Is a Long War Desirable?
Optimal Debt Concessions in Attrition Warfare

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Abstract

Why reducing budget deficits policies are delaying the debt stabilization? The answer of this paper is that such policies appear as concession programs in order to extend the distributional conflict generated by the process leading to a stabilization. Thus, the present paper presents an extension of the standard war of attrition model, extended by including an exogenous time where each political group optimally concedes by accepting to finance more deficits. I obtain a Nash symmetric equilibrium which positively connects the optimal level of concession with the degree of political polarization. More generally, the more the peace, namely the debt stabilization, is expected to be costly for all political groups, the more they concede to stay longer in war, in the pre-stabilization period. Finally, this paper exemplifies this model in the recent Greek debt crisis.

1. Introduction

As a reaction to the Great Recession, many Europe countries adopted vast fiscal policies often including austerity measures, traditionally designed to reduce the stabilization deadline of their public debt by dropping the debt burden. Especially, in the famous Greek sovereign debt crisis, in return of European bailouts (May 2010, and July 2011), major austerity plans were adopted, in particular by the large coalition of A. Samaras in November 2012 and January 2013, which planned 18 billion euros of savings by 2016. Nevertheless, the Greek debt has continued to grow: rising from 105% of GDP in 2008, to 177% of GDP end-2014, while at the same time, austerity measures have generated a primary budget surplus since 2013. Finally, these policies failed to reduce the debt stabilization delay, as the widely known Italian case. Indeed, the strong Italian debt, which was 116% of GDP in 2011, led the new Prime Minister C. Monti to adopt drastic austerity measures which generated a primary budget surplus since 2011, but also failed to stabilize debt, since it was 123% a year later.[1]

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[1]For a global view of the European sovereign debt crisis see Lane (2012).

Consequently, some Eurozone countries which adopted great austerity measures delayed the stabilization of their debt. A first intuitive explanation is that austerity measures can inhibit the economic growth by a decline in aggregate demand, or that the debt burden is too high to generate a budget surplus. Nevertheless, a second explanation is provided by the political economy framework, such policies can actually appear as a strategic variable for different political groups with partisan considerations. This comes back to the intuition of Ardagna and Caselli (2014), for whom the management of the Greek crisis "*to be more affected by electoral concerns*". Thus, the two bailouts in favour of Greece were obtained by political-economy equilibria, such that, for each signatory, it was individually rational to agree to it, and has finally resulted to delay the stabilization of Greek debt. Following this intuition, in order to determine the impact of such policies on the timing of stabilization, the present paper is based upon the seminal contribution of Alesina and Drazen (1991). According to these authors, the process leading to a stabilization can be described as a War of Attrition between different socio-economic groups with conflicting distributional objectives. Thus, this paper argues that the reducing budget deficit policies can appear as concession programs which extend the debt stabilization by delaying the resolution of the distributional conflict generated by the stabilization process, namely the War of Attrition.

More precisely, Alesina and Drazen (1991) develop a model where the delay of stabilization is the result of a distributional conflict about the burden of the fiscal policy that is required to eliminate budget deficits and growing public debt. Indeed, the period in which public debt grows is seen as a time of war among political groups, which have an interest to wait because the group that concedes first is assumed to bear more than half of the fiscal burden. Consequently, even if all groups wish the debt stabilization (i.e. if the stabilization is a common goal of the society), they also fight among each other to avoid assuming the largest share of the fiscal burden which generates the stabilization. In addition, in incomplete information setup, each group revises their beliefs during the conflict about the opponents capacity to stabilize (i.e. to agree to pay the greater fraction of fiscal burden) the first one. In this way, the debt stabilization moment is given by a symmetric Nash Equilibrium which defined the shortest instant such that the expected gain of waiting before leaving the conflict becomes smaller for one group than the gain to stabilize now.

This paper modifies that game by introducing an exogenous date into Alesina-Drazen model, which is called the "concession time", at which each political group can strategically concede to reduce a share of "the purpose of the conflict" in order to limit the "intensity of war". In the public finance framework, this concession corresponds across each group to an agreement to pay a additional part of distortionary taxation in the aim to reduce public deficits and the amount of public debt. This date of concession can be imposed on all groups by an national institution as the government or more likely by an international institution such that the European Union, as e.g. in the Pact of stability and growth setup,[2] or the IFM, especially in the Greek debt crisis where each bailout is subjected in austerity measures to obtain a primary budget surplus.[3] More precisely, the concession appears as a sovereign debt restructuring, since each political group can reduce the nominal value of debt by accepting to finance more deficits by taxes.[4] The intuition is the following. Each political group can concede to pay an little additional share of taxation in order to delay the very costly moment of stabilization, which requires to assume the totality of tax burden, and not a little part. In this way, the program of each group is not only to determine the exit instant of conflict, as in Alesina and Drazen, but also in the same time, to compute the optimal level of concession to stay longer in the war. Thus, there is a trade-off which comes from two conflicting forces: the cost to assume more taxes, and the earning to delay the end of the war.

The modification of the standard war of attrition model has interesting consequences. First of all, this paper establishes that the concession is a strategic variable which actually delays the debt stabilization. More exactly, the optimal level of concession increases with regard to the probability to become the loser of the war (namely, to pay the greater part of stabilization burden). The intuition is the following. The more the end of the conflict (i.e. the debt stabilization), is expected to be costly by each political group, the more these groups concede in order to stay longer in the war.[5]

Besides, the main result of our model is that the debt stabilization is more delayed with coalition government rather than single party government. This finding contraries the famous result of Spolaore (1993). Indeed, the author shows that coalition government delays adjustment, while a single party government react "too much", relative to what a social planer would do. This result is due to the fact that each political parties by their partisan considerations, would like to be spared from taxes. The present model, following Alesina and Drazen, assimilates the gap from the stabilization burden which is borne between both

[2]In this setup, member countries with a deficit upper to 3% can be forced by the European Commission to adopt fiscal reforms.

[3]Effectively, on the one hand, regarding the international bailout in April 2010, the Greek government obtained 110 billion euros credits of European Union and the IMF over three years. In return, the Greek government adopted a lot of austerity measures to obtain a primary budget surplus. The accomplishment of this program was closely supervised and monitored by investors: the IMF and the Eurozone countries. On the other hand, regarding the last European bailout in June 2015, the European Commission proposed 15.5 billion euros loans against the commitment by the Greek government to obtain a primary budgetary surplus of 1% of the GDP at end-2015.

[4]This paper don't focus on a change in the debt structure, but simply on the reduction of nominal value of public debt, which is also the real value, since following Alesina and Drazen (1991), the model is without money. Contrary to Ardagna and Caselli (2014) which study the mechanism of international negotiation between Athens and international institutions, this paper focuses on effects of internal political mechanisms on the delay of debt stabilization.

[5]The football competition is a clear example. Effectively, traditionally it is know that the more the stake of a football match is high, the more both teams "close the game" by adopting a very defensive strategy, which appears as the "concession strategy". That explain the little scores or the multiplicity of the penalty kick shootout that occurred in the last three finals of football world cup in 2006, 2010 and 2014.

groups as the degree of political polarization in the society. By this definition, contrary to the result of Spolaore, this paper argues that the more the society is politically polarized, the more the stabilization burden is unequally distributed, and the more both groups concede in order to delay the stabilization.

Finally, our results allow to explain many political contexts. Especially, I will exemplify this model in the Greek debt crisis framework. Effectively, the different austerity programs voted by the Hellenic Parliament can be assimilated to concessions plans. Notably, during the government of A. Samaras between November 2012 and January 2015, two austerity programs were supported by a great political coalition. Their aims were to obtain massive public spending cuts to generate a great primary budget surplus in order to repay the loans of European Institutions and the IMF. In this way, despite the difficulty to master the Greek debt burden, our model establishes a new explanation of this multiplicity of austerity plans. Indeed, this paper argues that these plans, by delaying the stabilization time, are related to strategic concessions to continue negotiations with the "Troka" for future bailouts, in order may be to maintain in power the current government (Samaras), weakened by a large coalition.

Consequently, our model argues that the stabilization generates a great loss of utility which leads all groups to pay a strategic part of taxes in order to stay longer in the war. Therefore, this result offers a new type of explanation for why fiscal policies delay the debt stabilization.

1.1. Related literature

This paper belongs to the political economy literature that focuses on the war of attrition model in the public finance framework.[6] From Alesina and Drazen (1991), Drazen and Grilli (1993) by developing a monetary version obtain in particular an inverted U-shape relationship between inflation and expected welfare for moderate to high levels of inflation. Besides, from Nalebuff and Riley (1985) and Fudenberg and Tirole (1986), numerous works are extending the standard model in the incomplete information, see e.g. Krishna and Morgan (1997), Bulow and Kemperer (1999), or more recently Martinelli and Escorza (2007), among other.

On the one hand, more connected to the political analysis, Spolaore (1993, 2004) shows that a coalition government delays more the stabilization than a single party government. In particular, this intuition is tested and developed by Padovano and Venturi (2001) in Italian political framework, during government coalitions in the period 1948-1994.

On the other hand, in following Casella and Eichengreen (1996) which study the effect of the announcement of a foreign transfer on the expected stabilization time, Carré (2000) introduces a exogenous deadline to represent the Maastricht

[6]War of Attrition models was initially formalized in a biological framework by Riley (1980), and further developed by Bliss and Nalebuff (1984). In the economic view, Backus and Driffill (1985) and Tabellini (1988) analysis a war of attrition between trade unions and central bank.

Treaty, beyond which a punishment. She argues that higher the punishment is, the higher the probability of stabilization before that date will be.

More generally, this paper is connected to the literature that focuses on the public debt as a commitment, or as the strategic variable for politicians (Persson and Svensson, 1989; Aghion and Bolton, 1990; Alesina and Tabellini, 1989, 1990b; Milesi-Ferretti and Spolaore, 1994; Drazen, 2000, among other). Besides, Alesina and Perotti (1994) present a large survey of this literature, while Persson and Tabellini (2000) develop a clear voting model with public debt. Especially, Alesina and Tabellini (1990a) show that political polarization and frequent government changes should be associated with larger debt.

Consequently, the contribution of our model is to introduce a exogenous time where each political groups can strategically "manipulates" (reduces in our case) the amount of public debt by assuming an additional share of taxes in the standard war of attrition models.

The rest of the paper is organized as follows. Section 2 resolves the model, Section 3 presents the social planner Equilibrium, Section 4 discusses the results in the Greek debt crisis framework, and finally, Section 5 concludes.

2. The model

We consider one economy with two political or interest groups, denoted by D and R respectively. Each group is populated by a continuum of Households (or Citizens) with measure normalized to unity. In addition, there is a government which provides public expenditure, either by levying taxes or by issuing a public debt, since our model is non-monetary.[7] Following Alesina and Drazen (1991), until $t = 0$ the government budget is supposed balanced, with external government debt constant at level $b(0)$. To start the conflict, at $t = 0$ a shock hits reducing available tax revenues. Therefore, we suppose that a fraction of the deficit $\gamma > 0$ is covered by distortionary taxation, and a fraction $1 - \gamma$ by issuing debt. Thus, the public debt $b(t)$ evolves according to

$$\dot{b}(t) = (1 - \gamma)d(t), \tag{1}$$

and the amount of distortionary taxes is

$$\tau(t) = \gamma d(t), \tag{2}$$

where the level of deficit $d(t)$ is simply given by

$$d(t) = rb(t) + g(t), \ \forall t \geq 0. \tag{3}$$

Finally, for simplicity throughout the paper we assume that $b(0) = 0$, and that public expenditures are constant over times, i.e. $g(t) = g(0) =: g_0$, $\forall t \geq 0$.

[7] Monetary versions of the model of Alesina and Drazen are considered by Drazen and Grilli (1993), and Guidotti and Vegh (1999).

As usual, debt stabilization consists of an increase in taxes sufficient to prevent further growth in the debt. Thus, at the time where the debt is stabilized, denoted by the date T, the deficit is totally covered by distortionary taxation, i.e. $\gamma = 1$, $\forall t \geq T$. In addition, this fiscal reform can be implemented only if one of the two groups accepts or proposes it. Consequently, taxes to be levied at the date of stabilization $t = T$ are $\tau(T) = d(T)$.

The key assumption in our model is that the group-i, $i \in \{D, R\}$, can reduce the government deficit before the stabilization by accepting to paid a additional share of distortionary taxation, denoted by β_i, during a period describe as the "concession time" (or reforms moment), denoted by the date $\tilde{T}$, where $0 \leq \tilde{T} < T$. Besides, this additional reducing deficit is likened to "a concession", since a drop of deficits decreases in turn the level of distortionary taxation. In addition, this concession corresponds to a indirect fiscal reform. Indeed, as public expenditures are supposed constant over time, when both groups accept to pay an additional tax, the issuing debt decreases. Thus, the choice to concede of assuming the part β_i at $t = \tilde{T}$, is equated to the choice to liquidate a amount of public debt. In this way, concession appears on a sovereign debt restructuring, since each group can reduce a nominal value of debt by financing more deficits. Thus, from the concession time $t = \tilde{T}$ until the stabilization time $t = T$, the new level of public deficit is simply $(1 - \beta_R - \beta_D)d(\tilde{T})$, where $\beta_i d(\tilde{T})$, $\beta_i \in [0, 1/2]$, denotes the additional level of tax assumed by the group-i, $i \in \{D, R\}$.

