Debt Renegotiation

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Franck Moraux[a], Florina Silaghi[a,*]

[a]*Université de Rennes 1, 11, Rue Jean Macé, 35000, Rennes, France*

Abstract

This paper develops a new model of debt renegotiation in a structural framework that accounts for both taxes and bankruptcy costs. Renegotiation consists of a permanent coupon reduction that occurs at an endogenous renegotiation threshold and that does not decrease the debt value, ensuring creditors to be at worse indifferent. We investigate the size of the optimal coupon reduction and show that the new coupon has to lie in a certain range due to Pareto efficiency and participation constraints. The exact size of the new coupon and the sharing of the renegotiation surplus depend on the bargaining power of claimants. The optimal number of renegotiations is also analyzed. As the renegotiation surplus is rapidly decreasing in the number of renegotiations, and renegotiation costs increase with the number of renegotiations, a firm can only have a limited very small number of renegotiations, which is in line with empirical evidence.

Keywords: Debt renegotiation, Debt pricing, Strategic contingent claim analysis
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*Corresponding author. Tel.: +33 (0)2 23 23 78 08; fax: +33 (0)2 23 23 78 34.
Email addresses: `franck.moraux@univ-rennes1.fr` (Franck Moraux),
`florina.silaghi@univ-rennes1.fr` (Florina Silaghi)

"The basic idea was to make debt relief acceptable to commercial bank creditors by offering a smaller but much safer payment stream in exchange for the original claim that clearly could not be serviced in full.", Sturzenegger and Zettelmeyer (2007)[1]

1. Introduction

Although court supervised restructuring is always a possible solution to consider, empirical studies demonstrate that private workouts are quite often considered by firms in financial distress. Debt renegotiation represents a simple and primary alternative to improve creditors' and equity holders situations. Gilson et al. (1990), for instance, find that about half of the 169 firms they consider resolve distress through private workouts. A very strong incentive is that costs of private workouts are significantly lower than formal bankruptcy costs (see again Gilson et al., 1990, and Franks and Torous, 1989). It is also worth noting that for both formal reorganizations and private workouts, the total number of renegotiations for the same firm/loan is small, with higher numbers for the latter. Few firms undertake a second formal reorganization through Chapter 11 after a first debt restructuring: Gilson (1997) finds a percentage of only 33%, LoPucki and Whitford (1993), as well as Hotchkiss (1995) find 32%, and Alderson and Betker (1999) 24%. As for private workouts, Roberts (2010) finds an average of 4 private renegotiations for the typical loan in his sample.

Leland (1994) offers *preliminary thoughts* on debt renegotiation in the eighth section of his famous article. Debt renegotiation was also investigated in depth by Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997) and Fan and Sundaresan (2000) to name only a few. All these three important papers focus on strategic debt service (SDS hereafter) and model debt renegotiation as arbitrary "take-it-or-leave-it" offers (from the equity holder to creditors or from creditors to the equity holder) that result in a

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[1]It is worth having a look at the rich economic literature on sovereign debt renegotiation as this issue has been extensively studied compared to corporate debt renegotiation. This clear-cut description of the Brady plan (a plan for sovereign debt restructuring at the end of the 1980s in Latin America) by Sturzenegger and Zettelmeyer (2007) captures the essence of our model. Suter (1992) reveals that for interwar sovereign defaults, interest payments suffered an average haircut of 34% and face values were reduced by 23%.

temporary reduction in the coupon. Debt renegotiation may however lead to other kinds of restructuring. Fan and Sundaresan (2000) analyze debt equity swaps as well, while Longstaff (1990) considers maturity extension. Debt renegotiation with multiple lenders is analyzed by Dumitrescu (2007), Hackbarth et al. (2007) and Hege and Mella-Barral (2005). Chen (2003) and Shibata and Tian (2010, 2012) study debt renegotiation in the presence of asymmetric information. Mella-Barral (1999), Hege and Mella-Barral (2000) are also important references to consider.

We share in common with Leland and SDS models that the firm is financed with a perpetual debt whose coupon is paid in full as long as an underlying state variable[2] remains above a certain renegotiation threshold and that, when this threshold is reached from above for the first time, the firm starts paying a lower coupon. A key difference relies on the nature of the reduction. In SDS models, the coupon reduction is systematically temporary, while it is permanent in our model. In a SDS setting, the firm reduces the coupon whenever this threshold is reached from above and retakes the full coupon payment as soon as it recovers, i.e. when the assets value or output price reaches the threshold from below. This view to proceed (infinite number of temporary coupon reductions) has a clear advantage for finding analytical formulae. However, we consider that SDS models emphasize effects of temporary missed interest payments rather than effects of the permanent solution debt and equity holders may look for. Moreover, in these models the firm can strategically avoid bankruptcy costs through an infinite number of renegotiations, which is not the case in the real world. We believe it is important to account for frictions that prevent firms from infinitely renegotiating.

In this paper, we propose a new continuous time model for debt renegotiation, where the coupon reduction is permanent, which allows us to derive and investigate the effects of renegotiation on equity, debt, firm value, capital structure and the optimal number of renegotiations. Following Leland (1994), the equity holder can default at an endogenously determined bankruptcy threshold. At any point in time between debt issuance and bankruptcy time, the equity holder can ask creditors to reduce their coupon permanently in a renegotiation. Rationally, creditors accept the coupon reduction as long

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[2]Assets value or output price.

as it does not decrease the value of their claim.[3] The new coupon is then paid "forever" or until bankruptcy, meaning that the new post-renegotiation debt is a standard Leland's consol. Renegotiation is therefore theoretically equivalent to a debt-for-debt swap. However, given the empirical evidence of Roberts and Sufi (2009) and Roberts (2010) that renegotiations are much more common than new loans, we favor debt renegotiation. We can fully characterize the conditions under which renegotiation is possible, the range of possible coupons, the magnitude of the optimal reduction and the renegotiation timing.

We first show that renegotiation can only occur once the assets value deteriorates enough. In other words, when the firm is financially healthy, the equity holder cannot extract concessions from creditors.[4] We prove that the equity holder will decide debt renegotiation exactly when Leland's model recommends a default. This in turn implies that, if renegotiation fails, liquidation is optimally decided by the equity holder. Consequently, debt renegotiation is a "take-it-or-leave-it" offer. This result actually differs from previous contributions that rather assume that, if renegotiation fails, the firm will be liquidated.[5,6] In many models, if creditors reject the offer, it is not optimal for the equity holder to liquidate the firm as the equity value is still positive. She will go on paying the full coupon and will not liquidate the firm. In this sense, take-it-or-leave-it offers are often not that credible.

