho)(1-\tau)v\right]}\leq 0$$

which is always true for all $\tau\in]\hat{\tau},1[$.

D Numerical simulation

E Proof of Proposition 4

RHS is not a function of q_x so its curve does not move following a move in q_x.

Since $\frac{\partial LHS}{\partial q_x}<0$, the response of τ to a change in q_x is represented in Figure 2.

It may be deduced from Figure 2 that an increase in q_x is associated to a lower extraction rate. Thus, $\frac{\partial\tau}{\partial q_x}<0$.

F Proof of Proposition 5

RHS is not a function of q_w so its curve does not move following a move in q_w.

Since $\frac{\partial LHS}{\partial q_w}>0$, the response of τ to a change in q_w is represented in Figure 3.

It may be deduced from Figure 3 that an increase in q_w is associated to a higher extraction rate. Thus, $\frac{\partial\tau}{\partial q_w}>0$.

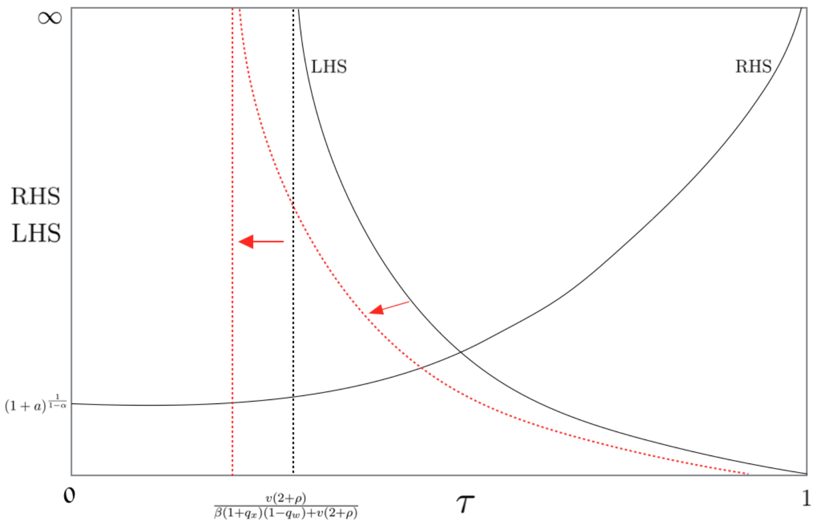

Figure 2: Effect of a change in q_x

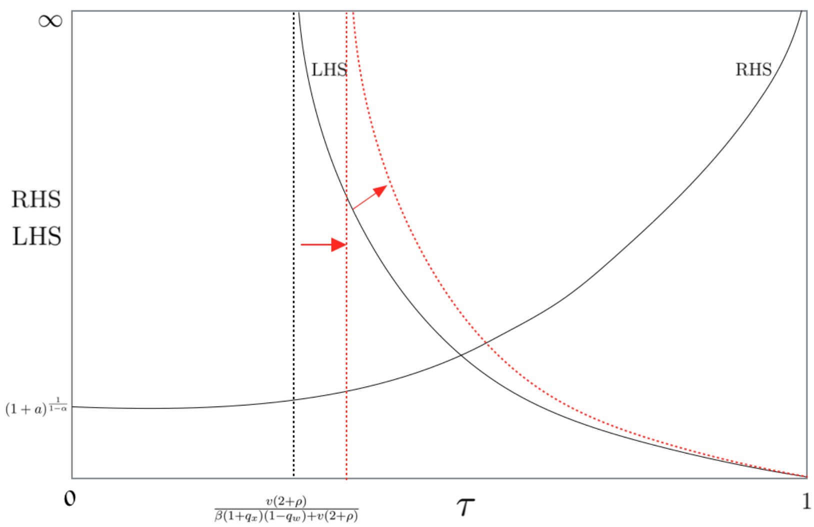

Figure 3: Effect of a change in q_w

G Proof of Proposition 6

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- (2) at the BGP leads to $\tilde{\mu}_m = 1 - \tilde{\tau}$.
- (1) at the BGP leads to $\tilde{\mu}_m = \tilde{\mu}_x$.
- (8) at the BGP leads to $\tilde{\mu}_e = \tilde{\mu}_x$.

- (27) at the BGP leads to $\tilde{\mu}_c = \tilde{\mu}_d$.
- (19) at the BGP implies that $\tilde{\mu}_y = (1+a)\tilde{\mu}_k^{\alpha}\tilde{\mu}_x^{v}\tilde{\mu}_e^{-\theta}$.
- (28) at the BGP gives $(1+a)\tilde{\mu}_k^{\alpha}\tilde{\mu}_x^{v}\tilde{\mu}_e^{-\theta} - (1-\delta) = A_{t+1}\alpha k_{t+1}^{\alpha-1}x_{t+1}^{v}e_{t+1}^{-\theta}$ At the BGP, it gives $1 = (1+a)\tilde{\mu}_k^{\alpha-1}\tilde{\mu}_x^{v}\tilde{\mu}_e^{-\theta}$ which implies that $\mu_k = \mu_y = \mu = (1+a)^{\frac{1}{1-\alpha}}(1-\tilde{\tau})^{\frac{v-\theta}{1-\alpha}}$.
- $\mu_k = \mu_y$ in (20) at the BGP imples that $\mu_c = \mu$.