In what follows, the concession is costly for each group. Indeed, concession requires embarking on a negotiation process within both groups that generates costs, as e.g. psychological or political costs, since the concession consists to find a agreement to assume an additional level of tax by each individuals belonging to the same political group. That is to say, the concession of the group-i is the result of a internal consensus on the willingness to pay an additional tax, $\beta_i \tau(\tilde{T})$. To this end, we define the cost function $c(\cdot)$, where $c' > 0$ and $c'' > 0$.

Besides, the concession decreases the amount of tax burden, which is necessary to generate the stabilization with probability ϵ, $\epsilon \in [0, 1]$. In this way, the reduction of tax burden from the stabilization time ($t \geq T$) is simply given by $\epsilon\beta_i$, where ϵ denotes the success probability of the concession β_i. The intuition is that at the time of assuming all of deficit ($t = T$) a hidden information can appear by generating any costs, such that e.g. the massaging public account by the government, that of the Greece in particular. Consequently, the probability ϵ can reflect some external shocks which affects, on the one hand, the consensus on the agreement about the additional amount of taxes, and the other hand, the true level of deficit at the stabilization time which can evolve between the concession time and the stabilization time ($\tilde{T} \leq t \leq T$), as e.g. the rating downgrades of the sovereign bonds from the rating agencies.

Finally, using (3) and (1), explicit forms of the level of public debt and deficit before the concession are

$$b(t) = \frac{g_0}{r}(e^{(1-\gamma)rt} - 1), \text{ and } d(t) = g_0 e^{(1-\gamma)rt}, \ \forall t \in [0, \tilde{T}[. \tag{4}$$

According to our assumptions, public deficit after the concession becomes:

$(1-\beta_R-\beta_D)d(t)$, $\forall t \in [\tilde{T}, T[$, and $(1-\epsilon\beta_R-\epsilon\beta_D)d(T)$, $\forall t \geq T$. Thus, by (2), the level of distortionary taxes is given by

$$\tau(t) = \begin{cases} \gamma d(t) = \gamma g_0 e^{(1-\gamma)rt}, & 0 \leq t < \tilde{T}, \\ \gamma(1-\beta_R-\beta_G)d(t) = (1-\beta_R-\beta_G)\gamma g_0 e^{(1-\gamma)rt}, & \tilde{T} \leq t < T, \\ (1-\epsilon\beta_R-\epsilon\beta_D)d(T) = (1-\epsilon\beta_R-\epsilon\beta_D)g_0 e^{(1-\gamma)rT}, & t \geq T. \end{cases} \tag{5}$$

Figure 2 shows the public deficit path. Initially ($t=0$), by the shock which reduces available tax revenues, public deficit is equivalent to public expenditures ($d(0)=g(0)$). Before the concession time ($t \leq \tilde{T}$), by (1) and (4), public deficit evolves to $rb(t)+g(0) = g(0)\exp((1-\gamma)rt)$. At the concession time appear the first threshold effect, since the group-i can reduce the part β_i of deficit by accepting to assume an additional fraction (β_i) of tax. From the stabilization time ($t \geq T$), taxes increases until to equalize the constant level of public deficit ($\tau(T) = d(T)$), the debt becomes then constant ($\dot{b}(t) = 0$). Finally, the second threshold effect comes from to the success probability ϵ: the more ϵ increases, the more concession reduces public deficit during the stabilization, and the more the constant level of deficit decreases.

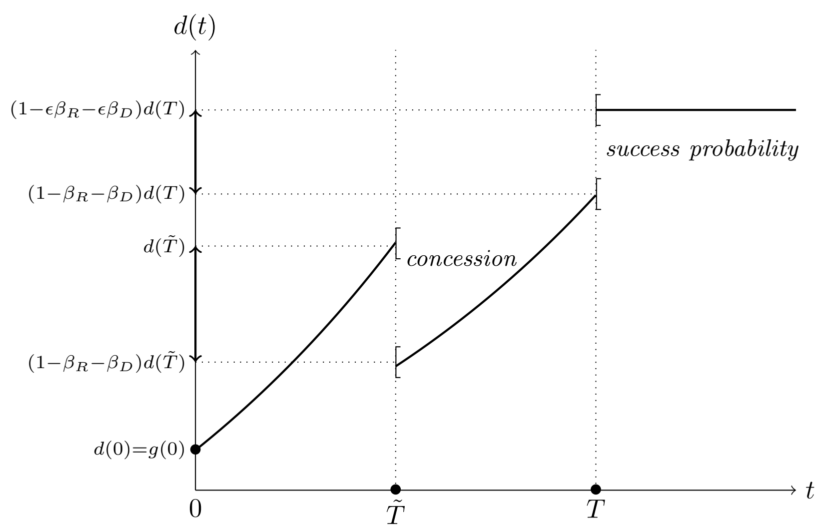

Figure 1: Public deficit path

2.1. Agent's preference

At each period, utility of representative individual of group-i decreases with distortionary taxes. We defined then θ_i the idiosyncratic random variable, which denotes the individual loss of utility of group-i due to distortionary taxes. The cost θ_i is independently drawn from a distribution $F(\cdot)$ on $[\underline{\theta}; \overline{\theta}]$, and we denote

$s := \overline{\theta} - \underline{\theta}$. What is critical is that θ_i is know only to the group-i itself,[8] other groups knowing only the distribution F. For simplicity the utility loss from distortionary taxes, denoted by K_i, is supposed linear in the level of taxes, namely $K_i(t) = \theta_i \tau(t)$. Let w_i be the utility level reached without distortions, which represent the constant level income.[9]Finally, the instantaneous utility of the representative individual of the group-i is simply: $u_i(t) = w_i - K_i(t)$, $\forall t [0; T[$.

Besides, agents consume their net income in each period. After stabilization welfare loss disappear: $K_i = 0$, since taxes after stabilization are assumed to be non-distortionary. However, stabilization requires an agreement among both groups on the distribution of tax burden. We suppose that one of the two group (which becomes the *loser*) has to agree to assume a part $\alpha > 1/2$, of tax burden while the other group (the *winner*) a fraction $1-\alpha$. In this way, α captures the "degree of polarization in the society": when α is close to one, the loser-group suffer the totality of tax burden, thus the degree of polarization is high.

As Alesina and Drazen (1991),[10] both groups pay half of tax during the pre-stabilization period, namely $w_i(t) = -\tau(t)/2, \forall t \leq T$. Now, representative agent'i s utility before the date of concession, denoted by $u_i^c(t)$, and until the date of concession and stabilization, $u_i^s(t)$, are given by

$$u_i^c(t) = -(\frac{1}{2} + \theta_i)\tau(t) = -\gamma g_0(\frac{1}{2} + \theta_i)e^{(1-\gamma)rt}, \forall t \in [0, \tilde{T}[, \qquad (6)$$

$$u_i^s(t) - \gamma g_0(1 - \beta_R - \beta_D)(\frac{1}{2} + \theta_i)e^{(1-\gamma)rt}, \qquad \forall t \in [\tilde{T}, T[, \qquad (7)$$

and the lifetime utility on the loser and the winner from the date of stabilization onward, denoted by $V^L(t)$ and $V^W(t)$ respectively, are defined by

$$V^L(t) = -\alpha g_0(1 - \epsilon\beta_R - \epsilon\beta_D)e^{(1-\gamma)rt}, \qquad \forall t \geq T, \qquad (8)$$

$$V^W(t) = -(1-\alpha)g_0(1 - \epsilon\beta_R - \epsilon\beta_D)e^{(1-\gamma)rt}, \forall t \geq T. \qquad (9)$$

The program of the group-i, $i \in \{R, D\}$, is to maximize its expected present discounted utility according to the behavior of its opponent by optimally determining: (i) the date, denoted by T_i, to agree to bear the fraction α of the tax burden if the other individual has not already volunteered, and (ii) the level of concession β_i at the date $\tilde{T}$.

Finally, let us denote by $H(T)$ the distribution of the opponent's optimal stabilization time, and by $h(T)$ the associated density function.[11] Thus, using

[8]If θ_i was public information, the player with the higher cost would concede immediately and there would be no war of attrition (Rubinstein, 1982). In the game of incomplete information, a player concedes immediately only if he has the highest possible parameter, $\overline{\theta}$.

[9]Their gross income is assumed constant over time and then neglected in the analysis

[10]In their model, Alesina and Drazen find this result later maximization of the consumers' utility.

[11]These will depend on $F(\theta)$ and on his strategy.

Eqs. from (6) to (9), the group i's expected lifetime utility is defined by

$$\begin{aligned} \mathrm{U}(T_i,\beta_i) =&[1-H(T_i)]\{W_i + \int_{\tilde{T}}^{T_i} u_i^s(z)\; e^{-rz}dz + V^L(T_i)e^{-rT_i}\} \\ &+ \int_{x=\tilde{T}}^{x=T_i} \{W_i + \int_{\tilde{T}}^{x} u_i^s(z)\; e^{-rz}dz + V^W(x)e^{-rx}\}\; h(x)dx \\ &- c(\beta_i)\tau(\tilde{T})e^{-r\tilde{T}},\; \forall i \in \{D,R\}, \end{aligned} \tag{10}$$

where $W_i := \int_0^{\tilde{T}} u_i^c(z)\; e^{-rz}dz$ is the level of utility before the date of concession.

The term in the first line represents the expected utility if the group-i will be the one to stabilize (at time T_i), while the term in second line is the expected utility deriving from the possibility that the other group will stabilize before T_i (at time x). Finally, the last line denotes the cost to concede β_i.

Now, the group-i's program is solving in three steps. The following Subsection determines the optimal time of stabilization, while the Subsection 2.3. gives the optimal level of concession. Finally, the Subsection 2.4 establishes the symmetric Nash equilibrium.

2.2. The optimal time of stabilization

Following Alesina and Drazen (1991), the optimal time of stabilization for the group-i, $i \in \{D,R\}$, denoted by $T(\theta_i)$, is obtained by a symmetric Nash equilibrium, such as if the other group is behaving according to the function $T(\cdot)$, it is optimal to concede according to $T(\theta_i)$. Therefore, the function $T(\cdot)$ is defined in the following Proposition.

Theorem 1. *The monotonically decreasing function $T(\cdot)$ is defined by*

$$T'(\theta) = -\frac{f(\theta)}{F(\theta)} \frac{(2\alpha-1)/\gamma}{(1/2+\theta)\Delta_\epsilon(\beta_R,\beta_D)-\alpha r}, \tag{11}$$

where $\Delta_\epsilon(\beta_R,\beta_D) := (1-\beta_R-\beta_D)/(1-\epsilon\beta_R-\epsilon\beta_D)$, and $T(\overline{\theta}) = 0$.

Proof: See Appendix A.

As usual, the relation (11) can be write[12]

$$\left[-\frac{f(\theta)}{F(\theta)}\frac{1}{T'(\theta)}\right](2\alpha-1) = \gamma[(\frac{1}{2}+\theta)\Delta_\epsilon(\beta_R,\beta_D)-\alpha r]. \tag{12}$$

[12] As Alesina and Drazen (1991), to obtain analytic results we suppose that $(1/2+\theta)\Delta_\epsilon(\beta_R,\beta_D)-\alpha r > 0$, that is the mild assumption since in the rest of the paper we suppose that the real interest rate is small (i.e. $r \approx 0$). Besides, note that according to the boundary condition, if the group-i is characterized by the maximum possible cost ($\theta_i = \overline{\theta}$), it will stabilize immediately, and so $\tilde{T} = 0$. Also note that $T(\theta)$ is monotonically decreasing, so that high cost groups stabilize first.

The right-hand side is the cost of waiting another instant to stabilize, that is the difference between the loss of utility due to distortionary taxation $\gamma(1/2+\theta)$ multiplied by the relative fraction of the residual tax ($\Delta_\epsilon(\beta_D, \beta_R)$), that we call "the relative cost of the war", and adjusted by the discount value of the fraction α of tax burden assumed by the loser-group ($\gamma\alpha r$). Indeed, $\Delta_\epsilon(\beta_D, \beta_R)$ denotes the ratio between "the cost of the war" (i.e. the fraction of the distortionary taxation not liquidated during the concession $(1-\beta_D-\beta_R)$) and the 'cost of the peace" (i.e. the fraction of tax burden which must be paid at stabilization $(1-\epsilon\beta_D-\epsilon\beta_D)$). Therefore, the more the war is expensive with regard to the peace, the more the ratio $\Delta_\epsilon(\cdot,\cdot)$ increases, and the more it is costly to wait another instant to stabilize. Besides, note that this ratio positively depends on the success probability ϵ: when $\epsilon = 1$, the ratio $\Delta_\epsilon(\cdot,\cdot)$ equals one, since the reduction of distortionary taxation is identical to the decrease of tax burden.

The left-hand side is the expected gain to waiting another instant to stabilize, which is the product of the conditional probability that one's opponent stabilizes (the hazard rate, in brackets) multiplied by the gain if the other groups stabilizes $(2\alpha-1)$. Effectively, $2\alpha-1$ represents the extra-gain from the stabilization time, namely the additional share of tax burden assumed by the loser-group, since, by (8) and (9), $(V^W(T) - V^L(T))/\tau(T) = 2\alpha - 1$. Finally, debt stabilization occurs when that cost of waiting just equals the expected benefit from waiting. This is a standard result in the war of attrition. In addition, we will work with the uniform distribution over $[\underline{\theta};\overline{\theta}]$, namely $f(\theta) = 1/(\overline{\theta}-\underline{\theta})$, and $F(\theta) = (\theta-\underline{\theta})/(\overline{\theta}-\underline{\theta})$. We obtain then an explicit form of optimal time $T(\cdot)$, which is given by the following Corollary.

