Is Government Debt a Vamp?
Public Finance in a Transylvanian Growth Model

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Abstract

The vampire metaphor has been used in numerous papers describing biological interactions between two populations. Such a metaphor translates well to a standard endogenous growth model with public debt. Public debt can be assimilated to a Vamp, whose blood-sucking behavior corresponds to the harmful effect of the debt burden on productive public expenditures. However, the complete destruction of public debt in the long-run is shown to be socially undesirable, because this would imply too much distortionary taxation, with damaging effects on the balanced growth path. By identifying ecological or biological processes with usual national account relationships, this analysis is one step further in the integration of macroeconomics and environmental economics.

Keywords: public debt, vampire, ecological process

1. Introduction

The vampire metaphor has a long tradition in economic though.[1] Several authors[2] have outlined the importance of this metaphor to Marx's work when describing the relationship between capital and labour:

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[1]More generally, Gordon and Hollinger (1997), by a great inventory of works, highlighted the importance of the Vampire metaphor in the modern literature and filmography.

[2]See for example, Moretti (1982) and Neocleous (2003).

> "*Capital is dead labour, that, Vampire-like, only lives by sucking living labour, and lives the more, the more labour it sucks*" (Marx, 1867).

Besides, the allegory of Vampires was also used by liberal authors, such as Bastiat:

> "*In recent times great pains have been taken to stir up public resentment against that infamous, that diabolical thing, capital. It is pictured to the masses as a ravenous and insatiable monster, more deadly than cholera, more terrifying than riots, as a Vampire whose insatiable' appetite is feed by more and more of the lifeblood of the body politic.*" (Bastiat, 1850).[3]

Closer to home, the vampire metaphor has been useful in representing the ecological process of two interacting populations. The classic paper of Hartl and Mehlmann (1982) for example deals with a typical problem of renewable resources (the depletion of Human population) described in terms of an optimal control problem that optimizing Vampires must solve. More recently, Munz et al. (2009) analyses the mathematical modelling of an outbreak of zombie infection by using an epidemiological model to investigate the dynamics of a zombie apocalypse. For his part, Farhat (2013) builds an agent-based model to study the "economics of Vampires" and to investigate the dynamics of Human-Vampire interaction with heterogenous agents.

The synthesis of vampirism and macroeconomics was developed by Snower (1982) , in a model where Humans are not only receptacles of blood at the discretion of Vampires but compute an optimal strategy for their destruction. This looked like a promising avenue of research. Surprisingly, this integration between macroeconomics and ecological processes through the Vampires did not lead to many works. The main reason is probably that the Human-Vampire dynamics is a very special case of biological interaction, because the Vampires, which are considered as predators, actually turn the preys, considered as the

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[3]The metaphor of Vampires recently culminates in Whitman & Dow' book Economics of the Undead: Zombies, Vampires, and the Dismal Science (2014).

Humans, into their own kind, thus increasing their own population by blood-sucking, characteristic that is only encountered in epidemiological models. This paper shows that such a feature applies well to a simple macroeconomic system based on the evolution of public debt and private capital.

In this paper, we consider two populations, namely public debt and private capital, that interact through public expenditures that offer productive services. The debt burden acts as a vampire bite, which reduces available resources for productive public spending in the Government budget constraint, and lowers output and the rate of capital accumulation. Simultaneously, the collection of income-tax vanishes, generating further increases in public debt. Thus, like a Vampire, the public debt transforms private capital into its own kind. The optimal rate of depletion of non-infected capital (the consumption of goods, which, in the vampire-equivalent system, is assimilable to the harvesting of healthy souls) can then be obtained from a standard optimal control problem. Like Snower (1982), but for different reason,[4] the complete destruction of the vampire species (the public debt) is shown to be socially undesirable. Effectively, public debt allows consumption-smoothing along the optimal path: Humans might have invented Vampires to help managing the problem of intertemporal harvesting of healthy agents.

Section 2 develops a simple model of Human-Vampire interaction, following Hartl and Mehlmann (1982), Section 3 identifies the ecological process of the two populations to a simple transylvanian macro-framework, and Section 4 computes the optimal path of the economy. Section 5 shows that the extinction of the public debt-vampire is generally not optimal. Finally, Section 6 concludes and suggests some areas for future research on ecological macro-modelling.

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[4]Although presented in a dynamic framework, the argument that produces Snower's trade-off is static (fighting Vampires today or producing goods today). In our set-up, the trade-off is intertemporal: Society agrees to increase the amount of infected individuals today in exchange for a greater quantity of tomorrow healthy individuals.

2. Angels and Vampires: a fable for macroeconomists

In the depths of Carpathian Mountains stands the world of Vlad Drakul, inhabited by two species doomed to a perpetual struggle: Humans and Vampires. The origin of Vampires is the subject of heated discussions in vampiristic science, but one prominent theory, according to the apocryphal Books of Enoch,[5] is that Vampires are the offspring of the union between "Watchers", initially appointed to protect the human kind and human women. However, while watching Humans (and particularly beautiful women), they decided to abandon Heaven, and join them on Earth. From this union resulted children, known as Vampires, damned to feed on human blood and to fear sun-lights.