This result also confirms and proves Leland's preliminary thoughts on debt renegotiation: *"Renegotiation of unprotected debt is particularly simple when bankruptcy is imminent [. . .] a small reduction in coupon will increase the value of both debt and equity [. . .]. This **assumes** stockholders can credibly make a "take-it-or-leave-it" offer to bondholders. Note that the firm **may** wait until the brink of bankruptcy before renegotiating, since this will minimize D(V)."* We show that indeed the equity holder does make a credible

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[3]A coupon reduction has a direct negative effect on debt value, and an indirect positive effect through default probability. Reducing the coupon decreases the probability of default and hence the present value of liquidation costs. Creditors accept renegotiation if the indirect positive effect dominates.

[4]See Mella-Barral and Perraudin (1997).

[5]See (Anderson and Sundaresan, 1996), Fan and Sundaresan (2000) and Mella-Barral and Perraudin (1997).

[6]While completing this paper, we found some related ideas developed in Lambrecht (2001), see section 4.1. Once again, a huge difference between Lambrecht (2001) and us is that we do not make the same assumptions, but rather deduce them from our model.

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offer and waits until the brink of bankruptcy to renegotiate.

Special features of our model concern the debt value and its behavior with respect to the coupon. It is worth reminding first that in Leland (1994) the debt value is hump-shaped with respect to the coupon and that consequently there exists an optimal coupon where the debt value is maximum. Interestingly, the firm's debt capacity critically depends on the contemporaneous value of the firm. So, if ever the coupon initially agreed with the creditors lies above the optimal coupon that currently prevails, the firm does not fully exploit its debt capacity. Moreover, the firm may clearly pay an unnecessarily high coupon for the debt value it has. In some extreme cases, the firm faces financial distress that is avoidable. We prove that such a situation arises when the firm's assets value deteriorates. We then investigate whether and how the total firm value increases when the coupon is permanently reduced. We find that renegotiation increases total firm value and that the renegotiation surplus can be shared among claimants according to their bargaining power. Additionally, we show that the coupon should be reduced at least to 67% of the initial value, and at most to 27% of the initial coupon level (for our baseline case). Any coupon value above 67% is Pareto dominated, as we could further reduce it and improve both the value of debt and equity. The coupon cannot be reduced beyond 27% as creditors would refuse renegotiation. The exact size of the coupon depends on the bargaining power of claimants. To our knowledge, this is the first paper to provide a range for the optimal coupon reduction.

To finally complete the analysis, we extend the model to account for several renegotiations and we introduce renegotiation costs. We show that the renegotiation surplus is rapidly decreasing in the number of renegotiations, and assume that renegotiation costs are most likely increasing in the number of renegotiations. We conclude, consistently with empirical evidence, that a firm could only have a very small number of renegotiations.

The rest of the paper is organized as follows. Section 2 describes our financial setup and valuation of financial claims. Section 3 develops the solution of the renegotiation process. Section 4 extends the model to multiple renegotiations and introduces renegotiation costs. Numerical simulations are discussed in section 5, while section 6 concludes.

2. Financial setup and valuation

This section describes first the continuous-time financial setup we use, introduces our model of debt renegotiation and then presents the valuation of financial claims. Our setting follows Leland (1994). Financial markets are perfect,[7] efficient and complete and trading takes place continuously. There exists a riskless asset paying a known and constant interest rate denoted by r. There are no transaction costs. Let us now consider a firm that is financed by equity and a consol debt only. Initially, creditors enjoy a coupon whose value is denoted by c. The firms' assets value is observable by all agents and is correctly described under the objective probability measure by a geometric Brownian motion:

$$dV_t = \mu V_t dt + \sigma V_t dW_t \tag{1}$$

where $W = (W_t)_t$ is a standard Brownian motion, μ and σ represent the drift and volatility terms, respectively. If ever the firms assets value falls below a certain threshold level, say V_B, the firm goes bankrupt.[8] The firm pays income taxes at a rate τ, at least until bankruptcy. In case of liquidation, a fraction of the assets value, denoted by $\alpha (0 \leq \alpha \leq 1)$ is lost and the absolute priority rule strictly applies. This means that creditors must be fully repaid before the equity holder can receive something.

In absence of renegotiation, we know how to price equity, unprotected debt, tax shields, bankruptcy costs and the firm as a whole when the equity holder can decide to stop paying the coupon (see Leland, 1994). The price of unprotected debt is e.g. given by:

$$d(V, c, V_B^*) = \frac{c}{r}\left(1 - \left(\frac{V}{V_B^*}\right)^{-X}\right) + (1-\alpha)V_B^*\left(\frac{V}{V_B^*}\right)^{-X}, \tag{2}$$

whereas that of the equity is:

$$e(V, c, V_B^*) = \left\{V + \frac{c}{r}\tau\left(1 - \left(\frac{V}{V_B^*}\right)^{-X}\right) - \alpha V_B^*\left(\frac{V}{V_B^*}\right)^{-X}\right\} \\ - d(V, c, V_B^*), \tag{3}$$

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[7]Perfection here does not mean that there is no liquidation cost.

[8]Throughout the paper, default, bankruptcy and liquidation occur simultaneously as differentiating them is out of the scope of the paper. See Bruche and Naqvi (2010), François and Morellec (2004) and Moraux (2002), for a discussion on this. This bankruptcy threshold is an endogenously determined one.

where the term in brackets in equation (3) stands for the total value of the firm. Here, the constant X is given by $X = 2r/\sigma^2$ and the value of the firm's assets V is higher than the default threshold $V_B^* = \frac{(1-\tau)c}{r}\frac{X}{1+X} = \frac{(1-\tau)c}{r+0.5\sigma^2} \equiv V_B(c)$, which is endogenously chosen by the equity holder. Note that V_B^* is the optimal default threshold in the absence of renegotiation and it solves the smooth-pasting condition $\left.\frac{\delta e(V,c,V_B)}{\delta V}\right|_{V=V_B^*} = 0$.

In presence of renegotiation, the equity holder aims at reducing the initial coupon c to a payment of c'. Hence the equity holder pays the initial coupon until the firm's assets value reaches a certain renegotiation threshold V_S (if ever). At V_S the equity holder proposes renegotiation and starts paying a lower coupon c' until a new (postponed) default threshold V_B' is reached.[9] One has $V_B' \leq V_S \leq V_0$, where V_0 is the initial assets value.[10] The following proposition then applies.