Corollary 1. *The function of optimal concession time $T(\theta)$ is*

$$T(\theta) = \frac{(2\alpha-1)/\gamma}{(1/2+\theta)\Delta_\epsilon(\beta_R,\beta_D)-\alpha r}\log\left(\left[\frac{\overline{\theta}-\underline{\theta}}{\theta-\underline{\theta}}\right]\left[\frac{(1/2+\theta)\Delta_\epsilon(\beta_R,\beta_D)-\alpha r}{(1/2+\overline{\theta})\Delta_\epsilon(\beta_R,\beta_D)-\alpha r}\right]\right). \tag{13}$$

Proof: See Appendix A.

Notice that when $\epsilon = 1$, the optimal time of stabilization $T(\cdot)$ is independent of the concession β_i, $i \in \{D, R\}$, since $\Delta_\epsilon(\cdot,\cdot) = 1$. Finally, the expected date of stabilization, denoted by T^{SE}, is usually defined by the expected minimum of the optimal concession time T. In addition, as $T(\theta) < +\infty$ a.e,[13] and $T^{SE} := \mathbb{E}[\min_\theta\{T(\theta_D), T(\theta_R)\}] \leq \mathbb{E}[T(\theta)] < +\infty$, there is a positive value of the expected date of stabilization, since $0 \leq T^{SE} < +\infty$.

Proposition 1. *The expected time of stabilization T^{SE} is given by*

$$T^{SE} = 2\int_{\underline{\theta}}^{\overline{\theta}} T(x)\; F(x)f(x)dx. \tag{14}$$

[13] Indeed, by (13), $T(\theta) \to +\infty$ when $\theta \to \underline{\theta}$, thus $\mathbb{P}\{T(\theta) = +\infty\} = \mathbb{P}\{\theta = \underline{\theta}\} = 0$.

Proof: See Appendix A.

However, in this Subsection, the fractions ($\beta_i,\ i \in \{D, R\}$) are exogenous, thus we can ask how changes in levels of concession affect the expected time of stabilization by defining the following case.

Definition 1. The case without reduction of deficit ($\beta_i = 0$) is call the "Alesina-Drazen (AD) case", and we note $T^{AD}(\theta) := T(\theta)\big|_{\beta=0}$.

Effectively, without concession, the level of distortionary taxation evolves to the same continuously dynamic between $\tilde{T}$ and T. Besides, the Alesina-Drazen case is also characterized when $\epsilon = 1$, since $\epsilon = 1 \Rightarrow \Delta_\epsilon(\cdot, \cdot) = 1$.[14] Finally, by (12), as $\partial\Delta_\epsilon(\beta_R, \beta_D)/\partial\beta_i \leq 0$, it is clear that the expected cost decreases with $\beta_i,\ i \in \{D, R\}$. Thus, the time of stabilization $T(\cdot)$ with the positive concession β_i is upper than the Alesina-Drazen's stabilization time $T^{AD}(\cdot)$. In this way, we can establish the following Corollary.

Corollary 2. *The expected time of stabilization T^{SE} increases with the share of concession $\beta_i,\ i \in \{D, R\}$.*

Proof: See Appendix A.

The intuition is the following. When the group-i accepts to paid an important additional share of distortionary taxes, the deficit and the public debt amount become lower, in return, by (12), the cost of waiting another instant to stabilize decreases. Thus, this reduction of the marginal cost involves to increase the optimal concession time, so that, by (14), the expected time of stabilization rises. In other words, in a war of attrition framework, as the “problem” of war is more liquidated (in our case the public debt), the “intensity of war” declines, and its duration lengthens.

2.3. The optimal level of concession

This Subsection determines the optimal amount of concession (β_i^*) of the group-i, $i \in \{D, R\}$, at the date $\tilde{T}$. To this end, as the Proof of Theorem 1, choosing a time T_i as above is equivalent to choosing a value $\hat{\theta}_i$ and conceding at time: $T_i = T(\hat{\theta}_i)$. Therefore, after this change of variables, the optimal amount β_i^* is defined by

$$\{\beta_i^*(\theta_i), \theta_i\} = \underset{(\beta_i, \hat{\theta}_i) \in [0;1/2[\times [\underline{\theta}, \overline{\theta}]}{\text{argmax}} \ \mathrm{U}(\beta_i, \hat{\theta}_i), \ \ \forall i \in \{D, R\}. \tag{15}$$

The first order condition of program (15) is given by the following Proposition.

[14] Therefore, according to Eq. (12), the expected cost to waiting becomes simply $(1/2 + \theta) - \alpha r$, as in the Alesina and Drazen (1991).

Proposition 2. *(FOC) The optimal level of concession ($\beta_i^*(\theta_i), i \in \{D, R\}$) satisfies the following first order condition:*

$$\frac{(\tilde{\theta} - \theta)(2\alpha - 1)(1/2 + \theta - \epsilon\alpha r)}{(\overline{\theta} - \underline{\theta})((1/2 + \theta)\Delta_\epsilon(\beta_R, \beta_D) - \alpha r)} + \epsilon\alpha F(\tilde{\theta}) e^{-\gamma r \tilde{T}} = c'(\beta_i) e^{-\gamma r \tilde{T}}, \tag{16}$$

where $\tilde{\theta}$ is defined by $T(\tilde{\theta}) = \tilde{T}$.

Proof. First, with the change of variables $T_i = T(\hat{\theta}_i)$, (10) becomes

$$\begin{aligned}
&\mathrm{U}(\hat{\theta}_i, \theta_i, \beta_i) = F(\hat{\theta}_i) \Big\{ W_i + \frac{g_0}{r}(1/2 + \theta_i)(1 - \beta_R - \beta_D)[e^{-\gamma r T(\hat{\theta}_i)} - e^{-\gamma r T(\tilde{\theta})}] \\
&-\alpha g_0 (1 - \epsilon\beta_R - \epsilon\beta_D) e^{-\gamma r T(\hat{\theta}_i)} \Big\} \\
&+ \int_{x=\tilde{\theta}}^{x=\hat{\theta}_i} \Big\{ W_i + \frac{g_0}{r}(1/2 + \theta_i)(1 - \beta_R - \beta_D)[e^{-\gamma r T(x)} - e^{-\gamma r T(\tilde{\theta})}] \\
&-(1 - \alpha) g_0 (1 - \epsilon\beta_R - \epsilon\beta_D) e^{-\gamma r T(x)} \Big\} f(x) dx - c(\beta_i) g_0 e^{-\gamma r T(\tilde{\theta})},
\end{aligned} \tag{17}$$

where $\{\beta_i, \hat{\theta}_i\}$ are chosen by the group-i, $i \in \{D, R\}$. Using (17), differentiating with respect to β_i, the first order condition is

$$\begin{aligned}
\frac{\partial \mathrm{U}}{\partial \beta_i} &= F(\hat{\theta}) \Big\{ -\frac{1}{r}(1/2 + \theta_i) g_0 [e^{-\gamma r T(\hat{\theta})} - e^{-\gamma r T(\tilde{\theta})}] + \alpha g_0 \epsilon e^{-\gamma r T(\hat{\theta})} \Big\} \\
&+ \int_{\hat{\theta}}^{\tilde{\theta}} \Big\{ -\frac{1}{r}(1/2 + \theta_i) g_0 [e^{-\gamma r T(x)} - e^{-\gamma r T(\tilde{\theta})}] + (1 - \alpha) g_0 \epsilon e^{-\gamma r T(x)} \Big\} f(x) dx \\
&- c'(\beta_i) g_0 e^{-\gamma r T(\tilde{\theta})} = 0.
\end{aligned} \tag{18}$$

Note that directly: $\partial^2 \mathrm{U} / \partial \beta_i^2 = -c''(\beta_i) g_0 e^{-\gamma r T(\hat{\theta})} < 0$, as $c(\cdot)$ is strictly convex. Therefore, in dividing by g_0, the FOC becomes

$$g(\hat{\theta}, \beta_i) = c'(\beta_i) e^{-\gamma r T(\tilde{\theta})}, \tag{19}$$

where the function g is given by the following Lemma.

Lemma 1. *The function $g : [\underline{\theta}, \overline{\theta}] \times [0, 1/2] \to \mathbb{R}$ is*

$$g(\hat{\theta}, \beta_i) = \frac{1}{\overline{\theta} - \underline{\theta}} \left\{ \frac{(\overline{\theta} - \hat{\theta})(2\alpha - 1)(1/2 + \theta - \epsilon\alpha r)}{(1/2 + \theta)\Delta_\epsilon(\beta_R, \beta_D) - \alpha r} + \epsilon\alpha(\tilde{\theta} - \underline{\theta}) e^{-\gamma r T(\tilde{\theta})} \right\}.$$

Proof: See Appendix B.

However, in the maximization with respect to $\hat{\theta}$, we have $\hat{\theta} = \theta$ when $\hat{\theta}$ is chosen optimally by the group with the cost θ, see the Proof of Theorem 1 in Appendix A. Thus, with this optimality condition, the FOC (19) becomes $g(\theta, \beta_i) = c'(\beta_i) e^{-\gamma r T(\tilde{\theta})}$, and finally we obtain the result (16). □

According to definition (11), and as $T(\tilde{\theta}) = \tilde{T}$, the FOC (16) can be write

$$\epsilon\alpha F(\tilde{\theta})e^{-\gamma r\tilde{T}} + \left[-\frac{(\tilde{\theta}-\theta)(\theta-\underline{\theta})}{\overline{\theta}-\underline{\theta}}T'(\theta)\right]\gamma(1/2+\theta-\epsilon\alpha r) = c'(\beta_i)e^{-\gamma r\tilde{T}}. \quad (20)$$

The left-hand side (LHS) denotes the expected marginal gain to concede, and depends on two terms.

First, if the group-i, $i \in \{D, R\}$, stabilizes at the time of concession, namely $\theta_i = \tilde{\theta}$, it stabilizes the first one, and so actually will become the loser-group with probability $F(\tilde{\theta})$, namely it will have to pay the fraction α of tax burden. In this case, this group will take advantage only of the reduction of tax burden that it has to pay ($\epsilon\alpha$). Finally, in discount value, if $\theta_i = \tilde{\theta}$, the marginal earning for the group-i is simply $\epsilon\alpha F(\tilde{T})e^{-\gamma r\tilde{T}}$.

Second, if the group-i stabilizes after the concession, namely $\theta_i < \tilde{\theta}$, the marginal gain is increased by a second component. Effectively, an additional amount of concession decreases the loss of utility due to distortionary taxation during the war, namely the "intensity of war" ($\gamma(1/2+\theta)$), net of the discount gain due to the peace ($\epsilon\alpha\gamma r$). However, this marginal earning is linked to the war's duration, namely to the stabilization delay: the more the war lasts for a long time, the more the group takes advantage of the reduction in the war's intensity. In this way, this marginal gain is multiplied by a hazard rate (in brackets), which represents the increase effect of the optimal stabilization delay ($T'(\theta)$) adjusted by the uncertainty rate on the outcome of the war.[15]

Finally, the right-hand side (RHS) represents the expected marginal cost to concede, which is usually the discount value of the marginal cost ($c'(\beta_i)$).

However, as the date of stabilization $\tilde{T}$ is exogenous, without loss of generality, we can easily suppose that the date of concession is close to initial instant ($t = 0$), i.e. $\tilde{T} \approx 0$, what implies $\tilde{\theta} \approx \overline{\theta}$, since $T(\tilde{\theta}) \approx \tilde{T} = 0$.[16] In addition, to obtain analytics results, we suppose a small discount rate. In this way, FOC (16) becomes

$$\mathscr{G}(\beta_i, \theta) = \mathscr{C}(\beta_i), \ \forall i \in \{D, R\}, \quad (21)$$

where $\mathscr{G}(\beta_i, \theta) := (2\alpha-1)F(\theta)/\Delta_\epsilon(\beta_R, \beta_D) + \epsilon\alpha$ is the expected marginal gain, while $\mathscr{C}(\beta_i) := c'(\beta_i)$ denotes the marginal cost to concede. First, we remark

[15] Indeed, $(\tilde{\theta}-\theta)(\theta-\underline{\theta})/(\overline{\theta}-\underline{\theta})$ can be equated with $f(\theta)F(\theta)[1-F(\theta)]$. Afterward it will be the case since we shall suppose $\tilde{T} = 0$, namely $\tilde{\theta} = \overline{\theta}$. Thus, as $F(\theta_i)$ (resp. $1 - F(\theta_i)$) denotes the probability that the group-i loses (resp. wins) the war, the more the outcome of war is clear (i.e. $F(\theta)$ is close to 0 or 1), the more the uncertainty rate is small. Effectively, $F(\theta_i) = 1 - H(T_i)$, where H is the distribution of the opponent's optimal stabilization time. Thus, as $1 - H(T_i)$ corresponds to the probability that the group-i stabilizes the first one, namely that the group-i loses the war, $F(\theta)$ denotes also the probability to lose the war. Besides, we consider the random variable Z_i which equals 1 if the group-i loses the war, 0 else. So, as Z_i follows a Bernoulli's law with the success probability $F(\theta_i)$, $F(\theta_i)(1-F(\theta_i)$ is the variance of Z_i. Consequently, the uncertainty rate $f(\theta_i)F(\theta_i)[1-F(\theta_i)]$ measures the dispersal of Z_i, namely the volatility of the outcome of war.