In Drakulworld, the human condition is quite unenviable: Humans must procreate, try to escape the Vampires or fight against them. But the fight is uneven, and many bitten Humans die and themselves become Vampires. The only escape is to die a natural death without being bitten, so as to ascend to Heaven and become Angels. In this way, at each period of time, a human stream is taken away from Drakulworld, turned into Angels and transferred to Heaven, while the other deaths are due to bites and swell the stock of Vampires. Thus, the problem of the Human specie is to optimally control the collection of healthy souls at each instant. In what follows we will show how Humans can use Vampires to carry out this task.

Let us somewhat formalize this discussion. By slightly modifying the set-up of Hartl and Mehlmann (1982) and Snower (1982), we can define the ecological process of the population of Vampires (V) and Humans (H) by

$$\dot{V} = (w - k)H + (\rho - \sigma)\,V, \tag{1}$$

$$\dot{H} = nH - \alpha\rho V - C. \tag{2}$$

Vampires feed on blood of Humans being alive, which is extracted from the Humans' neck through a blood-sucking. Therefore, ρ denotes the average of the

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[5]The "Book of Enoch" was declared apocryphal, and was denied entrance into the "approved" version of the original Bible by the Nicene Council of 325 A.D.

bitting frequency per period. Besides, we assume that at each instant a constant part of Vampires, denoted by σ, is exposed to sunlight, and so dies naturally.[6] Once bitten, the Human dies and becomes a member of the Vampire race with the probability α (happily, all Humans do not die following a biting). Beyond blood-sucking, it is well-known that Vampires can emerge through the charm of some evil wizards,[7] which we assume to be a constant fraction of the Human population. In this way, w denotes the odious task of evil wizards in fashioning werewolves, zombies, ghouls and other creatures generating a favorable environment for the reproduction of Vampires.

Furthermore, regarding Eq. (1), Humans can get rid of Vampires by recruiting vampire-hunters, assumed to be a constant proportion k of the human population. At this stage, we suppose that the technology to kill Vampires (as e.g. the preparation of stakes, crucifies, garlic necklaces and so on) is a scarce resource: only some Humans (the vampire hunters) control this technology, so that $w > k$ (as we will see below, there are reasons to suppose that $w > k$, because the optimally chosen k will really be less than w).

Finally, Eq. (2) describes the law of motion of the human population. At each instant, there are nH births and a flow of Humans dies as a result of Vampire bites ($\alpha \rho V$). In addition, Humans who are not bitten during their lifetime can die naturally. C denotes the flow of natural deceased, whose souls are collected to generate Angels. Finally, for the human race will survive, it must be the case that $nK \geq C$.

So, by (1) and (2), the ecological processes of both populations are

$$\frac{\dot{V}}{V} = \frac{w - k}{b} + \rho - \sigma, \text{ and } \frac{\dot{H}}{H} = n - \alpha\rho b - c, \tag{3}$$

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[6]Following Snower (1982), σ also corresponds to the rate of "vampire attrition".

[7]In the Russian folklore, for example, Vampires are perceived as old wizards or as Humans who have rebelled against the Orthodox Church (see Reader's Digest Association, 1988). In his famous book concerning the Vampire mythology, Créméné and Zemmal (1981) point out that, in the Romanian folklore, Vampires were known as "*strigoi*", which were described as living witches. More recently, during the years 2009-2010, in the famous TV series *Vampire Diaries*, the main character (Esther Mikaelson) is a powerful witch, who changes her husband and her children in Vampires by using a magic spell.

where $b = V/H$ is the Vampire-to-Human ratio, and $c = C/H$ denotes the Angel-to-Human ratio. This Human-Vampire ecology is a special case of the more general Lotka-Volterra specification common for modelling predator-prey interactions in biological system.[8] The growth rate of the vampire population ($\dot{V}/V$) positively depends on the Human-to-Vampire ratio ($1/b$), and the growth rate of the human population ($\dot{H}/H$) negatively depends on the Vampire-to-Human ratio (b).

So, by (3), the growth rate of the vampire ratio evolves according to

$$\frac{\dot{b}}{b} = \frac{w-k}{b} + \alpha\rho b + c + \rho - n - \sigma. \tag{4}$$

The behavior of this ecological process is described in the following Theorem.

Theorem 1. *(The Vampire Epidemic Theorem) There is a critical level of Angel per Human $\bar{c} = n + \sigma - \rho - 2\sqrt{\alpha\rho(w-k)}$, such that*[9]

  i.   *If $c > \bar{c}$, it is impossible for the human race to survive (i.e. $b \to +\infty$).*

  ii. *If $c \leq \bar{c}$, there is a fragile ecological equilibrium between Humans and Vampires. Indeed, there are two positive steady-states (b_1^* and b_2^*), where the low steady-state b_1^* is stable, while the high steady-state b_2^* is unstable.*

Proof: See Appendix A.

On the one hand, when the flow of angel collection is "high" ($c > \bar{c}$), the vampire population increases indefinitely relative to the human population, generating an epidemic of Vampires (i.e. $b \to +\infty$), as shows the Figure 1a. Indeed, when the Angel-to-Human ratio c is upper the critical level $\bar{c}$, the collection of

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[8]In such a specification, the usual ecological processes of the predator population (x) and the prey population (y) are given by the following relationships: $\dot{x} = xf(x,y)$, $\dot{y} = yg(x,y)$. Besides, for a global view on the prey-predator models, see especially Abrams (2000), and Berryman (1992).