Proposition 1. *For all $V_B' \leq V_S \leq V$*

(i) *The debt value is given by:*

$$
\begin{aligned}
D(V,c,V_S,c',V_B') = &\frac{c}{r}\left(1 - \left(\frac{V}{V_S}\right)^{-X}\right) + \frac{c'}{r}\left(1 - \left(\frac{V_S}{V_B'}\right)^{-X}\right) \\
&* \left(\frac{V}{V_S}\right)^{-X} + (1-\alpha)V_B'\left(\frac{V}{V_B'}\right)^{-X}
\end{aligned}
\tag{4}
$$

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[9]We highlight that we distinguish between two different situations: no renegotiation, which corresponds to Leland's case in which the firm defaults at $V_B^* \equiv V_B(c)$ and renegotiation (the model proposed in this paper) in which the firm renegotiates at V_S and defaults at a new default threshold V_B'.

[10]Notice that the new default threshold V_B' accounts for the existence of renegotiation, thus it will be lower or equal to V_S

(ii) *The equity value is given by:*

$$
E(V, c, V_S, c', V_B') = \left\{ V + \frac{c}{r}\tau \left(1 - \left(\frac{V}{V_S} \right)^{-X} \right) + \frac{c'}{r}\tau \left(1 - \left(\frac{V_S}{V_B'} \right)^{-X} \right) \right.
$$

$$
\left. * \left(\frac{V}{V_S} \right)^{-X} - \alpha V_B' \left(\frac{V}{V_B'} \right)^{-X} \right\} - D(V, c, V_S, c', V_B')
$$

$$
= V - (1 - \tau)\frac{c}{r} \left(1 - \left(\frac{V}{V_S} \right)^{-X} \right) - (1 - \tau)\frac{c'}{r}
$$

$$
* \left(1 - \left(\frac{V_S}{V_B'} \right)^{-X} \right) \left(\frac{V}{V_S} \right)^{-X} - V_B' \left(\frac{V}{V_B'} \right)^{-X},
$$

(5)

PROOF OF PROPOSITION 1. See appendix.

Other highlighting equivalent expressions to equations (4) and (5) are respectively:

$$
D(V, c, V_S, c', V_B') = \frac{c}{r} \left(1 - \left(\frac{V}{V_S} \right)^{-X} \right) + d(V_S, c', V_B') \left(\frac{V}{V_S} \right)^{-X}
$$

(6)

$$
E(V, c, V_S, c', V_B') = \left(V - \frac{(1 - \tau)c}{r} \right) + \left\{ e(V_S, c', V_B') \right.
$$

$$
\left. - \left(V_S - \frac{(1 - \tau)c}{r} \right) \right\} \left(\frac{V}{V_S} \right)^{-X},
$$

(7)

Equation (6) reveals that, until the first hitting time of V_S, creditors receive (for sure) the initial coupon flow. Equation (7) highlights that the debtor owns the firm's assets minus the coupon flow paid to creditors (net of tax). As soon as the renegotiation threshold is hit, stakeholders simply swap their initial share for a new Leland-type equity claim, which depends on the new reduced coupon.

Both the renegotiation and the default thresholds (V_S and V_B') are determined strategically by the equity holder, just as they choose the default one in Leland (1994). Since the coupon change is unique and permanent,

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the equity just after the single reorganization is that studied by Leland (1994): his findings therefore apply and the optimal default threshold is $V'_B = V_B(c') = \frac{(1-\tau)c'}{r} \frac{X}{1+X} = \frac{(1-\tau)c'}{r+0.5\sigma^2}$. The post-renegotiation endogenous default threshold V'_B is therefore a consequence of the choice of c'.

To convince creditors to accept a coupon reduction, the equity holder proposes them a new coupon such that the value of their new claim is at least as big as their reservation value, assuming that creditors may always refuse any change. Creditors may be at worse indifferent between accepting and rejecting the offer, when the value of their new claim is equal to their reservation value. However, they can expect the new lower debt service to be financially easier to sustain.[11] Equations (6) and (7) reveal that, in the presence of renegotiation, values of equity and debt do not *directly* depend on V_B^*. We will show below that this default threshold plays an essential role and impacts indirectly through its effect on the reservation value of the creditors. When the equity holder designs a new (convincing) coupon, a couple of situations arise depending on whether the value of V_S is above or below V_B^*.

For $V_S > V_B^*$, the reservation value of the creditors is the debt value in the absence of renegotiation (with full coupon payments). This is because if creditors refuse renegotiation, it is optimal for the equity holder to continue paying the full coupon (equity is positive above V_B^*), and not to default (the equity holder would get zero in default). The new coupon c' has to satisfy the following condition:[12]

$$D(V_S, c, V_S, c', V_B(c')) = \beta d(V_S, c, V_B^*). \tag{8}$$

where $\beta \geq 1$ and $d(V_S, c, V_B^*)$ stands for the value of the debt creditors own if they refuse renegotiation, which depends on the default threshold V_B^*. Using equation (6), the left hand side, debt value just after renegotiation, is equal to

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[11]We assume without loss of generality that whenever creditors are indifferent they will accept renegotiation. A reason for this is that, in anticipation of rejection, the equity holder can always slightly increase the coupon such that the creditors strictly prefer acceptance.

[12]Notice that this is highly different from what is usually assumed in the literature. Creditors are usually posited indifferent between accepting or rejecting the offer because the debt value is equal to the liquidation value. Notice also that at renegotiation creditors receive a lower coupon. However, by reducing the coupon, creditors avoid the bankruptcy costs and decrease the probability of a future default. Overall, creditors benefit from a coupon reduction.

$d(V_S, c', V_B(c'))$. Hence, the above equation is equivalent to $d(V_S, c', V_B(c')) = \beta d(V_S, c, V_B^*)$, and the debt value after renegotiation is greater or equal to the reservation value of the creditors.

For $V_S \leq V_B^*$, the reservation value of the creditors is the liquidation value. Indeed, if they refuse renegotiation, it is optimal for the equity holder to default since equity value is zero in the absence of renegotiation. The new coupon c' satisfies the following condition:

$$D(V_S, c, V_S, c', V_B(c')) = \beta(1 - \alpha)V_S. \tag{9}$$

The default threshold V_B^* does not appear in the right hand side of equation (9), but it characterizes the domain of definition of this equation. For the special case $V_S = V_B^*$, one has $d(V_S, c', V_B(c')) = \beta(1 - \alpha)V_B^*$.