[16] Effectively, if $\tilde{T} > 0$, our results are unchanged, because the period until the initial time ($t = 0$) and the concession time ($t = \tilde{T}$) is neglected in the analysis. Indeed, the utility $u_i^c(t)$, $\forall t < \tilde{T}$ is not present in the Nash equilibrium (12).

that the marginal gain positively depends on the probability to lose the war $F(\theta)$. Indeed, when θ_i increases, the group-i is more likely to lose, since its optimal concession time $T(\theta_i)$ decreases sharply. At the limit, when the group-i stabilizes at the time of concession, i.e. $\theta_i = \tilde{\theta}$, it becomes the loser-group, and so its marginal gain is only composed by the reduction of the part of tax burden that it has to pay ($\epsilon\alpha$). Second, note that $\mathscr{C}(\beta_i)$ is independent of the individual loss θ_i, because the density of θ is uniformly distributed on $[\underline{\theta};\overline{\theta}]$, namely the density function is independent of θ.

Finally, as usual, concession β_i occurs when that the marginal cost equals the marginal gain, i.e. $\mathscr{C}(\beta_i) = \mathscr{G}(\beta_i, \theta_i)$, $\forall i \in \{D, R\}$. As $\mathscr{G}$ negatively depends on θ_i, the fraction β_i also depends on θ_i, i.e. $\beta_i = \beta_i(\theta_i)$. Besides, as in symmetric equilibrium both groups stabilize according to the same function $T(\cdot)$, they will concede according to the same function $\beta(\cdot)$, i.e. $\beta_D(\theta_D) = \beta_R(\theta_R) =: \beta(\theta_i)$, with $\beta'(\cdot) \leq 0$. Thus, the maximization program involves to find the function $\beta(\theta)$ which maximizes the utility function.

In the rest of the paper we will work with the quadratic cost function, which is given by $c(\beta_i) = \beta_i^2$, $i \in \{D, R\}$. Under one condition, there are two levels $(\underline{\beta}(\theta), \overline{\beta}(\theta))$ given by the following Proposition, which verify the FOC (21).

Proposition 3. *If* $1 - \epsilon \geq 2\alpha - 1$ *(H1), there are two shares of concession* $(\underline{\beta}(\theta), \overline{\beta}(\theta))$ *which respect the first order condition* (21)*, where* $\overline{\beta}(\cdot) \geq \underline{\beta}(\cdot)$.

Proof. The optimal condition (21) can be rewritten by the implicit function $\varphi : [\underline{\theta};\overline{\theta}] \times [0; 1/2] \to \mathbb{R}^*$. So, (21) becomes

$$\varphi(\theta, \beta_i) := \left[\frac{\overline{\theta} - \theta}{\overline{\theta} - \underline{\theta}}\right] \frac{2\alpha - 1}{\Delta_\epsilon(\beta_R, \beta_D)} - c'(\beta_i) + \epsilon\alpha = 0. \tag{22}$$

Yet anticipating the symmetric equilibrium, we note the symmetric level of concession by β, i.e. $\beta_R = \beta_D =: \beta$. Therefore, as $\Delta_\epsilon(\beta_R, \beta_D) = (1 - 2\epsilon\beta)/(1 - 2\beta)$, and by the quadratic function cost ($c'(\beta) = 2\beta$), (22) becomes

$$\varphi(\theta, \beta) = \left[\frac{\overline{\theta} - \theta}{\overline{\theta} - \underline{\theta}}\right] \frac{(2\alpha - 1)(1 - 2\epsilon\beta)}{1 - 2\beta} - 2\beta + \epsilon\alpha = 0. \tag{23}$$

Differentiating with respect to β, we obtain $\partial_\beta \varphi(\theta, \beta) = 2[1 - F(\theta)(2\alpha - 1)(1 - \epsilon)/(1 - 2\beta)^2]$. Hence, $\partial_\beta \varphi(\theta, \beta) \geq 0 \Leftrightarrow \beta \geq X(\theta) := \{1 - \rho(\theta)\}/2$, where

$$\rho(\theta) := \sqrt{\left[\frac{\overline{\theta} - \theta}{\overline{\theta} - \underline{\theta}}\right] (2\alpha - 1)(1 - \epsilon)}. \tag{24}$$

Lemma 2.
i. $0 \leq X(\cdot) \leq 1/2$.
ii. $1 - \epsilon \geq 2\alpha - 1 \Rightarrow \varphi(\theta, X(\theta)) \leq 0$, $\forall \theta \in [\underline{\theta};\overline{\theta}]$.
iii. $\varphi(\cdot, 0) > 0$.

Proof: See Appendix B.

On the one hand, it is clear that $\varphi(\theta,\cdot) \in C^1([0,1/2[)$, and that $\varphi(\theta,\cdot)$ increases on $[X(\theta);1/2[$ and decreases on $[0;X(\theta)]$, since the result i. of Lemma 2 assures that $X(\theta) \in [0;1/2], \forall\theta \in [\overline{\theta};\underline{\theta}]$. On the other hand, using the result ii., $\varphi(\theta, X(\theta)) \leq 0$, and by (23), we have $\varphi(\theta,\beta) \to +\infty$ when $\beta \to 1/2$. Thus, by the result iii. and according to the intermediate value Theorem, there are $\overline{\beta}(\cdot) \in [X(\cdot);1/2[$, and $\underline{\beta}(\cdot) \in [0;X(\cdot)]$, such as: $\varphi(\cdot,\overline{\beta}(\cdot)) = 0$, and $\varphi(\cdot,\underline{\beta}(\cdot)) = 0$. □

The sufficiently condition (H1) involes that the the extra-gain of the winner-group $(2\alpha - 1)$ is lower than the failure probability $1-\epsilon$. Therefore, as α measures the "degree of polarization in the society", this condition assures that the society is weakly polarize, since $2\alpha - 1 \leq 1 - \epsilon \Leftrightarrow \alpha \leq 1 - \epsilon/2$. In this way, higher the probability ϵ is, the higher the society must be weakly polarized. Effectively, when $\epsilon = 0$, there is no constraint since $\alpha \in [1/2, 1]$, while if $\epsilon = 1$, each group must assume the same fraction of tax burden, namely $\alpha = 1/2$.

The intuition is the following. When ϵ increases, the cost to waiting another instant in the conflict also increases for both groups. In symmetric equilibrium, the differential of tax burden assumed by both groups (α) must decrease to obtain a optimal concession. In other words, the more the peace generates an great earnings for all groups (i.e. ϵ increases), the more these earnings must be uniformly allocated among all groups (i.e. α decreases). Finally, if the polarization of society is small, namely $\alpha \in \mathscr{E}_\epsilon := [1/2, 1-\epsilon/2]$, there are two levels $(\underline{\beta}(\cdot), \overline{\beta}(\cdot))$ which respect the FOC (21), as shows the Figure 2.3.

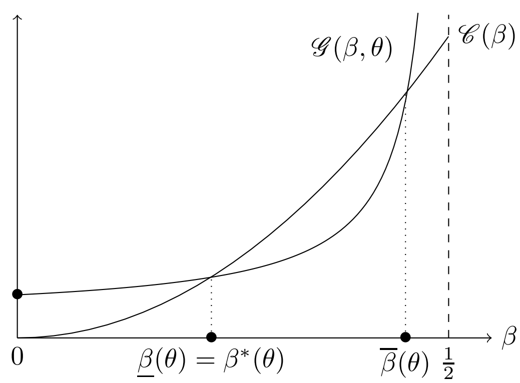

Figure 2: Optimally condition for $\theta \in [\underline{\theta}, \overline{\theta}]$.

Each marginal function ($\mathscr{G}(\cdot,\theta)$, and $\mathscr{C}(\cdot)$, $\forall\theta \in [\overline{\theta},\underline{\theta}]$) is described by a increasing and convex curve, where the marginal gain is initially upper than the marginal cost, since $\mathscr{G}(0,\theta) = F(\theta)(2\alpha - 1) + \epsilon\alpha \geq 0 = \mathscr{C}(0)$.[17]

[17]Nevertheless, for small values of concession ($\beta \to 0$) the slope of marginal gain is less than the slope of marginal cost, while when the value of concession is close to half of deficit

Finally, if the polarization of the society is limited, $\alpha \in \mathscr{E}_\epsilon$, there are two levels $(\underline{\beta}(\cdot), \overline{\beta}(\cdot))$ which respect the FOC (21). However, there are upper limits of the individual cost θ and of the polarization degree α which assure that at least one of both levels respects the second order condition. Thanks to this selection condition, the Symmetric Nash Equilibrium which solves program (15) is given by the following Theorem.

Theorem 2. *(Symmetric Nash Equilibrium) If $\alpha \in \mathscr{E}_\epsilon$, there exists upper limits $\overline{\theta}^*$ and α^* which assure that $(T^*(\theta), \beta^*(\theta))$ is the unique maximum of the utility function, where*

$$\beta^*(\theta) = \underline{\beta}(\theta), \tag{25}$$

$$T^*(\theta) = \frac{(\overline{\theta} - \underline{\theta})\mathscr{C}(\beta^*(\theta))}{(\overline{\theta} - \theta)(1/2 + \theta)\gamma} \log\left(\frac{(\overline{\theta} - \underline{\theta})(1/2 + \theta)}{(\theta - \underline{\theta})(1/2 + \overline{\theta})}\right), \ \forall \theta \in [\underline{\theta}; \overline{\theta}]. \tag{26}$$

Proof: See Appendix B.

According to Theorem 2, the existence of the Nash Equilibrium is provided by two sufficient conditions. First, the upper limit $\overline{\theta}^*$ implies that the growth rate of marginal cost $c'(\cdot)$ is weaker than the loss of utility due to distortionary taxation.[18] Second, the upper limit of the degree of polarization (α^*) provides that the loser-group does not pay the total amount of tax burden.That is to say, the society's cohesion is enough brought up. Indeed, despite the conflict, each group must assume a non-zero fraction of the common problem (the tax burden). In this way, α^* corresponds to the maximal degree of society's polarization, and we note $\mathscr{E}_\epsilon^* := \mathscr{E}_\epsilon \cap [1/2, \alpha^*]$, and $\Theta^* := [\overline{\theta}^*, \underline{\theta}]$.

Consequently, if $(\alpha, \theta) \in \mathscr{E}_\epsilon^* \times \Theta^*$, there is a global maximum of the utility function $(\beta^*(\theta), T^*(\theta))$, where $\beta^*(\theta)$ is defined by the lower value $\underline{\beta}(\theta)$. First of all, we remark that the optimal stabilization time is connected to $\beta^*(\theta)$ through the marginal cost to concede: the more one unit of concession is costly, the more this unit rises the optimal time of stabilization. Effectively, the goal of the concession is to increase the date of stabilization in order to delay the time where each group will assume a part of tax burden. Thus, the more the expected loss of utility from the stabilization increases, the more the marginal cost to concede rises, and the more the concession delays the debt stabilization.

The figure 2.4 sum the Equilibrium. The optimal concession $\beta^*(\theta)$ is represented in the bottom graph, the optimal time of stabilization $T^*(\theta)$ is drawn on the upper graph, while the time of stabilization of Alesina-Drazen $T^{AD}(\theta)$ is represented by the dashed curve.

$(\beta \to 1/2)$, the marginal gain shoots up ($\mathscr{G}(\beta, \theta) \to +\infty$) since the "relative cost of the war" becomes very high, namely $\Delta_\epsilon(\beta, \beta) \to +\infty$.

[18] Effectively, by (B.4): $\theta < \overline{\theta}^* = 3/2 \Leftrightarrow 1/2 + \theta < 2 \Leftrightarrow -(1/2 + \theta) > -c''(\beta), \ \forall \theta \in [\overline{\theta}, \underline{\theta}]$.That is to say, the loss of utility from distortionary taxes for both groups during the conflict $-(1/2 + \theta)$ is greater than the second-order partial derivatives with respect to the amount β, net of the public expenditure: $-c''(\beta)g_0 = -2g_0$.

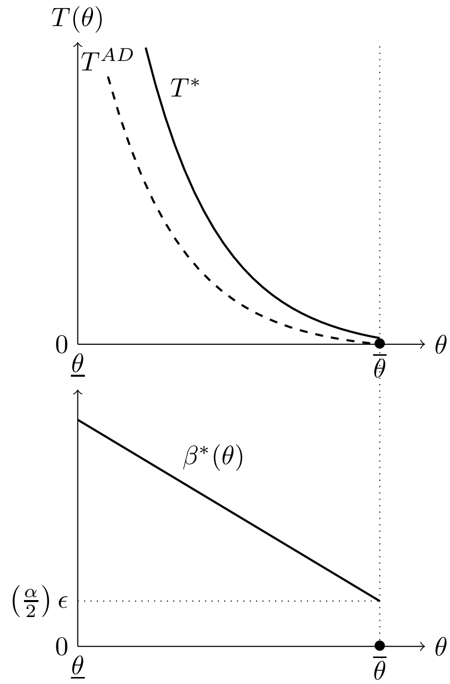

Figure 3: The symmetric Nash equilibrium.