[9]We assume that the Human birth rate is sufficiently high to cover the effective net blood-sucking frequency, i.e. $n > \rho - \sigma$, and that the difference $w - k$ is small. In this way, the critical ratio $\bar{c}_k$ is positive.

Angels is so large that the rate of depletion of the human population becomes unsustainable relative to the rate of growth of the vampire population.

On the other hand, when the flow of angel collection is "low" (i.e. $c \leq \bar{c}$), in particular in the case without any angel collection ($C = 0$), a fragile ecological equilibrium between human and vampire populations can emerge, as shows the Figure 1b. If the initial Vampire-to-Human ratio $b(0)$ is greater than the unstable steady-state b_2^*, the extinction of the Human race is inevitable, but if $b(0) < b_2^*$, the Vampire-to-Human ratio converge to the steady-state b_1^*: the Human race survives in the long-run, despite Vampires. Therefore, according to Theorem 1, the control of the collection of healthy souls is a necessary but not sufficient (if $b(0) > b_2^*$) condition to induce a dynamically stable steady-state compatible with the survival of the human race.
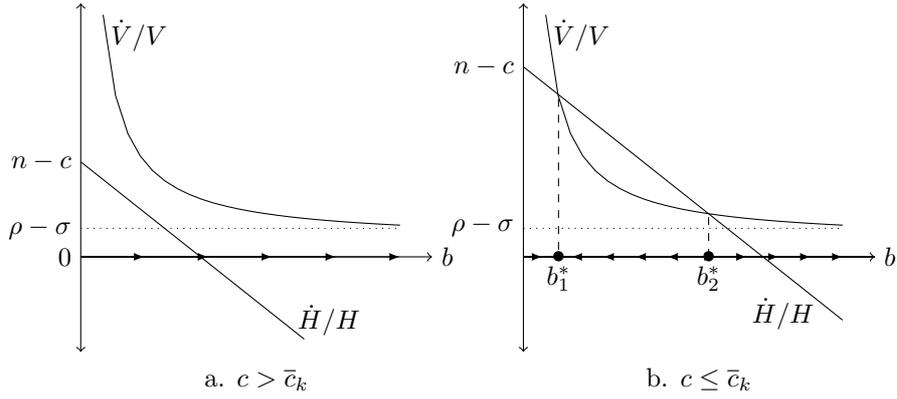

a. $c > \bar{c}_k$ b. $c \leq \bar{c}_k$

Figure 1: The ecological dynamics of the Human-Vampire model

3. A Transylvanian macro-model

Our Transylvanian economy consists of two sectors: a public sector (the Government) and a representative Household (that stands for the private sector). In equilibrium, the economy is described in a rather standard way by two differential equations: the Government budget constraint and the goods equilibrium. On the one hand, the Government budget constraint depicts the law

of motion of the public debt (B):

$$\dot{B} = rB + Z - \tau Y, \tag{5}$$

where r denotes the real interest rate, Z represents public spending, which is assumed to provide productive services, and $\tau < 1$ is the tax-rate on output (Y). On the other hand, the IS equilibrium provides the simple dynamics of capital accumulation:

$$\dot{K} = Y - Z - C \tag{6}$$

where K is private capital, and C denotes consumption. All variables are in real terms and per capita, with population normalized to be one. Finally, following Barro (1990), output is produced by private capital and the flow of productive services provided by public expenditures, namely

$$Y = F(K, Z) \tag{7}$$

For simplicity, the production function is described by a simple linear form: $F(K, Z) = A(K + Z)$, with A a technological parameter. In addition, the Government follows a fiscal rule imposing a target $\overline{G}$ for total public spending:

$$rB + Z = \overline{G}. \tag{8}$$

In this way, by introducing (8) in (5) and (6), we obtain the ecological process of the public debt and private capital

$$\dot{B} = (1 - \tau A)\overline{G} - \tau A\ K + \tau A r B, \tag{9}$$

$$\dot{K} = AK + (A - 1)(\overline{G} - rB) - C. \tag{10}$$

This system mimics the Human-Vampire framework of the preceding Section. Public debt can be consider as a Vamp, which bites private capital (the human population), through the debt burden (rB).[10] Effectively, given the fiscal rule

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[10]The fact that the public debt burden crowds out productive public expenditures, or public investment, is well documented in the literature. See, for example, Oxley and Martin (1991) and, Alesina et al. (2014) or IMF (2014, chapter 3).

(8), any increase in the debt burden reduces productive public expenditures (Z), and output (Y).

On the one hand, the Vampires' sucking impact on capital accumulation depends on coefficient $A - 1$. If $A > 1$, the blood-sucking hampers the capital path, because the net impact of the productive public spending on goods market equilibrium is positive (i.e. $\partial(Y - Z)/\partial Z > 0$). Conversely, if $A < 1$, the Vampires' sucking favors capital accumulation by exerting a life-saving bloodletting that bled the patient, as in medicine of ancient times. Effectively, this case corresponds to an over accumulation of public goods[11] (i.e. $\partial(Y - Z)/\partial Z < 0$). In what follows, we exclude the latter situation by focusing on efficient solutions.