Equations (8) and (9) deserve few comments. First of all, for the particular case of $\beta = 1$ and $V_S \geq V_B^*$, the new debt is designed such that $D(V_S, c, V_S, c', V_B(c')) = d(V_S, c, V_B^*)$ and the debt price at time zero coincides with what Leland (1994) finds. Second, equations (8) and (9) can only be solved numerically. Anyway, the new coupon c' is a function of V_S and we denote it by $c'(V_S)$. Hence, equity is worth at time 0:

$$E(V_0, c, V_S, c'(V_S), V_B(c'(V_S))) \equiv Eq(V_0, c, V_S)$$

and we observe this is a function of c and V_S only. We question in the next section whether an optimal renegotiation threshold V_S^* exists for the equity holder. Finally, one could interpret β as a proxy for the bargaining power of creditors. When $\beta = 1$, creditors have zero bargaining power as they obtain no surplus from renegotiation. As β increases, the bargaining power of creditors increases. Actually their share of renegotiation surplus increases, until it reaches a maximum. Debt being a hump-shaped function of the coupon level, there exists a maximum value that debt after renegotiation can take, which corresponds to a coupon value equal to $C_{max}(V_S)$. So there exists a maximum value for β given by $\beta_{max} = \frac{d(V_S, C_{max}(V_S), V_B(C_{max}(V_S)))}{d(V_S, c, V_B^*)}$ for $V_S \geq V_B^*$ and given by $\beta_{max} = \frac{d(V_S, C_{max}(V_S), V_B(C_{max}(V_S)))}{(1-\alpha)V_S}$ for $V_S \leq V_B^*$,[13] which depends on the risk-free interest rate, volatility, tax rate, liquidation costs and on

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[13]Further manipulating this equation we obtain for the case $V_S \leq V_B^*$:
$\beta_{max} = \frac{d(V_S, C_{max}(V_S), V_B(C_{max}(V_S)))}{(1-\alpha)V_S} = \frac{[1 + X - (1-\alpha)(1-\tau)X]^{-1/X}}{(1-\alpha)(1-\tau)}$.

the renegotiation threshold.[14] We must solve now the renegotiation problem, that is, find both the optimal renegotiation threshold and the optimal reduced coupon.

3. Solving the renegotiation problem

We start by deriving the specific conditions under which renegotiation can occur in this model. Proposition 2 essentially formalizes the intuition that renegotiation can only occur if the assets value deteriorates enough.

Proposition 2. *Renegotiation can only occur if $V < \tilde{V}(c)$, where:*
$c = C_{max}(\tilde{V})$, $C_{max}(V) = V[(1 + X)k]^{-1/X}$,
$k = [1 + X - (1 - \alpha)(1 - \tau)X]m$, *and* $m = [(1 - \tau)X/r(1 + X)]^X/(1 + X)$.

We remind that in Leland (1994) $C_{max}(V)$ captures the debt capacity of the firm, i.e. the coupon rate that maximizes the debt value.

PROOF OF PROPOSITION 2. As the firm starts with an initial coupon inferior to $C_{max}(V_0)$, it is not optimal for the manager to make an offer at a high V_S, since this would actually imply to increase the coupon in order to maintain the debt value at least constant, which damages the equity holder (see panel a) of Figure 1). We show that there exists a lower threshold above which the firm cannot enter renegotiation as defined in this model. This threshold $\tilde{V}$ corresponds to the point in which $c = C_{max}(\tilde{V})$ (see panel b) of Figure 1). Below this threshold, the initial coupon will be higher than the coupon corresponding to the firm's debt capacity. Therefore, for any V_S between V_B' and $\tilde{V}$, there exists a $c' \leq C_{max}(V_S) < c$ such that equation (8) or (9) is satisfied, implying that the firm can make an offer of a reduced coupon that does not hurt the creditors (see panel c) of Figure 1). □

[Figure 1 about here.]

As the firm assets value starts to deteriorate, it reaches a point at which the initial coupon level becomes unsustainable (or in other words, unnecessarily high). From that point on, renegotiation is desirable, as it can at least

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[14]See Table 4 that illustrates the values that β_{max} can take depending on these parameters for $V_S \geq V_B^*$.

improve the situation of one party without harming the other. Note that this is different from previous results that show that renegotiation is possible even when the firm is healthy and that the equity holder may exploit this opportunity for her own advantage.[15]

Next, we analyze two extreme situations that will give us a range for the new coupon, and therefore for the coupon reduction. Consider first the case in which $\beta = 1$. This implies that debt value stays constant at renegotiation, and that the equity holder captures all the surplus from renegotiation. Let us denote the new coupon obtained in this case by c'_{min}. This coupon corresponds to the maximum coupon reduction possible at renegotiation. Any further reduction of the coupon is not possible, since the creditor will not accept any coupon below c'_{min}. At the opposite extreme we have the case in which $\beta = \beta_{max}$, in which the creditors obtain the highest possible increase in their debt value.[16] Let us denote by c'_{max} the corresponding coupon value. We argue that this matches the minimum coupon reduction at renegotiation. Any coupon in the interval (c'_{max}, c) would be Pareto dominated, as a decrease in the coupon would make both the creditors and the equity holder better off. Therefore, the coupon has to be reduced at least until c'_{max}. We have thus provided a range for the new coupon: $c' \in [c'_{min}, c'_{max}]$. See Figure 2 for an illustration.

[Figure 2 about here.]

The following corollary analyzes the existence and uniqueness of c'.

Corollary 1. *For any V_S with $V'_B \leq V_S < \tilde{V}(c)$, there exists a unique coupon $c'(V_S)$, with $c' \in [c'_{min}, c'_{max}]$, such that equation (8) (for $V_S > V^*_B$) or equation (9) (for $V_S \leq V^*_B$) is satisfied. The exact size of the coupon depends on the bargaining power of the claimants, β.*

We now solve for the optimal renegotiation threshold. This threshold is endogenously chosen by the equity holder to maximize equity value.[17] Proposition 3 characterizes the optimal renegotiation threshold.

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[15]See Mella-Barral and Perraudin (1997), and Anderson and Sundaresan (1996).

[16]Note that this surplus is equal to the firm value minus $(1 - \alpha)V_S$, which is higher than αV_S due to the existence of tax shields. In particular, the firm value is higher than the assets value due to the tax advantage of debt. By renegotiating, one saves not only the liquidation costs, but also the future tax benefits.

[17]Ex ante (before debt issuance) the equity holder wants to maximize the total firm value denoted by v, i.e. the equity value (E) plus debt proceeds (D). Ex post, the equity holder

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Proposition 3. *The optimal renegotiation threshold is given by $V_S^* = V_B^*$.*

PROOF OF PROPOSITION 3. The renegotiation threshold has two opposite effects on equity value: a positive direct effect and a negative indirect effect through its impact on the reduced coupon. The reasoning is as follows: on the one hand the equity holder would like to renegotiate as soon as possible in order to obtain a coupon reduction (positive direct effect). On the other hand, the later you renegotiate, the higher the coupon reduction obtained (negative indirect effect).