Notice that the optimal concession β^* positively depends on the cost θ. Indeed, by (21), the expected marginal gain to concede rises with the probability to lose the war ($F(\theta)$). Therefore, the more the loss of group-i's utility due to distortionary taxation θ_i decreases, the more it is likely to lose, and so the more it delays the wars term by reducing deficits during the concession. The intuition is that the concession is a strategic variable for both groups to control the stabilization delay (i.e. the war's duration). In addition to the decision to assume the tax burden, i.e. the choice to stabilize the debt, each political group can strategically reduce a little level of deficit for the purpose of repel the peace period which requires to assume a great part of tax burden. Notably, when $\theta = \overline{\theta}$, namely when each group stabilizes at the time of concession, there is not war: $T^*(\overline{\theta}) = T^{AD}(\overline{\theta}) = 0$, but there is a positive amount of concession since $\beta(\overline{\theta}) = \epsilon\alpha/2$. Effectively, if $\theta = \overline{\theta}$, according to Theorem 2, the stabilization of debt takes place immediately $T^* = 0$, but each group concedes in order to take advantage to the reduction of tax burden. Thus, the optimal concession (β^*) is simply the share of stabilization burden assumed by each group ($\alpha/2$) which is reduced with probability ϵ. In this way, $\beta^*(\theta) > 0$, $\forall\theta \in [\underline{\theta}, \overline{\theta}]$, so the more the group-$i$ concedes a strong fraction of deficit, the more the optimal stabilization time T^* is upper than the date of stabilization of Alesina-Drazen.

So, we can now ask how various changes in the parameters affect the optimal amount of concession, namely the delay of stabilization.[19]

Proposition 4. *The optimal concession rises with the degree of polarization α.*

Proof: See Appendix B.

Following Alesina and Drazen (1991), the difference in fraction of stabilization burden (α) denotes the degree of political cohesion in the society. If α is closes to 1/2, the society is characterized by a high political cohesion, since the burden of stabilization is shared relatively equally, while if α is closes to α^*, the society is more polarized (namely, less cohesive), because the burden is very unequal. In this way, when the share of stabilization burden is very unequally distributed, the expected marginal gain to concede is larger, and both groups stay longer in the war. This result is connecting with a number works in political economy (see the great survey of Alesina and Perotti, 1994). For example, Roubini and Sachs (1989) argue that countries characterized by a short tenure of government and by the presence of many political parties in coalition have larger deficits.[20] More recently, Huber et al. (2003) suggest that coalitions with equally strong parties provide more deficits than coalitions with one dominating party. Thus, our model argues that the more the society is politically polarized, the more each group concedes by assuming a great part of distortionary taxation, the more the level of public debt decreases at the time of concession, what connect to conclusion of Huber et al. (2003), and the more the debt stabilization is delayed, contrary to the results of Spolaore (1993).

Proposition 5. *The optimal concession linearly rises with the probability ϵ.*

Proof: See Appendix B.

Notice that there is a dual effect of the success probability ϵ on the marginal gain to concede. By (20), on the one hand, ϵ increases the fraction of tax burden which is reduced by the concession. One the other hand, ϵ decreases the net discount value of loss of utility during the war, and decreases the advantage to concede an additional fraction of deficit. Thus, Proposition 5 argues that the positive effect offsets the negative effect.

Finally, according to Theorem 2, as $\partial T^*(\cdot)/\partial \beta^*(\cdot) \geq 0$, the results of Propositions 4, and 5 are also true regarding the effect on the optimal stabilization time. Now, the following Section determines the Social Welfare Equilibrium.

3. Social Welfare

To establish a benchmark with the results given by the Nash symmetric equilibrium, this Section computes the social first best by a proof in two steps.

[19] Formally, $\partial||T^*(\cdot) - T^{AD}(\cdot)||/\partial\beta \geq 0$, where usually $||\cdot||$ is the standard L^1-norm.

[20] Some articles question the conclusions of Roubini and Sachs, see e.g. De Haan and Sturm (1997).

Step 1. We consider a social planner who weights each of the two groups equally and we ignoring the direct dependance of groups' utility on the cost θ. Thus, following Drazen and Grilli (1993), in the symmetric equilibrium, the social welfare if the debt stabilization comes at time T, is given by

$$L(T, \beta) = \int_0^T u_i^s(z) e^{-rs}\, ds - 2\tau(\tilde{T}) c \left(\frac{\beta}{2}\right) e^{-r\tilde{T}} + \Delta V(\beta) e^{-rT}, \tag{27}$$

where $\Delta V(\beta) := (V^L + V^W)/2$ is the mean of the lifetime utility from the date of stabilization, which is supposed for simplicity independent of the time T. Thus, the expected social welfare, denoted by $\text{ESW}(\beta)$, is given by the expectation of $L(T, \beta)$ taken over the distribution of possible time T. So, we have

$$\text{ESW}(\beta) = \int_0^{+\infty} L(T, \beta)\; g(T) dT, \tag{28}$$

where $G(T)$ is the cumulative distribution of one group stabilizes at the time T, and $g(T)$ is the density function associated.

Step 2. To obtain the analytic form of the distribution $g(\cdot)$, we change the variable T on θ, i.e. $T = T(\theta)$. Thus, $1 - G(T(\theta))$ becomes the probability that both groups stabilize after the time $T(\theta)$. Therefore, as $g(T) = dG(T)/DT$, it is clear that $g(T(\theta)) = 2F(\theta)f(\theta)$.[21] So (28) becomes

$$\text{ESW}(\beta) = 2 \int_{\underline{\theta}}^{\overline{\theta}} L(T(\theta), \beta)\; F(\theta) f(\theta) d\theta. \tag{29}$$

Using (29), the social planner determines the amount β^{SW} which maximizes the expected social welfare, namely $\beta^{SW} := \text{argmax}_{\beta \in [0,1/2]} \text{ESW}(\beta)$. This optimal level is given in the following Proposition.

Proposition 6. *(First best) If $\gamma \leq 24\epsilon$, $\beta^{SW} = 1/2$.*

Proof: See Appendix C.

Intuitively, the social planner liquidated all deficits at the time of concession, i.e. $\beta = 1/2$. This result is due to the fact that the increase effect of utility before the stabilization offsets the intrinsic cost to concede. That is to say, the public debt is stabilized during the date of concession since in this period deficits are totally cover by taxes. In other word, the social welfare is maximized when the stabilization takes place as soon as possible without loser and winner: a draw characterized the war result. Besides, the optimal level of concession obtaining in symmetric Nash equilibrium is lower than the first best, since $\beta^*(\cdot) < \beta^{SW} = 1/2$. Thus, the Nash equilibrium is characterized by a sub-liquidation of debt.

Finally, the results given by the symmetric Nash equilibrium and by the social welfare equilibrium contribute to explain the recent crisis of Greek debt.

[21] Let us suppose $\theta_0 \in [\underline{\theta}, \overline{\theta}]$. Thus, we can write $1 - G(T(\theta_0)) = \mathbb{P}(\{\forall i \in \{D, R\},\ T(\theta_i) > T(\theta_0)\}) = \mathbb{P}(\theta_R < \theta_0,\ \theta_D < \theta_0)$, since $T(\cdot)$ is an decreasing function. As θ_R and θ_D are independent variables, we obtain $1 - G(T(\theta_0)) = F(\theta_0)^2$.

4. Discussion: the Greek Debt Crisis

Our setup can be clearly exemplified in the Greek political framework.

First of all, concessions can be easily identified in the different austerity programs recently voted by the Hellenic Parliament. Regarding political groups, following Buiter et al. (2015a), the referendum about the agreement of the European bailout in July 2015 highlighted the fact that there is, for simplicity, two political groups. The first group which supports the bailout by accepting to assume austerity measures such as e.g. the increase of VAT about 23%, or the reduction of 0.5% of GDP of social spending. In opposite, the second group pro-SYRIZA, which was against the European project, obtained 61.31% of the vote. Regarding our model, the first group (assimilated to the subscript R) would be ready to assume a great fraction of additional distortionary taxation, namely a high β_R, to reduce the government deficit, while the second group (denoted by the subscript D) will not wish paid a additional tax, i.e. $\beta_D \approx 0$.

Regarding assumptions of the model, the great protests against austerity measures between 2011 and 2015 are prime examples of concession costs. Besides, the consensus defining each group in the Greek referendum (in July 2015) is fragile, as shows the burst of the SYRIZA majority in the Greek parliament, which obliged the Prime Minister A. Tsípras resigned in August, 2015. Regarding the success probability of concession, it can be represented by two items. First, by the massaging public account by the Greek government. Effectively, in October 2009, shortly after the general election, the newly elected Prime Minister George Papandréou sheds light on the Greek deficit: it is not 6% as announced by the previous government, but of 12.7% of the GDP for 2009. More recently, Buiter et al. (2015b) in their paper "Why Greece's Third Bailout Will Probably Fail" argue that "*Greece is unlikely, barring deep political and economic changes, to be able and willing to implement the conditionality of such a program*" about the last bailout (June 2015). Second, by the rating downgrades of debt which appear as a external shock. Indeed, after the announcement of G. Papandréou, all rating agencies downgraded the Greek debt rating.[22]

Regarding our finding, the present model provides two main results. The first is that the optimal level of concession can appear as a strategic variable in order to delay the stabilization.

The period (June 2012-January 2015) of the large coalition government of A. Samarás is a clear example. During this period, a first great austerity plan was adopted by the Hellenic Parliament on November 7th, 2012. It consist of vast austerity measures to obtain a primary budget surplus, which is seen as a precondition for the bailout of 31.5 million euros from the European Union and the IMF. Nevertheless, one of the goals of this plan is actually not to reduce the level of Greek debt, since it continues to grow, but rather to allow the Samarás's government to continue the negotiations with European institutions and the IMF: the famous "Troka". Effectively, this first plan adopted early-

[22]The rate of the Greek 10 years bond was about 7%.

November allowed the continuation of negotiations, which reached an agreement on November 26th, 2012 about the decline of 40 billion euros of Greek debt, which must be reduced to 124% of the GDP in 2020, against 120% requested by the IMF initially. Besides, this bailout, insufficient to solve the crisis, in turn enabled the continuity of negotiations with Athens. The consequence is a new austerity program in January 2013 of 2.5 billion euros with sharp tax increases which was supported by the Samaras's government.[23] Finally, this multiplicity of concession plans are mainly aimed to delay the debt stabilization to continue negotiations, in order may be to maintain in power the current government (Samarás), weakened by a large coalition.

The second main result of our model is that the optimal concession increases with the degree of political polarization, as shows the Proposition 4.

Regarding Greek governments between 2011 and 2015, we can distinguish, for simplicity, two period of time: a first period (November 2011-January 2015) characterized by large fragile coalition governments, especially the administration of A. Samarás composed by three major political parties.[24] The second period (January 2015- August 2015) corresponds to the administration of A. Tsípras which is a small coalition clearly dominated by only one party (SYRIZA).[25] Intuitively, in the second period, the political polarization (α) is relatively higher than in the first period.

In each period, austerity programs are voted. In the first period, a austerity plan which has already been discussed, was voted with great difficulty by the Parliament on 7th November, 2012.[26] In the second period, on 16th July, 2015, the Hellenic Parliament voted the reform program of Tsípras's government by a comfortable majority.[27] This plan is characterized by great austerity measures: the VAT increase (23%), increase of the tax on compagny from 16% to 28%, cut of military spending, pension reform,... Consequently, in spite of the fact that the government is left-wing political, the level of concession is very high, since the aim of the plan is to obtain a primary budget surplus of 1% in 2015, and 3% in 2016. To sum up, the program of Samarás was voted with great

[23]Therefore, the Greek Finance Minister, Mr Stournaras, stressed that the vote of this plan is a prerequisite for the payment of future loans to Greece by its creditors. He argues in particular that "*the payment of the next tranche of the loan to the country will be decided at the next Eurogroup meeting* [ministers of eurozone finance]".

[24]After the Gi. Papandréou's administration, the economist L. Papadímos is appointed Prime Minister to solve the debt crisis on November 10th, 2011. His government is then a large coalition, composed by the social-democratic party: the Panhellenic Socialist Movement (PASOK), by the centre-right party: the New Democracy (ND), and by the radical right-wing populist party: Popular Orthodox Rally (LAOS). The following elections do not allow to obtain a clear majority. Nevertheless, during the general election on June 17th, 2012, the New Democratic party obtains a relative majority, what allowed A. Samarás to form a government of national union with PASOK and the democratic Left.

[25]Among 16 ministers of the government, 12 are member of SYRYZA, 3 are without political party, and only one is member of the Independent Greeks party (ANEL).

[26]Effectively, the government was supported by a short majority: 153 members of Parliament, while the theoretical majority is 176

[27]223 members have voted in favor to the Tsípras measures.

difficulty, while the Tsípras plan, by means of the votes of opposition, was relatively easily accepted by the Hellenic Parliament. This simple intuitive example shows in the period with a coalition dominated by only one party (i.e. high α), on the one hand, the concession to reduce deficit happen easier, what connects in conclusions of Huber et al. (2003), and the other hand, the level of this concession is also high delaying actually the stabilization, contrary to the result of Spolaore (1993).

Thus, beyond the explanation linked to international pressures (in particular, the question of belonging of Greek in Eurozone ("Grexit"), Tsípras plan exemplifies our model by two interpretations. On the one hand, the various measures would be a strategic concession to extend the deadline of the "problem" (namely, to stabilize and to repay debt). On the other hand, according to Buiter et al. (2015c), the goal of this concession is also to enable to continue negotiations with European institutions.