On the other hand, the tax-rate τ, which allows reducing public debt, represents the part of the vampire hunters on the human population (first term of Eq.(9)). Nevertheless, blood-sucking increases the stock of Vampires (public debt) by reducing output and the associated level of tax, hence the biting efficiency $\tau A r$ (last term of Eq.(9)). Thus, everything happens as if each unit of sucked capital became Vampires, notwithstanding the efficiency parameters.

To solve the Transylvanian model, it is not unrealistic to set $\overline{G} = \theta Y$, where $\theta < 1$ is the public spending target, as a function of GDP. Thus, system (9)-(10) becomes formally equivalent to the Human-Vampire system (1)-(2), namely

$$\dot{B} = (w - k)K + (\rho - \sigma) B, \tag{11}$$

$$\dot{K} = nK - \alpha\rho B - C. \tag{12}$$

with the following identification of parameters

$$w := \frac{\theta A}{1 - \theta A}, \ k := \frac{\tau w}{\theta}, \ \rho := rw, \ \sigma := rk, \ n := \frac{A\left(1 - \theta\right)}{1 - \theta A} > 1, \ \alpha := \frac{A - 1}{\theta A} > 0.$$

In Eq. (12), consumption and the public debt burden can be assimilated to withdrawn flows from a renewable resource (the stock of private capital), in link with the evolution of public debt in Eq. (11). Therefore, according to Theorem 1, too much harvesting (i.e. a high consumption ratio $C/K > \bar{c}$) conducts to the

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[11]See Arrow and Kurz (1970).

asymptotical elimination of the capital species. Only a suitable withdrawal of consumption (i.e. a low consumption ratio $C/K < \bar{c}$) may give rise to a stable stationary equilibrium with positive long-run stock of capital, conditionally to a not-to-high initial debt-to-capital ratio $(b(0) < b_2^*)$. Now, let us turn our attention to the optimal consumption policy.

4. Thwarting the Vampires: the optimal collection of healthy-souls

According to previous assumptions, the souls of uninfected deceased ascend to Heaven and become Angels. In this way, the human social welfare, denoted by the function $u(\cdot)$, depends solely on the angel consumption flows per period (C), where as usual, $u'(C) > 0$ and, $u''(C) < 0$.

Suppose that a benevolent authority attempts to maximize the intertemporal welfare from present to infinite future associated with the optimal angel collection at each instant, subject to the ecological process of the two species (5) and (6). By substituting (5) into (6), the optimal depletion of human population and the optimal evolution of vampire race are found by solving the following optimal control problem

$$\max \left\{ W := \int_0^{+\infty} e^{-\beta t} u(C) \, dt \right\}, \tag{13}$$

$$\text{subject to:} \quad \dot{K} + \dot{B} = (1-\tau)Y + rB - C,$$

$$B(0), K(0) \in \mathbb{R}^+,$$

where β is the discount rate. Using a standard isoelastic utility function

$$u(c) = \begin{cases} S\left(c^{(S-1)/S} - 1\right)/(S-1), & \text{if } S \neq 1, \\ \log(c), & \text{if } S = 1, \end{cases} \tag{14}$$

where $S > 0$ denotes the intertemporal elasticity of substitution, the first order condition of the programm (13) conducts to the well-known Keynes-Ramsey relationship

$$\frac{\dot{C}}{C} = -\frac{u'(C)}{Cu''(C)}(r - \beta) = S(r - \beta), \tag{15}$$

where, according to the usual no-arbitrage condition: $r = (1-\tau)\frac{\partial Y}{\partial K} = (1-\tau)A$.

Relation (15) has a standard interpretation. By reducing the souls-collection by one unit today, Society looses the current marginal utility of consumption $(u'(C_t))$. This increases the reproduction of the human race, which depends on the rate of regeneration (A) and on the probability of the surviving agents not to be requisitioned for the fight against Vampires (that roughly corresponds to the fraction of Humans that is allocated to reproduction $(1-\tau)$), thus allowing more soul consumption tomorrow. Thus the return of current souls-saving is $(1-\tau)Au'(C_{t+1})$ tomorrow, that is to say, in current value: $(1-\tau)Au'(C_{t+1})/(1+\beta)$. Finally, Eq. (15) corresponds to the continuous-time version of this no-arbitrage condition.

The Transylvanian macro-model can now be solved by defining variables that are constant in steady-state. Let $b = B/K$ define the public debt to capital ratio and $c = C/K$ define the consumption to capital ratio, as in previous Sections, the law of motion of these two variables are the following, substituting A with n in system (11)-(12)

$$\frac{\dot{c}}{c} = S\left[(1-\tau)A - \beta\right] - n + (n-1)rb + c, \tag{16}$$

$$\frac{\dot{b}}{b} = \frac{(\theta - \tau)n}{(1-\theta)b} - \frac{(\theta - \tau)n}{1-\theta}r - n + (n-1)rb + c. \tag{17}$$

The behavior of this ecological process is described in the following Theorem.

Theorem 2. *The unique steady-state (b^*, c^*) is characterized by saddle-path stability, where*

$$b^* = \frac{(\theta - \tau)n}{S(r - \beta)(1-\theta) + (\theta - \tau)rn} \geq 0, \tag{18}$$

$$c^* = n - (n-1)rb^* - S(r - \beta) > 0. \tag{19}$$

Proof: First, by (16) and (17), the conditions of steady-state $\dot{c} = 0$ and $\dot{b} = 0$ directly implies the relation (18) and (19).