We show in the appendix that in the region $V_S \geq V_B^*$ (where equation (8) applies), the indirect effect dominates and equity is a decreasing function of V_S, so that the equity holder has incentives to choose the lowest possible value. If $V_B^* \leq V_S < \tilde{V}(c)$, renegotiation is possible and the optimal renegotiation threshold is the naive default barrier V_B^*. For the region $V_S \leq V_B^*$, an analytic proof is not available, however it can be shown numerically that $\frac{dE}{dV_S} > 0$ for reasonable parameter values. Combining this with the previous result that $\frac{dE}{dV_S} < 0$ for $V_S \geq V_B^*$, we obtain that the optimal renegotiation threshold is Leland's default threshold. $\square$

Several important results are implied by the optimal renegotiation threshold we obtain above. We list them in the next corollary.

Corollary 2. (i) *The equity holder's offer is a take-it-or-leave-it-offer.*
(ii) *The new reduced coupon c'^* verifies*

$$d(V_B^*, c'^*, V_B(c'^*)) = \beta(1-\alpha)V_B^* \qquad (10)$$

and is a constant fraction of the initial coupon, irrespective of the level of the initial coupon. The ratio $\frac{c'^*}{c}$ verifies

$$\frac{r + \sigma^2/2}{r(1-\alpha)(1-\tau)} - \beta\frac{c}{c'^*} + \left(1 - \frac{r + \sigma^2/2}{r(1-\alpha)(1-\tau)}\right)\left(\frac{c}{c'^*}\right)^{-X} = 0. \quad (11)$$

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wishes to maximize equity value. It can be shown however that the optimal renegotiation threshold is not always incentive compatible in the sense that once debt is in place, the equity holder sometimes has incentives to deviate and renegotiate at a different threshold, as the ex ante set V_S^* does not maximize his equity value ex post. Since the equity holder does not have to commit ex ante to a certain renegotiation threshold, we assume he can deviate ex post, this is why we consider that the optimal renegotiation threshold is given by the ex post V_S^*.

(iii) *The new optimal default threshold is such that $V_B(c'^*) = V_B'^* = V_B^* \frac{c'^*}{c}$.*

(iv) *When the renegotiation threshold is endogenously chosen, the debt value at time 0 is*

$$D(V_0, c, V_S^*) = D(V_0, c, V_B^*)$$
$$= c/r + [\beta(1-\alpha)V_B^* - c/r] (V/V_B^*)^{-X} . \qquad (12)$$

(v) *Creditors save part of liquidation costs equal to $(\beta - 1)(1 - \alpha)V_B^*$.*

PROOF OF COROLLARY 2. (i), (ii), and (iii) are immediate, yet they are important qualitative consequences of Proposition 3. Details are given in the appendix. As a consequence of (i), debt renegotiation appears not to be an arbitrary assumption as previously considered in the literature. At V_B^* the equity holder avoids bankruptcy through renegotiation. She therefore makes a take-it-or-leave-it-offer, meaning that the naive default barrier V_B^* is V_S^*. A new default threshold arises too ($V_B'^*$). The ratio $\frac{c'^*}{c}$ depends only on structural parameters such as the level of the interest rate, the firm's assets volatility, the tax rate and the bankruptcy costs. It does not depend on the initial coupon level c. So no matter the coupon level that the firm starts with, the coupon reduction (expressed in percentages) will always be the same for a given firm. The coupon reduction ratio $\frac{c'^*}{c}$ also characterizes the default threshold reduction ratio. Equation (12) looks very similar to the standard Leland's equation (2) except that now the recovery component depends on β. The debt value with renegotiation is actually identical to the debt value of a firm that enjoys no renegotiation, but which recovers a higher fraction of the assets in default: $\beta(1-\alpha)$ instead of $(1-\alpha)$. Creditors can save a part of liquidation costs through renegotiation. The additional recovery they obtain is a function of their bargaining power and amounts to $\beta(1-\alpha) - (1-\alpha) = (\beta - 1)(1-\alpha)$. □

The model presented here assumes a single permanent coupon reduction and a possible subsequent default after renegotiation in case the firm value deteriorates dramatically after a successful renegotiation, with neither renegotiation nor reputation costs. We now generalize this setup and analyze what happens when we allow for several renegotiations and we account for reputation costs.

4. Multiple renegotiations and reputation costs

We have assumed so far one single renegotiation. One can wonder however why the firm would not renegotiate again, once it has reached the post-renegotiation bankruptcy threshold, and thus avoid bankruptcy costs through infinite renegotiation. There are two important things to notice. First, infinite renegotiation is not possible in our model, as we only consider permanent coupon reductions (as opposed to temporary coupon reductions or alternating coupon reductions with coupon increases). Eventually, after a finite number of renegotiations, the coupon would become insignificant and the firm would be a full-equity firm. Indeed, if the same result as above (see equation (11)) applies twice, three times, etc., then the computed ratio is powered by 2, 3, etc. leading to a final negligible coupon value.

Second, renegotiation costs might prevent firms from infinitely renegotiating their debt, leading to a small finite number of renegotiations. Quite similar to this concern, Fischer et al. (1989) consider in their study of dynamic recapitalization policies that costs of recapitalization prevent a firm from adjusting its capital structure continuously. Actually, costs of renegotiation as well as reputation costs could matter for both parties for different reasons. The firm's reputation and its value might be strongly affected as the private workout becomes common knowledge to suppliers. These latter can refuse to grant supplier credit facilities. At the same time, creditors' reputation might suffer as current renegotiation sends a "friendly-debtor" signal to other firms which might want the same concessions. Our setting can easily account for renegotiation and reputation costs. A firm will renegotiate its debt as long as the surplus from renegotiation is higher than renegotiation costs. We abstract from the bargaining power of claimants and analyze the optimal number of renegotiations from the point of view of the total firm value. We index the renegotiation number by i. The overall surplus is the sum of the increase in equity value due to renegotiation and the increase in debt value, so at each renegotiation it is then given by:

$$S_i = [E_i^+(V_S) - E_i(V_S)] + [D_i^+(V_S) - D_i(V_S)] = E_i^+(V_S) + (\beta - 1)D_i(V_S), \quad (13)$$

where E_i^+, E_i represent the equity value just after, respectively just before the ith renegotiation. As the renegotiation threshold coincides with Leland's default threshold, equity just before renegotiation is equal to zero. Debt value increases by a factor β, as shown by equation (8). The renegotiation