5. Conclusion

In this paper, the concession is defined as an agreement on a additional level of distortionary taxes assumed by each group before the debt stabilization. Our main result is that this concession delays the debt stabilization by deficit cutting. Intuitively, the stabilization requires for all groups to pay the amount of tax which covers all deficits. In this way, the debt stabilization generates a high loss of utility for each political group, all the more raised that it believes that it will have to pay the largest share of tax burden. Therefore, each group accepts to assume a little more taxes during the pre-stabilization period in order to delay the very costly moment: the debt stabilization.

Besides, each group optimally computes: (i) the time of stabilization, and (ii) the level of concession, according to the behavior of his opponent. The result is a symmetric Nash equilibrium where the optimal stabilization time is positively connected on the optimal conceding amount through the marginal cost to concede.

The main result of the present paper is that the optimal level of concession and the stabilization delay increase with the degree of political polarization of the society. This finding is especially interesting since it contrary the famous result of Spolaore (1993). Besides, this result follows the traditionally literature (Alesina and Tabellini, 1989, 1990a; Roubini and Sachs, 1989; Huber et al., 2003, among other), which argues that large coalition governments provide a larger public deficit, and a greater public debt than governments dominated by only one party. Thus, in our model, the more the government is composed by a larger coalition, the less each group concedes to cut deficits, since the share of stabilization burden is more equally distributed, and *in fine*, the more the stabilization is delayed.

Our setup may lead to interesting prospects for future research. First, the introduction of a "concession time" in the standard war of attrition model might be studied in other contexts, specifically in international negotiations framework. In our model, one political group can be identified as the Greek government, while the second group can be assimilated to the international institutions as European Union, or the IMF. Thus, according to Ardagna and Caselli (2014), a "war of attrition" takes place regarding the negotiation of bailouts, where the concession can be understood for the second group as a agreement on the defect of a part of the Greek debt. Thus, our setup would allow a fresh look on the debates between the "Troika" and the Athens about the debt sustainability.

Second, the introduction of a strategic concession would allow to connect the war of attrition models with the standard voting models. Intuitively, we consider an political party in power which maximizes his probability of reelection. As the debt stabilization requires a great loss of voters' utility by the tax burden, it lowers the probability of reelection of party in office. Thus, in return, the incumbent party can delay the stabilization after the next election by a strategic concession, what delays the resolution of the "problem" (i.e. the debt stabilization). In this way, an interesting extension would be to introduce the war of attrition in a standard probability voting model of Lindbeck and Weibull (1987), and Persson and Tabellini (2000), where the incumbent politician will maximize its probability of reelection by managing the delay of stabilization.

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Appendix A

Proof of Theorem 1.

This proof follows Alesina and Drazen (1991), with two steps. The first step proves that the optimal concession time T_i is monotonically decreasing in θ_i, while the second step determines the symmetric Nash equilibrium.

Step 1. First, differentiating (10) with respect to T_i, $i \in \{D, R\}$, one obtains

$$\begin{aligned}\frac{\partial \mathrm{U}}{\partial T_i} =& e^{-rT_i}\{h(T_i)[V^W(T_i) - V^L(T_i)] \\ &+ [1 - H(T_i)][u_i^s(T_i) + (V^L)'(T_i) - rV^L(T_i)]\}. \end{aligned} \tag{A.1}$$

By (6) until (9), (A.1) becomes $\partial \mathrm{U}/\partial T_i = g_0 e^{-\gamma r T_i}\{h(T_i)(2\alpha - 1)(1 - \epsilon\beta_1 - \epsilon\beta_2)$ $+ \gamma[1 - H(T_i)][\alpha r(1 - \epsilon\beta_R - \epsilon\beta_D) - (1/2 + \theta_i)(1 - \beta_R - \beta_D)]\}$. Hence;

$$\frac{\partial^2 \mathrm{U}}{\partial T_i^2} = -\gamma g_0[1 - H(T_i)](1 - \beta_R - \beta_D)e^{-\gamma r T_i} < 0. \tag{A.2}$$

Thus, by Eq. (A.2), $\partial \mathrm{U}/\partial T_i$ is deceasing in θ_i, so that the optimal concession time T_i is then monotonically decreasing in θ_i.

Step 2. First, using (6) until (9), the expected utility (10) becomes

$$\begin{aligned}\mathrm{U}(T_i,\beta_i) = [1-H(T_i)]\Big\{W_i + \frac{g_0}{r}(1/2+\theta_i)(1-\beta_R-\beta_D)[e^{-\gamma r T_i} - e^{-\gamma r \tilde{T}}]\\ -\alpha g_0(1-\epsilon\beta_R-\epsilon\beta_D)e^{-\gamma r T_i}\Big\}\\ +\int_{x=\tilde{T}}^{x=T_i}\Big\{W_i + \frac{g_0}{r}(1/2+\theta_i)(1-\beta_R-\beta_D)[e^{-\gamma r x} - e^{-\gamma r \tilde{T}}]\\ -(1-\alpha)g_0(1-\epsilon\beta_R-\epsilon\beta_D)e^{-\gamma r x}\Big\}h(x)dx - c(\beta_i)g_0 e^{-\gamma r\tilde{T}}.\end{aligned} \quad \text{(A.3)}$$

Now, we suppose the other group, with personnal cost θ, stabilizes according to $T(\theta)$. Thus, choosing a time T_i as above would be equivalent to choosing a value $\hat{\theta}_i$ and stabilizing at time $T_i = T(\hat{\theta}_i)$. Expected utility (A.3) becomes, after the change of variables:

$$\begin{aligned}\mathrm{U}(\hat{\theta}_i,\theta_i,\beta_i) = F(\hat{\theta}_i)\Big\{W_i + \frac{g_0}{r}(1/2+\theta_i)(1-\beta_R-\beta_D)[e^{-\gamma r T(\hat{\theta}_i)} - e^{-\gamma r T(\tilde{\theta})}]\\ -\alpha g_0(1-\epsilon\beta_R-\epsilon\beta_D)e^{-\gamma r T(\hat{\theta}_i)}\Big\}\\ +\int_{x=\tilde{\theta}}^{x=\hat{\theta}_i}\Big\{W_i + \frac{g_0}{r}(1/2+\theta_i)(1-\beta_R-\beta_D)[e^{-\gamma r T(x)} - e^{-\gamma r T(\tilde{\theta})}]\\ -(1-\alpha)g_0(1-\epsilon\beta_R-\epsilon\beta_D)e^{-\gamma r T(x)}\Big\}f(x)dx - c(\beta_i)g_0 e^{-\gamma r T(\tilde{\theta})},\end{aligned} \quad \text{(A.4)}$$

where $\tilde{\theta}$ is given by $T(\tilde{\theta}) = \tilde{T}$. And as $T(\cdot)$ is deceasing, $\tilde{\theta} \leq \hat{\theta}_i$.
Therefore, differentiating with respect to $\hat{\theta}_i$, the first order condition implies now: (where we drop the i subscript)

$$\begin{aligned}\frac{\partial \mathrm{U}}{\partial\hat{\theta}} = -F(\hat{\theta})\gamma g_0 T'(\hat{\theta})e^{-\gamma r T(\hat{\theta})}[(1/2+\theta)(1-\beta_R-\beta_D) - \alpha r(1-\epsilon\beta_R-\epsilon\beta_D)]\\ - f(\hat{\theta})g_0 e^{-\gamma r T(\hat{\theta}}(1-\epsilon\beta_R-\epsilon\beta_D)(2\alpha-1) = 0,\end{aligned} \quad \text{(A.5)}$$

which becomes, using the definition $\Delta_\epsilon(\beta_R,\beta_D) = (1-\beta_R-\beta_D)/(1-\epsilon\beta_R-\epsilon\beta_D)$:

$$\frac{\partial \mathrm{U}}{\partial\hat{\theta}} = -F(\hat{\theta})\gamma T'(\hat{\theta})[(1/2+\theta)\Delta_\epsilon(\beta_R,\beta_D) - \alpha r] - f(\hat{\theta})(2\alpha-1) = 0. \quad \text{(A.6)}$$

Finally, as $T(\theta)$ is the optimal time of concession for a group with cost θ, then $\hat{\theta} = \theta$ when $\hat{\theta}$ is chosen optimally. Thus, the FOC (A.6) evaluated at $\hat{\theta} = \theta$ implies the result (11). Yet, substituting $T'(\theta)$ evaluated at $\hat{\theta}$ from (11) into (A.5), we obtain:

$$\frac{\partial \mathrm{U}}{\partial\hat{\theta}} = \frac{g_0 e^{-\gamma r T(\hat{\theta})}f(\hat{\theta})(2\alpha-1)(1-\beta_R-\beta_D)(\theta-\hat{\theta})}{(1/2+\hat{\theta})\Delta_\epsilon(\beta_R,\beta_D) - \alpha r}.$$

As $(1/2+\theta)\Delta_\epsilon(\beta_R,\beta_D)-\alpha r>0$, $\forall\theta\in[\underline{\theta};\overline{\theta}]$, $\text{sign}(\partial_{\hat{\theta}}\text{U})=\text{sign}(\theta-\hat{\theta})$, so that the second order condition is satisfied.

As usual, for any $\theta\in[\underline{\theta};\overline{\theta}]$, the gain to having the opponent stabilize is positive. Thus as long as $f(\overline{\theta})>0$, groups with $\theta<\overline{\theta}$ will not stabilize immediately. This in turn a group with $\theta=\overline{\theta}$ will stabilize immediately, i.e. $T(\overline{\theta})=0$. □

Proof of Corollary 1.
As$f(\theta)$ is uniform over $[\underline{\theta},\overline{\theta}]$, then $f(\theta)/F(\theta)=1/(\theta-\underline{\theta})$ $\forall\theta\in[\underline{\theta},\overline{\theta}]$. By (11), as $T(\overline{\theta})=0$ by the Theorem 1, $T(\theta)$ can be write

$$T(\theta)=\int_\theta^{\overline{\theta}}\frac{(2\alpha-1)/\gamma}{(s-\underline{\theta})[(1/2+s)\Delta_\epsilon(\beta_R,\beta_D)-\alpha r]}\,ds. \tag{A.7}$$

By using the method of partial fractions and integrating, (A.7) becomes

$$T(\theta)=\frac{(2\alpha-1)/\gamma}{(1/2+\theta)\Delta_\epsilon(\beta_R,\beta_D)-\alpha r}\log\left\{\frac{(\overline{\theta}-\underline{\theta})(|(1/2+\theta)\Delta_\epsilon(\beta_R,\beta_D)-\alpha r|)}{(\theta-\underline{\theta})(|(1/2+\overline{\theta})\Delta_\epsilon(\beta_R,\beta_D)-\alpha r|)}\right\}$$

□

Proof of Proposition 1.
The expected date of stabilization (T^{SE}) is the minimum of the both groups' optimal concession time ($T(\theta_i)$ for the group i, $i\in\{D,R\}$). Thus, $T^{SE}=\mathbb{E}[\min\{T(\theta_R),T(\theta_D)\}]$. First, by dividing into two cases ($\theta_R<\theta_D$, or $\theta_R\geq\theta_D$), we can write: $T^{SE}=\mathbb{E}[\min\{T(\theta_R);T(\theta_D)\}(\mathbf{1}_{\theta_R<\theta_D}+\mathbf{1}_{\theta_R\geq\theta_D})]$, where $\mathbf{1}$ denotes the indicator function. Yet, $T(\cdot)$ is the decreasing function, thus: $T^{SE}=\mathbb{E}[T(\theta_D)\mathbf{1}_{\theta_R<\theta_D}]+\mathbb{E}[T(\theta_R)\mathbf{1}_{\theta_R\geq\theta_D}]$. As θ_R and θ_D follow the same distribution F, we have: $T^{SE}=2\mathbb{E}[T(\theta_D)\mathbf{1}_{\theta_R<\theta_D}]$. Now, let us fix $s\in[\underline{\theta};\overline{\theta}]$. Therefore, as θ_R and θ_D are independent, we have

$$\begin{aligned}\mathbb{E}[T(\theta_D)\mathbf{1}_{\theta_R<\theta_D}|\theta_D=s]&=\mathbb{E}[T(s)\mathbf{1}_{\theta_R<s}|\theta_D=s]\\&=T(s)\mathbb{P}(\theta_R<s|\theta_D=s)=T(s)F(s).\end{aligned} \tag{A.8}$$

Finally, by the expectation of Eq. (A.8), it is easy to obtain that $T^{SE}=2\mathbb{E}[T(\theta_R)F(\theta_R)]=2\int_{\underline{\theta}}^{\overline{\theta}}T(s)\,F(s)f(s)\,ds$. □

Proof of Corollary 2.
On the one hand, as the integral is finite, we can write

$$\partial_\Delta T(\theta)=\int_\theta^{\overline{\theta}}\partial_\Delta\left\{\frac{(2\alpha-1)/\gamma}{(s-\underline{\theta})[(1/2+s)\Delta-\alpha r]}\right\}\,ds,$$

where to save the notation, $\Delta:=\Delta_\epsilon(\beta_R,\beta_D)$. Hence,

$$\partial_\Delta T(\theta)=\int_\theta^{\overline{\theta}}-\frac{(2\alpha-1)(1/2+s)/\gamma}{(s-\underline{\theta})[(1/2+s)\Delta-\alpha r]^2}\,ds.$$