Second, we assume that parameters are such that $c^* > 0$ and $b^* \geqslant 0$. The last inequality is true in particular if the quantity of "debt-killers" (the tax-rate) is not too large, *i.e.* $\tau \leqslant \theta$. By linearizing system (16)-(17) in the neighborhood

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of the steady-state, we obtain

$$\begin{pmatrix} \dot{c} \\ \dot{b} \end{pmatrix} = \mathbf{J} \begin{pmatrix} c - c^* \\ b - b^* \end{pmatrix}, \text{ where } \mathbf{J} := \begin{bmatrix} c^* & (n-1)\, rc^* \\ b^* & (n-1)\, rb^* - \frac{(\theta - \tau)n}{(1-\theta)b^*} \end{bmatrix},$$

and $\text{Det}\,(\mathbf{J}) = -c^* \left[S\,(r - \beta) + \left(\frac{\theta - \tau}{1 - \theta} \right) nr \right] < 0$.

Consequently, as the determinant of the Jacobian matrix is negative, the steady-state is characterized by saddle-path stability, as in Figure 2.

In addition, the saddle-point path can be reached if $c\,(0) > 0$, namely if $b\,(0) < b_2$, with: $b_2 := b^* + c^*(c^* - \lambda)/(n-1)\, rc^*$, where $\lambda < 0$ stands for the unique stable eigenvalue of system (16)- (17). $\qquad\qquad\square$

Figure 2 displays the phase diagram of system (16)- (17), for $\theta > \tau$ [12]
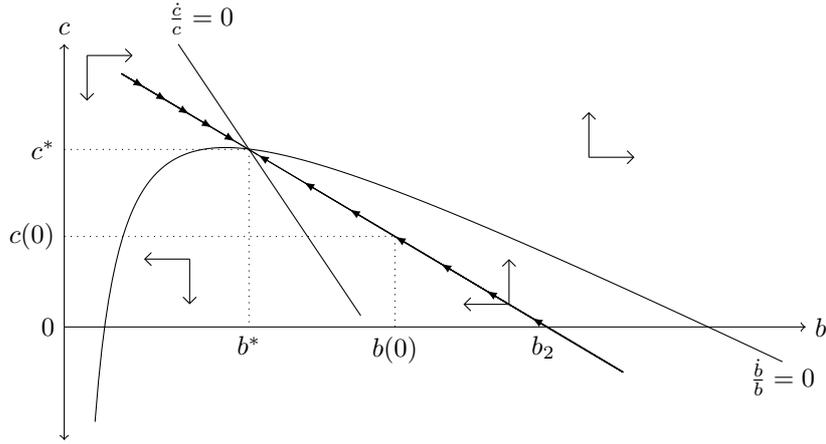

Figure 2: Stability of steady-state

Given an initial debt-to-capital ratio $b\,(0)$, the initial consumption ratio $c\,(0)$ must jump in order to place the economy on the saddle-point path. In the long-run, the economy approaches the steady-state with constant debt and con-

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[12]The $\dot{c} = 0$ function is downward-sloping in the $(c - b)$ space, while the $\dot{b} = 0$ function is hump-shaped, effectively: $\left.\frac{dc}{db}\right|_{\dot{c}=0} = -r\,(n-1)$ and, $\left.\frac{dc}{db}\right|_{\dot{b}=0} = -r\,(n-1) + \left(\frac{\theta - \tau}{1 - \theta} \right) \frac{n}{(b^*)^2}$. Notice that the latter is higher than the former since $\theta > \tau$.

sumption ratios b^* and c^*. In conformity with Theorem 1, a very high initial debt-to capital ratio (i.e. $b(0) > b_2$) is not compatible with the convergence to the steady-state.

As it must now be clear, the simple model of the Human-Vampire conflict has very interesting implications for the dynamics of consumption and public debt in a standard endogenous growth model. Like Vampires, public debt is renewed by sucking the substance of capital, through the eviction of productive public spending. The analogy can be pushed a little further. Snower (1982) offers a "Vampire neutrality theorem", stating that the spontaneous generation of Vampires (an exogenous increase in the Vampires to Humans ratio) is neutral in the long-run. The same property arises in our model: any debt relief, in the form of a reduction in the debt-to-capital ratio at some initial date will cause no long-term effect (and, particular will leave unchanged the debt-to-capital ratio in steady-state). This property comes from the fact that there is no hysteresis: the steady-state in independent on initial values.[13] Thus, the only effect of a debt relief will be to produce an immediate upwards jump in consumption, as in Figure 2, and variables adjust to the unchanged steady-state along the unchanged saddle path. Such a configuration might describe the difficulties faced by many developing countries to stabilize public debt without structural changes in their public stance (*i.e.* without change in some parameter).

Should public debt be removed in the long run? In Snower' s analysis, eliminating Vampires in the long-run is not optimal, because the fight against Vampires raises costs (Vampire hunters are subtracted from the production of goods that provide utility flows –"*widgets*"). In our framework the fight against public debt-Vamp does not entails such direct cost, and, by judiciously choosing the coefficient of debt hunters (the tax-rate), Humans can definitively get rid of Vampires in the long-run: $b^* = 0$ if $\tau = \theta$ in Eq.(17). Nevertheless, the following

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[13] More precisely, our model offers no hysteresis when variables are expressed as ratio of the capital stock. This is not the case in terms of variables expressed in level: a debt relief at t=0 will exert an effect on the public debt path in the long-run, because the whole path of public debt will lower at any time.