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surplus decreases rapidly with the number of renegotiations. We find the following relationship:

$$S_{i+1} = \gamma S_i = \gamma^i S_1, \tag{14}$$

where $\gamma = \frac{c_{i+1}}{c_i} < 1$. This implies that at each new renegotiation the new surplus will be equal to a proportion of the previous surplus. This proportion coincides with the coupon reduction at each renegotiation, which is constant across renegotiations.[18]

We model renegotiation costs in the simplest form possible. We assume a fix cost for the first renegotiation and then a linearly increasing cost in the number of renegotiations:

$$C_i = Ri, \tag{15}$$

where R is the fix cost of the first renegotiation. The optimal number of renegotiations, n^*, is such that the renegotiation surplus equals the renegotiation costs: $\frac{\gamma^{n^*-1}}{n^*} = \frac{R}{S_1}$.[19]

5. Numerical analysis and discussion

We now study numerical implications of our model on the default and renegotiation decisions, firm value, capital structure and the number of renegotiations. Regarding parameter values, we choose order of magnitudes similar to those assumed by previous models of debt renegotiation, in order to facilitate comparison between models. For our baseline case, we set the riskless interest rate to 6% (as Leland, 1994 and Mella-Barral and Perraudin, 1997 did), the tax rate to 35%, volatility to 25% (both as in Leland, 1994 and Fan and Sundaresan, 2000), bankruptcy costs to 40% (Mella-Barral and Perraudin, 1997 chose 20%, while Leland, 1994 chose 50%) and we consider a coupon rate of 7%. Without loss of generality, we set the initial assets value equal to 100. Our baseline case parameter values are presented in Table 1. These parameter values are used in all the tables and figures presented in this paper, unless specified otherwise.

[Table 1 about here.]

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[18] See appendix for a proof of this result.

[19] To make sure that this is an integer number, we round down to the highest integer that is lower or equal than the solution of this equation.

5.1. Renegotiation outcome

We start by examining the outcome of renegotiation at different levels of initial leverage of the firm. Table 2 illustrates the variables of interest for coupon rates varying from 5 to 9%. We present as well two special coupon rates that correspond to the optimal leverage and to the firm's debt capacity respectively in the Leland (1994) model, i.e. C^* that maximizes the value of the firm and C_{max} which maximizes the debt value respectively, as defined by Leland.[20] The variables we focus on are: the renegotiation threshold V_S^* (which coincides with the initial bankruptcy threshold, V_B^*), a proxy for the probability of renegotiation (the present value of \$1 to be obtained conditional on renegotiation) and the range for the optimal reduced coupon c'_{min} and c'_{max}.

[Table 2 about here.]

As shown in Table 2, the renegotiation threshold varies from 36 to 67 when we vary the coupon rate from 5% to the coupon corresponding to the firm's debt capacity of 9.37%. The higher is the coupon they have to pay, the less is the equity holder willing to continue running the firm at this coupon level. The renegotiation probability increases with the coupon level, reaching almost 50% for a coupon as high as 9%. In all the cases the coupon is reduced at most to 27% and at least to 67% of the initial coupon value.[21] The results suggest that it is precisely because of the huge decrease in coupon that the equity holder obtains at this renegotiation threshold that she chooses to pay the full coupon until reaching V_B^*, and does not renegotiate earlier. An earlier renegotiation would imply a smaller decrease in the coupon value.

[Table 3 about here.]

We next examine the comparative statics of these variables with respect to the parameters τ, σ, α, and r. As the renegotiation threshold coincides with Leland (1994)'s bankruptcy threshold, we obtain the same comparative statics. Namely, the renegotiation threshold decreases as the tax rate, riskiness of the firm or interest rate rise, and is independent of the bankruptcy

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[20]Note that the optimal coupon and the coupon that corresponds to the firm's debt capacity in this model are in general different from those from Leland's model.

[21]As mentioned previously, the relative coupon reduction does not depend on the initial coupon level.

costs. The probability of renegotiation decreases with the tax and interest rate, does not depend on bankruptcy costs and has an ambiguous sign with respect to volatility. The optimal reduced coupon set at renegotiation decreases with tax rate, volatility and bankruptcy costs, while it increases with the interest rate.

[Figure 3 about here.]

In order to understand these results, we analyze Figure 3, which represents the debt value evaluated at the renegotiation threshold, as a function of the new coupon, for different values of the tax rate/volatility/bankruptcy costs/interest rate (panels a), b), c), and d)). We consider the baseline case with an initial coupon rate of 7%. Therefore the new reduced coupon rate has to lie in the interval (0,7). An increase in any of the four parameters has a direct effect on debt value (negative for all except for the tax rate, for which it is positive) and an indirect effect through the renegotiation threshold (negative for all except for the bankruptcy costs, for which it is zero). Overall, an increase in any of the four parameters leads to a decrease in debt value at renegotiation. At the same time, we observe that a higher tax rate/volatility/bankruptcy cost leads to a lower $C_{max}(V_S)$ (the coupon that maximizes debt value at renegotiation, which is equal to c'_{max}, the optimal reduced coupon for $\beta = \beta_{max}$). However, an increase in the interest rate has the opposite effect, leading to an increase in $C_{max}(V_S)$. The minimum possible reduced coupon, $c'^* = c'_{min}$ corresponding to $\beta = 1$ is determined as the coupon for which the debt value at renegotiation equals the value of debt at renegotiation under the initial coupon rate of 7%. Thus, its comparative statics will be the same as those of c'_{max}: we obtain a new coupon that is decreasing in tax rate/volatility/bankruptcy cost, while increasing in the interest rate. Any coupon within this range will have the same comparative statics.

[Figure 4 about here.]

Figure 4 illustrates the maximum and the minimum coupon reduction for different values of volatility and bankruptcy costs. We can see that for low volatility levels the coupon range is tight, while as volatility increases this range for the new coupon widens. The coupon will be reduced at least to 86% of the initial coupon level and at most to 54% for $\sigma = 0.1$ and $\alpha = 0.1$.

For very risky firms, with $\sigma = 0.5$ and $\alpha = 0.9$, the coupon is reduced at least to 46% and at most to 3% of the initial level. Of course, these are extreme values, but they serve for an illustrative purpose.

[Figure 5 about here.]

We finally offer further insights on the renegotiation's outcome and effects in Figure 5, for the case of the maximum coupon reduction. We provide two distinct measures of the "extra life" that the firm obtains through renegotiation: the difference between the two default thresholds and a proxy for the probability of default at renegotiation.[22] The difference between the two default thresholds plotted in panel a) of Figure 5 is logically a linear increasing function of c for a given volatility (as c' is proportional to c). For a given coupon level, this distance is decreasing in volatility. The proxy for the default probability plotted in panel b) is increasing with volatility. Of course, the lower this probability is, the higher the extra life of the firm.