In addition, since $s \in [\theta; \overline{\theta}]$ in this integral, we establish the following inequality:

$$\partial_\Delta T(\theta) = \int_\theta^{\overline{\theta}} -\frac{(2\alpha-1)(1/2+s)/\gamma}{(s-\underline{\theta})[(1/2+s)\Delta - \alpha r]^2}\, ds \leq -\frac{(2\alpha-1)(1/2+\theta)/\gamma}{(\overline{\theta}-\underline{\theta})[(1/2+\overline{\theta})\Delta - \alpha r]^2} \leq 0,\ \forall \theta \in [\underline{\theta}; \overline{\theta}]. \tag{A.9}$$

On the other hand, as $\Delta = (1-\beta_R-\beta_D)/(1-\epsilon\beta_R-\epsilon\beta_D)$, and $\epsilon \leq 1$, we have

$$\frac{\partial \Delta}{\partial \beta_i} = -\frac{1-\epsilon}{(1-\epsilon\beta_R-\epsilon\beta_D)^2} \leq 0. \tag{A.10}$$

Finally, by (A.9) and (A.10), we obtain: $\partial_{\beta_i} T(\theta) = (\partial_{\beta_i}\Delta)(\partial_\Delta T(\theta)) \geq 0$, $\forall \theta \in \underline{\theta}; \overline{\theta}]$. Yet, from Corollary 1, the expected time of stabilization T^{SE} is defined by: $T^{SE} = \mathbb{E}[\min\{T(\theta_R), T(\theta_D)\}]$. Thus, we establish that $\partial_{\beta_i} T^{SE} \geq 0$, $i \in \{D, R\}$. □

Appendix B

Proof of Lemma 1.
Using (19), the function g is defined by, $\forall i \in \{D, R\}$,

$$g(\hat{\theta}, \beta_i) = F(\hat{\theta})\left\{\gamma(1/2+\theta)[T(\hat{\theta}) - T(\tilde{\theta})] + \alpha\epsilon[1-\gamma r T(\hat{\theta})]\right\} + \int_{\hat{\theta}}^{\tilde{\theta}} \left\{\gamma(1/2+\theta)[T(x) - T(\tilde{\theta})] + (1-\alpha)\epsilon[1-\gamma r T(x)]\right\} f(x)dx. \tag{B.1}$$

Differentiating with respect to $\hat{\theta}$, we obtain that $\partial g(\hat{\theta}, \beta_i)/\partial\hat{\theta} = \gamma F(\hat{\theta})T'(\hat{\theta})[1/2 + \theta - \epsilon\alpha r]$. Thus, by (11), we have

$$\frac{\partial g(\hat{\theta}, \beta_i)}{\partial \hat{\theta}} = -\frac{f(\hat{\theta})(2\alpha-1)(1/2+\theta-\epsilon\alpha r)}{(1/2+\theta)\Delta_\epsilon(\beta_R, \beta_D) - \alpha r}. \tag{B.2}$$

By (B.2), with the uniform distribution over $[\underline{\theta}; \overline{\theta}]$, the function g is given by

$$g(\hat{\theta}, \beta_i) = \int_{\hat{\theta}}^{\tilde{\theta}} \frac{(2\alpha-1)(1/2+\theta-\epsilon\alpha r)}{(\overline{\theta}-\underline{\theta})[(1/2+\theta)\Delta_\epsilon(\beta_R, \beta_D) - \alpha r]}\, ds + g(\tilde{\theta}, \beta_i),$$

hence,

$$g(\hat{\theta}, \beta_i) = \frac{(\tilde{\theta}-\hat{\theta})(2\alpha-1)(1/2+\theta-\epsilon\alpha r)}{(\overline{\theta}-\underline{\theta})[(1/2+\theta)\Delta_\epsilon(\beta_R, \beta_D) - \alpha r]} + g(\tilde{\theta}, \beta_i).$$

Yet, by (B.1), the constant $g(\tilde{\theta}, \beta_i)$ is $g(\tilde{\theta}, \beta_i) = \epsilon\alpha(\tilde{\theta}-\underline{\theta})[1-\gamma r T(\tilde{\theta})]/(\overline{\theta}-\underline{\theta})$. Note that as $\tilde{\theta} \leq \overline{\theta}$, the assumption where $\overline{\theta} - \underline{\theta} \geq 1$ assures that $g(\overline{\theta}, \beta_i) \geq 0$. Thus, we obtain the explicit form of function g

$$g(\hat{\theta}, \beta_i) = \frac{1}{\overline{\theta}-\underline{\theta}}\left\{\frac{(\overline{\theta}-\hat{\theta})(2\alpha-1)(1/2+\theta-\epsilon\alpha r)}{(1/2+\theta)\Delta_\epsilon(\beta_R, \beta_D) - \alpha r} + \rho_\epsilon(\tilde{\theta})\right\},$$

where $\rho_\epsilon(\tilde{\theta}) := \epsilon\alpha(\tilde{\theta}-\underline{\theta})[1-\gamma r T(\tilde{\theta})]$. □

Proof of Lemma 2.

Proof i. As $0 \leq 2\alpha - 1 \leq 1$, $0 \leq \epsilon \leq 1$, and $0 < \overline{\theta} - \theta \leq \overline{\theta} - \underline{\theta}$, $\forall \theta \in [\underline{\theta}; \overline{\theta}]$, by (24), it is obvious that $0 < \rho(\cdot) \leq 1$, namely $0 < X(\cdot) < 1/2$.

Proof ii. By (23), we can write

$$s(1-2\beta)\varphi(\theta, \beta) = (\overline{\theta} - \theta)(2\alpha - 1)(1 - 2\epsilon\beta) - 2\beta s(1 - 2\beta) + \epsilon\alpha(1 - 2\beta),$$

where $s := \overline{\theta} - \underline{\theta}$. Evaluated at $\beta = X(\theta) := \frac{1}{2}\{1 - \rho(\theta)\}$, we have $(1 - 2\beta) = \rho(\theta)$, hence (where we drop the argument of the function $\rho(\cdot) = \rho$)

$$\begin{aligned} s(1-2\beta)\varphi(\theta, X(\theta)) &= (\overline{\theta} - \theta)(2\alpha - 1)(1 - \epsilon + \epsilon\rho) - s\rho(1 - \rho) + \epsilon\rho\alpha. \\ &= (\overline{\theta} - \theta)(2\alpha - 1)(1 - \epsilon) + \epsilon\rho[(\overline{\theta} - \theta)(2\alpha - 1) + \alpha] - s\rho(1 - \rho). \end{aligned} \tag{B.3}$$

Yet, by (24), $(\overline{\theta} - \theta)(2\alpha - 1)(1 - \epsilon) = s\rho^2$. In replacing in (B.3), the function $\varphi(\theta, X(\theta))$ can be write as a quadratic polynomial in the variable ρ, namely

$$s(1-2\beta)\varphi(\theta, X(\theta)) = 2s\rho^2 - s\rho + \epsilon[(\overline{\theta} - \theta)(2\alpha - 1) + \alpha]\rho.$$

As $(\overline{\theta} - \theta) \leq s, \forall \theta \in [\underline{\theta}; \overline{\theta}]$, we have $\varphi(\theta, X(\theta)) \leq \xi(\epsilon)$, where $\xi(\epsilon) = 2s\rho - s + \epsilon s(2\alpha - 1) + \epsilon\alpha$. Now, we show that $\xi(\epsilon) \leq 0$ if $1 - \epsilon \geq 2\alpha - 1$, denoted by the condition (H1). First, $\xi'(\epsilon) = 2s\frac{\partial \rho}{\partial \epsilon} + s(2\alpha - 1) + \alpha$. Thus, by (24), we can write: $\xi'(\epsilon) = -\rho s/(1 - \epsilon) + s(2\alpha - 1) + \alpha$. Finally, as $1 \geq \alpha$, and $s > 1$, we obtain $\xi'(\epsilon) \geq -\rho/(1 - \epsilon) + 2\alpha$. Thus, as $\rho \leq \sqrt{(2\alpha - 1)(1 - \epsilon)}$, we can establish

$$\text{(H1): } 1 - \epsilon \geq 2\alpha - 1 \Rightarrow 2\alpha\sqrt{1 - \epsilon} \geq \sqrt{2\alpha - 1} \Rightarrow 2\alpha \geq \rho/(1 - \epsilon) \Rightarrow \xi'(\epsilon) \geq 0.$$

Second, as $\alpha \geq 1/2$, $\xi(1) = -s + s(2\alpha - 1) + \alpha \leq -s + 1/2 \leq 0$. Thus, $\xi(1) \leq 0$, since $s > 1$. Finally, as ξ is increasing and continuously function, under (H1), $\xi(\epsilon) \leq \xi(1) \leq 0$, $\forall \epsilon \in [0, 1]$. Consequently, if $1 - \epsilon \geq 2\alpha - 1$, $\varphi(\cdot, X(\cdot)) \leq 0$.

Proof iii. By (23), the function $\varphi(\theta, \beta)$ evaluated at $\beta = 0$ is given by $\varphi(\theta, 0) = \epsilon\alpha + (\overline{\theta} - \theta)(2\alpha - 1)/s \geq 0$, $\forall \theta \in [\underline{\theta}, \overline{\theta}]$. □

Proof of Theorem 2.

First of all, the optimization program is defined by: $\max_{(\beta,\hat{\theta}) \in [0;1/2[\times [\underline{\theta},\overline{\theta}]} \mathrm{U}(\beta, \hat{\theta})$, where the critical points are given by (18) and (A.5). Now, we shows that the Hessian matrix, denoted by $\mathbf{H}(\beta, \theta)$, is negative definite at $(\underline{\beta}(\theta), \theta)$, $\forall \theta \in [\underline{\theta}, \overline{\theta}]$. We start to search the components of the hessian matrix, when $r \approx 0$ and $\tilde{T} \approx 0$.

First, as $c''(\beta) = -2$, by (18), we have

$$\frac{\partial^2}{\partial \beta^2}\mathrm{U}(\beta, \theta) = -c''(\beta)g_0 = -2g_0 < 0, \tag{B.4}$$

Second, using (B.1) and (B.2), the mixed derivatives is simply

$$\left.\frac{\partial^2}{\partial \beta \partial \hat{\theta}}\mathrm{U}(\beta, \theta)\right|_{\hat{\theta}=\theta} = g_0\frac{\partial}{\partial \theta}g(\beta, \theta) = -g_0\frac{f(\theta)(2\alpha - 1)(1 - 2\epsilon\beta)}{1 - 2\beta} < 0. \tag{B.5}$$

Finally, by (A.6), the last second-order partial derivatives is given by

$$\frac{\partial^2}{\partial\hat{\theta}^2}\mathrm{U}(\beta,\theta) = -g_0\frac{f(\hat{\theta})(2\alpha-1)(1-2\epsilon\beta)(1/2+\theta)}{(1/2+\hat{\theta})^2}.$$

Hence, as $\hat{\theta} = \theta$ when $\hat{\theta}$ is chosen optimally, we obtain

$$\frac{\partial^2}{\partial\hat{\theta}^2}\mathrm{U}(\beta,\theta)\bigg|_{\hat{\theta}=\theta} = -g_0\frac{f(\theta)(2\alpha-1)(1-2\epsilon\beta)}{1/2+\theta} < 0. \tag{B.6}$$

Thus, by (B.4) and (B.6), it is clear that $\mathrm{tr}(\mathbf{H}(\beta,\theta)) < 0, \forall(\beta,\theta) \in [0,1/2[\times[\underline{\theta},\overline{\theta}]$.

Now, to determine the sign of the determinant of Hessian matrix, we defined the function $\upsilon : [0,1/2[\times[\underline{\theta},\overline{\theta}] \to \mathbb{R}$, by $\upsilon(\beta,\theta) := \det(\mathbf{H}(\beta,\theta))$. Using (B.4), (B.5), and (B.6), the function υ is given by

$$\upsilon(\beta,\theta) = 2g_0^2\frac{f(\theta)(2\alpha-1)(1-\epsilon\beta)}{1/2+\theta} - g_0^2\frac{f(\theta)^2(2\alpha-1)^2(1-2\epsilon\beta)^2}{(1-2\beta)^2}.$$

Hence, $\det(\mathbf{H}(\beta,\theta)) > 0 \Leftrightarrow \upsilon(\beta,\theta) > 0 \Leftrightarrow \Upsilon > 0$, where

$$\Upsilon(\beta,\theta) := \frac{2}{1/2+\theta} - \frac{f(\theta)(2\alpha-1)(1-2\epsilon\beta)}{(1-2\beta)^2}. \tag{B.7}$$

On the one hand, it is clear that $\Upsilon \in C^1([0,1/2[\times[\underline{\theta},\overline{\theta}])$, and $\lim_{\beta\to1/2}\Upsilon(\beta,\theta) = -\infty, \forall\theta \in [\underline{\theta},\overline{\theta}]$. On the other hand, $\Upsilon(\cdot,\theta)$ decreases on $[0,1/2[$, since

$$\frac{\partial}{\partial\beta}\Upsilon(\beta,\theta) = -\frac{2f(\theta)(2\alpha-1)[(1-\epsilon)+(1-2\epsilon\beta)]}{(1-2\beta)^3} \leq 0,\ \forall(\beta,\theta) \in [0,1/2[\times[\underline{\theta},\overline{\theta}].$$

In addition, $\Upsilon(0,\beta) = \frac{2}{1/2+\theta} - f(\theta)(2\alpha-1) > 0 \Leftrightarrow 2 > (1/2+\theta)f(\theta)(2\alpha-1)$. Yet, as $f(\theta) \leq 1$ and $2\alpha-1 \leq 1$, an sufficient condition is that $1/2+\overline{\theta} < 2$, namely $\overline{\theta} < \overline{\theta}^* := 3/2$. Thus, if $\overline{\theta} < \overline{\theta}^*$, according to the intermediate value Theorem, there is $\check{\beta}(\theta) \in [0,1/2[$, such as: $\Upsilon(\beta,\theta) > 0$, $\forall\beta \in [0,\check{\beta}(\theta)[$, and $\Upsilon(\beta,\theta) \leq 0$, $\forall\beta \in [\check{\beta}(\theta),1/2[$. Therefore, the determinant of the Hessian matrix is positive when $\beta \leq \check{\beta}(\theta)$, in this way, the Lemma 3 shows that $\underline{\beta}(\theta) < \check{\beta}(\theta)$.