Section shows that such a balanced budget rule would not be socially desirable.

5. Why does Vampires perpetuate? The optimal destruction of public debt in the long-run

This Section shows that the complete destruction of public debt in the long-run is generally suboptimal, from a second best perspective.[14] To this end, we suppose that a benevolent Government can fix the tax-rate (*i.e.* the rate of debt hunters or the fraction of the human population that is devoted to the fight against Vampires) in order to maximize Households' welfare. Our main result is that, under quite general assumptions, the optimal tax rate is less than the public expenditure ratio, namely: $\hat{\tau} < \theta$. Therefore, positive ratios of public deficit and public debt are desirable in the long-run. In order to more easily demonstrate this result, we define the deficit ratio as $\delta := \dot{B}/Y = \theta - \tau$. It follows that: $\tau = \theta - \delta \leq \theta$ if $\delta \geqslant 0$, and we prove below that the optimal deficit ratio is positive ($\hat{\delta} > 0 \Rightarrow \hat{\tau} < \theta$).

Since public debt comes from the accumulation of deficits, its growth rate is simply: $\dot{B}/B = \delta Y/B$, and, in steady-state, the public debt ratio becomes:[15] $b^* = \delta A / [\gamma^* (1 - \theta A) + \delta A r]$, where $\gamma^* := \dot{B}/B = \dot{K}/K = \dot{C}/C < r$ stands for the Balanced Growth Path (hereafter BGP) in the long-run.[16] In steady-state, for a positive rate of economic growth ($\gamma^* > 0$), public debt is positive as long as the deficit ratio is positive ($\delta \geqslant 0 \Leftrightarrow b^* \geqslant 0$). From (18), we obtain:

$$b^* (\delta) = \frac{\delta A}{S \left[(1 - \theta + \delta) A - \beta \right] (1 - \theta A) + \delta A^2 (1 - \theta + \delta)}, \qquad (20)$$

Intuitively, any increase of deficit ratio reduces the tax rate ($d\tau/d\delta < 0$) and, for small deficit ratio (formally: $\delta \to 0$), rises the debt-to-capital ratio

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[14]The first best equilibrium is associated to the feasibility of lump-sum taxation. First best welfare is thus independent of the level public debt, from the Ricardian equivalence theorem. Therefore, we consider here a second best Ramsey problem for a benevolent Government.

[15]We use the fact that: $b^*\gamma^* = \delta Y/K = \delta A(1 - rb^*)/(1 - \theta A)$.

[16]The standard transversality condition imposes that: $\gamma^* < r$, which corresponds to a No-Ponzi game condition for public debt (i.e. the public debt growth rate has to be lower than the real interest rate).

$(db^*/d\delta > 0)$.[17] Furthermore, the consumption ratio is, in the long-run

$$c^* = n - f(\delta) - \gamma^*, \tag{21}$$

where $f(\delta) := (n-1)(1-\theta+\delta)Ab^*(\delta)$, and $c^* > 0$ for "small" deficit ratio (see Appendix B).

In a second best perspective, the benevolent Government chooses the optimal deficit ratio in order to maximize Household's welfare, subject to equilibrium conditions arising from optimization behaviour in a decentralized economy. Along the BGP, Household's welfare (13) becomes[18]

$$U = \begin{cases} \dfrac{1}{\beta^2}\{\beta[\log(c^*) + \log(K(0))] + \gamma^*\}, & \text{if } S = 1, \\[3mm] \dfrac{S}{S-1}\left\{\dfrac{(c^*K(0))^{(S-1)/S}}{\beta - \gamma^*(S-1)/S} - \dfrac{1}{\beta}\right\}, & \text{if } S \neq 1, \end{cases} \tag{22}$$

where $K(0)$ is the initial capital stock (that we normalize to be one).

Theorem 3. *(Optimal public debt ratio in the long-run) In the long-run, the optimal public debt ratio is positive. In other words, the total elimination of Vampires is undesirable.*

Proof: From (21), we compute

$$\frac{dc^*}{d\delta} = -f'(\delta) - \frac{d\gamma^*}{d\delta}, \tag{23}$$

where $f'(\delta) = (n-1)A\left[b^*(\delta) + (1-\theta+\delta)\frac{db^*}{d\delta}\right] > 0$. The First Order Condition for the maximization of welfare (22) is the following:

$$P(\delta) := \left[\beta - \left(\frac{S-1}{S}\right)\gamma^*\right]\frac{dc^*}{d\delta} + c^*\frac{d\gamma^*}{d\delta} = 0, \tag{24}$$

with the associated second order condition $P'(\delta) < 0$.

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[17]Remark that: $\left.\dfrac{db^*}{d\delta}\right|_{\delta \to 0} = A/\{S[(1-\theta)A - \beta](1-\theta A)\} > 0$.

[18]We focus on steady-state welfare effects, namely we compare different BGPs associated with different values of the deficit ratio. In other words, we are not interested in the transition from a steady-state to another, and we do not study transitional dynamics following a change of parameters but only perform comparative statics among different BGPs.