5.2. Optimal capital structure

We now study the impact of renegotiation on debt, equity, and firm value, as well as on the optimal capital structure at time 0. We plot in Figure 6 debt, equity and firm value for Leland's model (panel a)) and for our model, the case of maximum coupon reduction with $\beta = 1$ (panel b)). As shown in Figure 6 (panel b)), in this model debt is a hump-shaped function of the coupon, and equity is a U-shaped function of the coupon. Debt value in our model is equal to Leland (1994)'s for $\beta = 1$, so logically we obtain similar results. Nevertheless, whereas Leland finds that equity is decreasing in the coupon rate, we obtain a U-shaped equity value. This is the consequence of the renegotiation process: the higher is the coupon, the more likely renegotiation becomes and the higher will be the increase in equity due to renegotiation. It turns out that at high (enough) levels of coupon this effect dominates the negative effect found by Leland (1994).[23] The total firm value

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[22]This is not exactly the default probability, as it includes the discount factor. It is in fact the present value (at renegotiation) of \$1 to be obtained conditional on default post-renegotiation.

[23]We mention that it could be the case that in order to avoid paying a high sustainable coupon, the equity holder has incentives to ask for an even higher unsustainable coupon. This is because at these levels of the coupon, renegotiation is very likely, and the coupon is significantly reduced. To this extent, it is possible that the reduced coupon is lower than the initial coupon the equity holder was trying to avoid in the first place.

is hump-shaped and it reaches its maximum at a C^* that is above the one from Leland (1994). Nevertheless, our model predicts lower leverage ratios than Leland's model. This is due to the fact that firm value is higher in our model, which leads to lower leverage ratios. This negative effect dominates the positive effect that the higher optimal coupon has on leverage.

[Figure 6 about here.]

5.3. Renegotiation surplus

Panel a) of Figure 7 overlaps the debt, equity and firm value derived in this model for $\beta = 1$ with the ones obtained by Leland (1994), in order to highlight the added value of renegotiation. We can notice first of all, that renegotiation is worthwhile, as the firm value in the presence of renegotiation is higher. Secondly, the extra value obtained through renegotiation is not shared at all among stakeholders, the equity holder captures everything, and creditors get nothing.

[Figure 7 about here.]

Panel b) of Figure 7 shows the sharing of the renegotiation surplus between creditors and the equity holder for $\beta = \beta_{max}$. Even when the creditors receive the maximum surplus from renegotiation, they do not capture the whole surplus. Thus, the equity holder's gain from renegotiation is still positive, as the gains of the creditors are limited by β_{max}.

Note however, that the renegotiation surplus (the pie shared) is not the same in the two situations as the new coupon differs. Which of the two surpluses is higher depends on the trade-off between tax benefits and bankruptcy costs. Figure 8, panel a), illustrates the percentage increase in firm value due to renegotiation for the case $\beta = 1$.[24] We can notice that renegotiation increases firm value by up to 12%.[25] Panel b) shows the difference between the renegotiation surplus for $\beta = \beta_{max}$ and the surplus for $\beta = 1$. For high bankruptcy costs, the difference in the surpluses is negative, which implies

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[24]Note that this is the present value of the increase in firm value due to renegotiation computed at the moment of debt issuance, that is at time 0. It thus takes into account the discount rate and the probability of renegotiation. The increase in firm value evaluated at the moment of renegotiation is much higher, around 50% for our baseline case.

[25]Of course this is true for the given parameter values, $\tau = 0.35$, $r = 0.06$, and $C = 7$. This surplus could be higher or lower, for different parameters.

that setting a lower coupon is better, in order to reduce the probability of reaching bankruptcy. For low bankruptcy costs, the difference is positive, which means that a higher coupon is better as the firm can take advantage of the tax benefit.

[Figure 8 about here.]

[Table 4 about here.]

5.4. Multiple renegotiations and reputational costs

For illustrative purposes, we provide a simple numerical example for the case in which we allow for multiple renegotiations and reputation costs. Consider the parameter values for the baseline case. Assume also that the equity holder has all bargaining power, so that at renegotiation we have the maximum coupon reduction. In this case, we obtain a surplus at renegotiation equal to approximately 29, on top of the pre-renegotiation firm value that is equal to approximately 29 as well (an increase of 50% in firm value at the moment of renegotiation). Remember that the maximum coupon reduction is in this case 27%. We further assume that after the first renegotiation the assets value deteriorates dramatically and reaches the post renegotiation bankruptcy threshold. We know that the surplus from a second renegotiation would be just a fraction of 27% of the first surplus, 29. This implies a second surplus of approximately 8, and a surplus from a third renegotiation of 2. The surplus decreases rapidly with the number of renegotiations. If the firm suffers some renegotiation costs above 8 for the second renegotiation, the firm will only renegotiate once and liquidate when the assets value reaches the post renegotiation bankruptcy threshold. With renegotiation costs and a rapidly decreasing surplus from renegotiation, the firm will have a finite small number of renegotiations.

While this example is very didactic and simplistic, we think it sheds some light on the dynamics of renegotiation and is in line with empirical evidence showing that the number of firms that undertake a second debt reorganization after a first debt restructuring is small.[26]

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[26]LoPucki and Whitford (1993), as well as Hotchkiss (1995) find only 32%, Gilson (1997) 33% and Alderson and Betker (1999) 24% of firms undertake a second reorganization. These studies refer to formal Chapter 11 reorganizations. One can expect these numbers to be slightly higher for private workouts, given that the cost of a private workout is smaller than the cost of a Chapter 11 reorganization. Indeed, Roberts (2010) finds an average of 4 private workouts for the typical loan.

5.5. Passing from junk to investment-grade bond

[Table 5 about here.]

As mentioned previously, our model of debt renegotiation implies a coupon reduction, while maintaining the value of debt at least constant. By entering renegotiation, the firm is able to get out of financial distress, without hurting the creditors. This special feature of the model makes it possible to transform a junk bond into an investment-grade bond. Table 5 presents the credit indicators of two different bonds with equal values: a junk bond that corresponds to the firm's debt just before renegotiation, and an investment-grade bond that corresponds to the firm's debt immediately after renegotiation. In the absence of renegotiation, when the asset value reaches V_B^*, the probability of default is equal to 1, leverage is equal to 100% and the credit spreads are extremely high. However, with renegotiation (and maximum coupon reduction, $\beta = 1$), the coupon reduction implies a significant decrease in the probability of default to 0.08, a leverage of 50.16% and an important decrease in the credit spreads to 40 basis points. The only credit indicator that worsens is the recovery rate, as now in case of default the creditors would recover a smaller value. Therefore, due to the renegotiation model presented here, a junk bond can climb several rates in the credit rating scale.