Thus according to Lemma 3, and as $\mathrm{tr}(\mathbf{H}(\beta,\theta)) < 0$, it is clear that $\underline{\beta}(\theta)$ is a maximum of the utility function U.

Now, let us show that the upper root $\overline{\beta}(\cdot)$ is a saddle point, that is to say $\det(\mathbf{H}(\overline{\beta}(\theta)),\theta) < 0$, namely $\Upsilon(\overline{\beta}(\theta),\theta) < 0$. For that purpose, we adopt a *reductio ad absurdum* by supposing $\check{\beta}(\theta) > \overline{\beta}(\theta)$. On the one hand, $\Upsilon(\cdot,\theta)$ is positive on $[0,\check{\beta}(\theta)[$, thus the utility function $\mathrm{U}(\cdot,\theta)$ is strictly concave on $[0,\check{\beta}(\theta)[$, since $\mathrm{tr}(\mathbf{H}(\beta,\theta)) < 0$. Consequently, there is a unique maximum $\beta_{max}(\theta) \in [0,\check{\beta}(\theta)[$. In addition, according to the Lemma 3, $\beta_{max}(\theta) = \underline{\beta}(\theta)$. On the other hand, if $\check{\beta}(\theta) > \overline{\beta}(\theta)$, $\det(\mathbf{H}(\overline{\beta}(\theta),\theta)) > 0$, and so $\overline{\beta}(\theta)$ is a maximum of the utility function. However, since the maximum is unique on $[0,\check{\beta}(\theta)[$, we obtain $\beta_{max}(\theta) = \underline{\beta}(\theta) < \overline{\beta}(\theta) = \beta_{max}(\theta)$, what establishes the absurdity. Thus, $\overline{\beta}(\theta) > \check{\beta}(\theta)$, namely $\overline{\beta}(\theta)$ is a saddle point. At the end, as there is in most two critical points, if $\overline{\theta} < \overline{\theta}^* = 3/2$ and, $\alpha < \alpha^* = 5/8$, $\underline{\beta}(\theta)$ is the unique maximum of the utility function. □

Lemma 3. *Let* $\alpha \in \mathscr{E}_\epsilon$. *There exists an upper limits* $\overline{\theta}^*$ *and* α^*, *such as* $\det(\mathbf{H}(\underline{\beta}(\theta), \theta)) > 0, \forall \theta \in [\underline{\theta}, \overline{\theta}]$, *where* $\overline{\theta}^* = 3/2$ *and,* $\alpha^* = 5/8$.

We prove the Lemma 3 by two steps. The first step establishes that $\underline{\beta}(\cdot) < 1/4$, while the second step proves that $\Upsilon(1/4, \cdot) > 0$, i.e. $\det(\mathbf{H}(1/4, \cdot)) > 0$.

Step 1. By (23), according to the implicit function Theorem, it is clear that $\underline{\beta}(\theta)$ is decreasing in θ, since $\partial_\theta \varphi(\theta, \beta) = -(2\alpha - 1)(1 - 2\epsilon\beta)/(\overline{\theta} - \underline{\theta})(1 - 2\beta) \leq 0$. Hence, $\underline{\beta}(\theta) \leq \underline{\beta}(\underline{\theta}) =: \hat{\underline{\beta}}$. Using (23), when $\theta = \underline{\theta}$, the FOC becomes $\chi_\beta(\epsilon) = 0$, where $\chi_\beta : [0, 1] \mapsto \mathbb{R}$ is given by

$$\chi_\beta(\epsilon) = \frac{(2\alpha - 1)(1 - 2\epsilon\beta)}{1 - 2\beta} - 2\beta + \epsilon\alpha. \tag{B.8}$$

By (B.8), we note that the function χ_β is linear with ϵ. Thus, χ_β is a monotonous function. In this way, we can distinguish two cases: either $\chi'_\beta \geq 0$ (the first case), or $\chi'_\beta < 0$ (the second case).

Case 1: $\chi'_\beta \geq 0$**.** By (B.8), $\chi_\beta(\epsilon) \leq \chi_\beta(1) = (2\alpha - 1) - 2\beta + \alpha$. Yet, as under (H1), $\epsilon = 1 \Rightarrow \alpha = 1/2$, we obtain that $\chi_\beta(1) = 1/2 - 2\beta$, hence; $\chi_\beta(1) = 0 \Leftrightarrow \beta = 1/4$. Consequently, as in this case the function φ and χ_β are increasing with ϵ, we can establish that $\hat{\underline{\beta}} \leq 1/4$.

Case 2: $\chi'_\beta < 0$**.** By (B.8), $\chi_\beta(\epsilon) \leq \chi_\beta(0) = (2\alpha - 1)/(1 - 2\beta) - 2\beta$. Thus, as $\beta < 1/2$, $\chi_\beta(0) = 0 \Leftrightarrow$

$$4\beta^2 - 2\beta + (2\alpha - 1) = 0. \tag{B.9}$$

Besides, if $\alpha \leq \alpha^* = 5/8$, the discriminant Δ associated to the quadratic relation (B.9) in β becomes positive and so there exists two real roots, since $\Delta = 4(5 - 8\alpha)$. Therefore, since $\underline{\beta}$ is defined as the lower root, $\hat{\underline{\beta}}$ is given by $\hat{\underline{\beta}} = (1 - \sqrt{5 - 8\alpha})/4 \leq 1/4$, where $\hat{\underline{\beta}} \geq 0$, since $\alpha \geq 1/2$.

Finally, to sum up, if $\alpha \leq \alpha^*$, in every case, we obtain that $\underline{\beta}(\theta) \leq \hat{\underline{\beta}} \leq 1/4$.

Step 2. Now, let us show that $\Upsilon(1/4, \theta) > 0$, i.e. $\det(\mathbf{H}(1/4, \theta)) > 0$. First, by (B.7) we can write $\Upsilon(1/4, \theta) > 0 \Leftrightarrow 1 > 2(1 - \epsilon/2)(2\alpha - 1)f(\theta)(1/2 + \theta)$. Second, on the one hand, under (H1), $2\alpha - 1 \leq 1 - \epsilon$, namely $\alpha \leq 1 - \epsilon/2$. Hence, $1 > 2\alpha(2\alpha - 1)f(\theta)(1/2 + \theta) \Rightarrow \Upsilon(1/4, \theta) > 0$. On the other hand, as $\alpha < \alpha^* = 5/8$, we have $2\alpha - 1 < 1/4$. Thus, we can establish that $1 > \alpha f(\theta)(1/2 + \theta)/2 \Rightarrow \Upsilon(1/4, \theta) > 0$. Finally, as the condition $\theta \leq \overline{\theta}^*$ assures that $(1/2 + \theta) < 2$, $\forall \theta$, it is clear that $1 > \alpha f(\theta)$, which is true since $\alpha < 1$, and $f(\theta) = 1/s < 1$, implies that $\Upsilon(1/4, \theta) > 0$.

Consequently, as on the one hand, $\Upsilon(\cdot, \theta)$ is monotonically decreasing on $[0, 1/2[$, and as, on the other hand, $\underline{\beta}(\theta) \leq 1/4$ and $\Upsilon(1/4, \theta) > 0$, we obtain that $\Upsilon(\underline{\beta}(\theta), \theta) > 0$, namely $\det(\mathbf{H}(\underline{\beta}(\theta), \theta)) > 0$. □

From FOC (23), we prove following Propositions, $\forall (\theta, \beta) \in [\underline{\theta}, \overline{\theta}] \times [0, 1/2[$.

Proof of Proposition 4: $\partial_\alpha \varphi(\theta, \beta) = \epsilon + 2F(\theta)(1 - 2\epsilon\beta)/(1 - 2\beta) \geq 0$.

Proof of Proposition 5: First, φ is linear with ϵ, since $\partial_\epsilon \varphi(\theta, \beta) = \alpha - 2\beta(2\alpha - 1)F(\theta)/(1 - 2\beta)$. Second, as $F(\theta)$ decreases with θ, $\partial_\epsilon \varphi$ is a increasing and continuously function in θ. In addition, $\partial_\epsilon \varphi|_{\theta=\overline{\theta}} = \alpha > 0$, and $\partial_\epsilon \varphi|_{\theta=\underline{\theta}} = -2\beta(2\alpha-1)/(1-2\beta)+\alpha \geq 0$. Indeed, as β is defined by the FOC (23), in the particular case $\epsilon = 1$, we can establish that $(2\alpha-1)/(1-2\beta) = 2\beta$. In this way, we have: $\partial_\epsilon \varphi|_{\theta=\underline{\theta}} = -2\beta(2\alpha-1)/(1-2\beta)+\alpha = -4\beta^2+\alpha \geq 0$, since $\beta \leq \alpha\epsilon/2$, and $-4(\alpha\epsilon/2)^2+\alpha = \alpha[1-\alpha\epsilon^2] \geq 0$. □

Appendix C: Proof of Proposition 6

Using Eqs. (6) from (9), and by (27), in symmetric equilibrium (i.e. $\beta_R = \beta_D =: \beta$), when $\tilde{T} \approx 0$, the social welfare function L evaluated at $(T(\theta), \beta)$ is

$$L(T(\theta),\beta) = -\gamma g_0(1-2\beta)(1/2+\theta)\int_0^{T(\theta)} e^{(1-\gamma)rs}e^{-rs}\,ds - \gamma g_0(1-2\beta)\frac{\beta^2}{2} + \Delta V(\beta)e^{-rT}, \quad \text{(C.1)}$$

where $\Delta V(\beta) := -g_0(1-2\epsilon\beta)$. Thus, when $r \approx 0$, (C.1) becomes

$$L(T(\theta),\beta) = -\gamma g_0 T(\theta)(1-2\beta)(1/2+\theta) - \gamma g_0(1-2\beta)\frac{\beta^2}{2} - g_0(1-2\epsilon\beta). \quad \text{(C.2)}$$

Yet, by (13), we obtain

$$T(\theta) = \frac{(2\alpha-1)(1-2\epsilon\beta)}{(1/2+\theta)(1-2\beta)\gamma}\log\left(X_\theta\right), \text{ where } X_\theta := \frac{(\overline{\theta}-\underline{\theta})(1/2+\theta)}{(\theta-\underline{\theta})(1/2+\overline{\theta})}. \quad \text{(C.3)}$$

By substituting (C.3) in (C.2), we have

$$L(\theta,\beta) = -g_0(2\alpha-1)(1-2\epsilon\beta)\log(X_\theta) - \frac{\gamma}{2}g_0(1-2\beta)\beta^2 - g_0(1-2\epsilon\beta).$$

For simplicity we introduce the function $\zeta(\cdot)$ defined by $\zeta(\beta) := -L(T(\theta),\beta)/g_0$. Therefore, $\zeta(0) = (2\alpha-1)\log(X_\theta) \geq 0$, and $\zeta(1/2) = (2\alpha-1)(1-\epsilon)\log(X_\theta) + (1-\epsilon) \leq \zeta(0)$. In addition, $\zeta'(\beta) \geq 0 \Leftrightarrow -3\gamma\beta^2+\gamma\beta-2\epsilon[1+(2\alpha-1)\log(X_\theta)] \geq 0$. Therefore, the discriminant Δ is given by $\Delta = \gamma[\gamma - 24\epsilon(1+(2\alpha-1)\log(X_\theta))]$. Hence, as $X_\theta \geq 1$, it is clear that $\Delta \leq 0$ if $\gamma \leq 24\epsilon$. Consequently, when $\gamma \leq 24\epsilon$, as the maximum of the social welfare function corresponds to the minimum of the function ζ, and as ζ is a continuously decreasing function: $\beta^{SW} := 1/2 = \text{argmax}_{\beta\in[0,1/2]} L(T(\theta),\beta)$, for each $\theta \in [\underline{\theta},\overline{\theta}]$. Finally, by (29), as the support of the density f is finite, it is clear that $\text{ESW}'(\beta) = 2\int_{\underline{\theta}}^{\overline{\theta}} \partial_\beta L(T(\theta),\beta)\; F(\theta)f(\theta)d\theta$. Thus, $1/2 = \text{argmax}_{\beta\in[0,1/2]} L(T(\cdot),\beta) = \text{argmax}_{[0,1/2]}\,\text{ESW}(\beta)$. □