Appendix C shows that, if β takes "small" values, which is the case because β is a discount rate, we have $P(0) > 0$ and $P'(0) < 0$. Thus, for reasonable values (close to zero) of the deficit ratio, we can use the following first order approximation of relation (24): $P(\delta) \approx P(0) + P'(0)\delta = 0$, so that the optimal deficit ratio is simply: $\hat{\delta} \approx -P(0)/P'(0) > 0$. In particular, if $S = 1$ and $\beta \to 0$, we obtain a very simple expression of the optimal deficit ratio:

$$\hat{\delta} = \frac{\theta A (1 - \theta)(1 - \theta A)}{A - 1 + (1 - \theta A)^2}.$$

$\square$

Theorem 3 shows that adopting a balanced-budget rule ($\delta = 0$) would be generally suboptimal in the long-run. Effectively, this would imply to set a too high tax-rate, which would reduce the incentive for saving and the long-run rate of economic growth. The optimal tax-rate (or the optimal proportion of vampire hunters) must be set at a level that maintains some Vampires alive. By removing less Humans from the sector of reproduction, this would increase the rate of growth of human population, hence allowing harvesting more healthy souls in the future, but at the cost of more human bites today, that reduces the current possibilities of soul consumption. This trade-off demonstrates the usefulness of Vampires, which allow the Humans to optimally manage their intertemporal harvesting policy. In some sense, Humans need to keep their enemies alive and, if the Vampires did not exist, they would gain to be created (but in low quantity, because, as shown by the fragile ecosystem in our model, a massive creation of public debt can trigger an epidemic of Vampires).

6. Conclusion and further remarks

Public debt can be viewed as a "Vamp" who sucks the productive flow of public expenditure and regenerates by weakening Governments' fiscal stance. However, in our model, a total elimination of public debt is shown to be sub-optimal, because this would involve to fix the tax-rate at such a level that economic growth would too much deteriorate. Thus, there is a trade-off between current consumption and future consumption, which optimally results in

a positive public debt ratio in second best equilibrium. On the one hand, our paper shows that Snower (1982)' s classic model of Vampires may relevantly be suited to economic policy considerations. On the other hand, it proves the interest of specifying simple macroeconomic relationships as ecological processes and shows that epidemiological models can accurately depict the overall balances of national accounts. One shortcut of our model is that the rate of economic growth does not depend on productive public expenditures, contrary to Barro (1990). This property comes the assumption of linear production function. However, considering a more general technology would complicate the analysis without jeopardizing the existence of an optimal level of public debt.[19]

Our model can be applied to many other contexts, involving the transformation of a "good" specie into a "bad" one. Environmental economics offers a natural extension, for example if the "good specie" is considered as the stock of environmental quality and the "bad" specie is identified to the stock of toxic waste. The emission of waste is a by-product of economic activities, so that pollution may act as a Vampire who contaminates environmental quality while reproducing the stock of toxic waste.[20] Applications to more distant areas of "sustainable development" like terrorism or corruption, for example, can also be considered. Terrorists can be seen like Vampires whose activities force States to take coercive measures that weaken democracy and that give rise to martyrs, thereby constituting a favorable environment for terrorism, therefore for the regeneration of their own specie.[21] Corruption, broadly defined, can produce a similar mechanism: the generalisation of illegal activities in some districts can lead some "honest" citizens to become Vampires, namely infected or corrupt ones, being attracted by the gains these activities generate. However, if taxing pollution reduces global factor productivity, or if too much productive resources

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[19]See Minea and Villieu (2010, 2012) for an extensive analysis of public debt in an endogenous growth model with productive public spending.

[20]Such contamination effects also operate in financial market, where speculative bubbles can transform high-quality assets into toxic or junk bonds, generating the explosion of vampire-assets.

[21]See Kama et al. (2015).

are devoted to the fight against terrorism or corruption, citizens incur a loss of welfare, hence the possibility of an optimal harvesting of pollution, terrorism or corruption.

Along these lines, a possible extension of our model would be to consider optimal strategies of Vampires. In our framework, this would not make sense, but one could imagine that another player (the Government) generates Vampires (public debt) to defend specific (electoral) interest. The interplay between citizens and Government, or any agent that takes benefits from terrorism, corruption, pollution etc, could then be captured through a differential game.

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Appendix A

PROOF OF THEOREM 3.

Using the ecological process (4), $\dot{b} = 0 \Leftrightarrow \phi(b) = 0$, where

$$\phi(b) := \frac{w - k}{b} + \alpha\rho b + c + \rho - n - \sigma. \tag{A1}$$

First, as $w > k$, it is clear that $\lim_{b \to 0} \phi(b) = +\infty$, $\lim_{b \to +\infty} \phi(b) = +\infty$, and $\phi \in C^{\infty}(]0, +\infty[)$. Second, as $b \geq 0$, differentiating with respect to d we have: $\phi'(b) \geq 0 \Leftrightarrow b \geq \bar{b} := \sqrt{(w-k)/\alpha\rho}$. Thus, ϕ is continuously decreasing (resp. increasing) on $]0, \bar{b}]$ (resp. on $[\bar{b}, +\infty[$). Third, by (A1), $\phi(\bar{b}) = 2\sqrt{\alpha\rho(w - k)} + c + \rho - n - \sigma \leq 0 \Leftrightarrow c \leq \bar{c}_K := n + \sigma - \rho - 2\sqrt{\alpha\rho(w - k)}$, where $\bar{c}_K$ is supposed positive.

Finally, if $c > \bar{c}_k$, $\phi(b) > 0$, $\forall b \geq 0$. Thus, there is no steady-state and the Vampire-to-Human ratio diverges to infinity, i.e. $b \to +\infty$. Conversely, when $c \leq \bar{c}_k$, according to the intermediate value theorem, it exists two positives levels: $b_1^* \in]0, \bar{b}]$, and $b_2^* \in [\bar{b}, +\infty[$, such as: $\phi(b_i^*) = 0$, $i = 1, 2$. Therefore, the steady-state b_1^* is stable, while b_2^* is unstable. $\qquad \square$

Appendix B

Lemma. There is a critical value $\bar{\delta} > 0$, such as: $\forall \delta \leq \bar{\delta}$, $c^* \geq 0$.

Proof. First, by (21), $\partial c^* / \partial \delta = -\partial f(\delta) / \partial \delta - \partial \gamma^* / \partial \delta$. Yet, it is clear that $\partial \gamma^* / \partial \delta \geq 0$, and $\partial f(\delta) / \partial \delta \geq 0$, since $db^* / d\delta > 0$. Thus, c^* is a continuous and decreasing function with respect on the deficit ratio δ.

Second, without deficit, namely $\delta \to 0$, by (20), $b^* \to 0$ and, $\gamma^* \to S[(1 - \theta)A - \beta] < +\infty$. Therefore, as $n > 1$, $c^*|_{\delta=0} \to n - \gamma^* > 0$, since the transversality condition assures that $\gamma^* < r < 1$. In addition, by (21), when the deficit ratio is very high, namely $\delta \to +\infty$, $\gamma^* = S[(1-\theta+\delta)A-\beta)] \to +\infty$, and by (20), $f(\delta) \to n - 1 < +\infty$. Consequently, when $\delta \to +\infty$, the transversality condition remains true since $r = A(1 - \theta + \delta) \to +\infty$, and $c^* \to -\infty$.

Finally, according to intermediate value theorem, there is a critical value of deficit ratio $\bar{\delta} > 0$, such as: $c^* \geq 0$, $\forall \delta \in [0, \bar{\delta}]$. $\qquad\qquad\square$

Appendix C

First, we show that there is a critical value $\bar{\beta} > 0$, such that: $P(0) > 0$. From Eq. (23), using the fact that: $\beta - \gamma^*(S - 1)/S = S(r - \beta) - r$, we find

$$P(\delta) = (\gamma^* - r) f'(\delta) + (c^* + \gamma^* - r) \frac{d\gamma^*}{d\delta} = 0, \qquad (C1)$$

where $f'(\delta) = (n - 1)A[b^*(\delta) + (1 - \theta + \delta) db^*/d\delta] > 0$. For $\delta = 0$, using $d\gamma^*/d\delta = SA$, it comes: $P(0) = [S(r - \beta) - r]f'(0) + (n - r)AS$, with

$$f'(0) = (n - 1)A(1 - \theta)\frac{db^*}{d\delta} \quad \text{and,} \quad \frac{db^*}{d\delta} = \frac{A}{S[(1 - \theta)A - \beta](1 - \theta A)}.$$

By substituting these values in (C1), we obtain

$$P(0) = \frac{An[(A - 1) + A\theta S(1 - \theta A)]}{(1 - \theta A)[(1 - \theta)A - \beta]}(\bar{\beta} - \beta),$$

hence; $P(0) > 0$ if $\beta < \bar{\beta} := (1 - \theta)A\{1 - (A - 1)/S[(A - 1) + A\theta S(1 - \theta A)]\}$.

Second, we show that $P'(0) < 0$. To this end, we consider a linear approximation of (20) in the neighborhood of $\delta = 0$. In this way: $\exists \epsilon > 0$, $\forall \delta \in [0, \epsilon]$,

$$b^*(\delta) = \frac{A\delta}{S[(1 - \theta)A - \beta](1 - \theta A)}. \qquad (C2)$$

By (C1), we compute

$$P'(\delta) = (\gamma^* - r) f''(\delta) + (c^* + \gamma^* - r)\frac{d^2\gamma^*}{d\delta^2}$$
$$+ \left[\frac{dc^*}{d\delta} + \left(\frac{S - 1}{S}\right)\left(\frac{d\gamma^*}{d\delta} + f'(\delta)\right)\right]\frac{d\gamma^*}{d\delta}.$$

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Now, as $dc^*/d\delta = -d\gamma^*/d\delta - f'(\delta)$ and, $d^2\gamma^*/d\delta^2 = 0$, we have

$$P'(\delta) = (\gamma^* - r) f''(\delta) - \frac{1}{S} \left(\frac{d\gamma^*}{d\delta} + f'(\delta) \right) \frac{d\gamma^*}{d\delta}.$$

Yet, $f'(\delta) = (n-1) A [b^*(\delta) + (1 - \theta + \delta) db^*/d\delta]$ and, since for small values of deficit ratio (formally if $\delta \to 0$), $d^2 b^*/d\delta^2 = 0$ in (C2), we have $f''(0) = 2(n-1) A db^*/d\delta > 0$, which implies that $P'(0) < 0$, so that the second order condition is verified in the neighborhood of $\delta = 0$. $\qquad \square$