6. Conclusions

This study presents an original model of debt renegotiation, in the framework of a structural model that accounts for both taxes and bankruptcy costs. We propose a permanent unique coupon reduction, at an optimal renegotiation threshold chosen by the equity holder, which we think provides a better description of private workouts (together with cuts in debt's face value) than strategic debt service (which we view as temporary missed interest payments).

We find that renegotiation can only occur at low enough levels of the firm's assets value. We show that the equity holder proposes renegotiation at the point at which the firm is liquidated in Leland (1994)'s model, and bankruptcy is postponed to a new post-renegotiation bankruptcy threshold. If renegotiation fails, the equity holder optimally liquidates the firm, meaning that the take-it-or-leave-it offer is credible.

To our knowledge, our paper is the first to provide a range for the coupon reduction. We show that the coupon should be reduced at least to 67% of the

initial coupon (otherwise there exist Pareto improvements) and at most to 27% (otherwise creditors refuse renegotiation) for reasonable parameter values. Renegotiation is worthwhile as it leads to a considerable increase in firm value (12% for our baseline case). The exact size of the reduced coupon and the sharing of the surplus stakeholders depend on their bargaining power. Finally, when accounting for multiple renegotiations, we show that the surplus rapidly decreases in the number of renegotiations. Furthermore, if we take into account renegotiation costs, possibly increasing in the number of renegotiations, the firm will have a finite small number of renegotiations, in line with empirical evidence.

Our framework could be extended to account for cash-flow based covenants. Given the significant reduction in the coupon payment, the renegotiation process proposed here could be a solution for any liquidity problems the firm might have that could impede it from satisfying such a covenant. Another interesting question is how the renegotiation process changes with heterogeneous debt (bank loan versus public debt, senior versus subordinated debt). This agenda is left for future research.

Acknowledgements

Appendix A.

PROOF OF PROPOSITION 1. Debt and equity prices are made of building blocks that can be priced by simple but lengthy application of the Law of iterated expectations. First of all, the present value of one euro received the first time a threshold K is hit is well known to be $\left(\frac{V_0}{K}\right)^{-X}$. As the default threshold fully characterizes the liquidated value of the firms assets, the present value of bankruptcy costs amounts to $(1-\alpha)V_B'\left(\frac{V_0}{V_B'}\right)^{-X}$. Next, a coupon to be paid or to be received between time zero and the first hitting

time of a threshold K is given by

$$E\left[\int_0^{\tau_K} c\exp(-rs)\,ds\right] = E\left[\int_0^\infty c\exp(-rs)\,ds - \int_{\tau_K}^\infty c\exp(-rs)\,ds\right]$$

$$= \frac{c}{r} - cE\left[\int_0^\infty \exp(-r(s+\tau_K))\,ds\right]$$

$$= \frac{c}{r}\{1 - E\left[\exp(-r\tau_K)\right]\}$$

$$= \frac{c}{r}\left(1 - \left(\frac{V_0}{K}\right)^{-X}\right).$$

So we can deduce from this that receiving c until V_S is reached is worth $\frac{c}{r}\left(1 - \left(\frac{V_0}{V_S}\right)^{-X}\right)$. Finally, receiving a coupon flow c' between the first hitting time of V_S and V'_B is worth:

$$E\left[\int_{\tau_{V_S}}^{\tau_{V'_B}} c'\exp(-rs)\,ds\right] = E\left[\int_0^{\tau_{V'_B}-\tau_{V_S}} c'\exp(-rs)\,ds\exp(-r\tau_{V_S})\right]$$

$$= \frac{c'}{r}E\left[\left(1 - \exp\left(-r\left(\tau_{V'_B} - \tau_{V_S}\right)\right)\right)\exp\left(-r\tau_{V_S}\right)\right]$$

$$= \frac{c'}{r}E\left[\left(1 - \exp\left(-r\left(\tau_{V'_B} - \tau_{V_S}\right)\right)\right)\right]E\left[\exp\left(-r\tau_{V_S}\right)\right]$$

$$= \frac{c'}{r}\left(1 - \left(\frac{V_S}{V'_B}\right)^{-X}\right)\left(\frac{V_0}{V_S}\right)^{-X}.$$

Summing these three terms, we obtain the debt value given in equation (4). The equity value is deduced in a similar way. □

PROOF OF EQUITY DECREASING IN VS. We know from equation (5) that:

$$E(V, c, V_S, c', V'_B) = V - (1-\tau)\frac{c}{r}\left(1 - \left(\frac{V}{V_S}\right)^{-X}\right) - (1-\tau)\frac{c'}{r}$$

$$* \left(1 - \left(\frac{V_S}{V'_B}\right)^{-X}\right)\left(\frac{V}{V_S}\right)^{-X} - V'_B\left(\frac{V}{V'_B}\right)^{-X},$$

where $V'_B = \frac{(1-\tau)c'}{r}\frac{X}{1+X}$ and c' is a function of V_S.

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Differentiating with respect to V_S we obtain:

$$\frac{dE}{dV_S} = \underbrace{\frac{\partial E}{\partial V_S}}_{direct\ effect} + \underbrace{\left(\frac{\partial E}{\partial c'} + \frac{\partial E}{\partial V_B'}\frac{\partial V_B'}{\partial c'}\right)\frac{\partial c'}{\partial V_S}}_{indirect\ effect}.$$

We have that:

$$\frac{\partial E}{\partial V_S} = \frac{(1-\tau)}{r}\left(\frac{V_0}{V_S}\right)^{-X}\frac{X}{V_S}(c-c'),$$

$$\frac{\partial E}{\partial c'} = -\frac{(1-\tau)}{r}\left(\frac{V_0}{V_S}\right)^{-X}\left(1-\left(\frac{V_S}{V_B'}\right)^{-X}\right),$$

and

$$\frac{\partial E}{\partial V_B'} = \left(\frac{(1-\tau)}{r}\frac{X}{V_B'}c' - (X+1)\right)\left(\frac{V_0}{V_B'}\right)^{-X}$$

$$= \left(\frac{(1-\tau)}{r}\frac{X}{\frac{(1-\tau)c'}{r}\frac{X}{1+X}}c' - (X+1)\right)\left(\frac{V_0}{V_B'}\right)^{-X} = 0.$$

Replacing the three terms in the equation above we obtain:

$$\frac{dE}{dV_S} = \underbrace{\frac{\partial E}{\partial V_S}}_{direct\ effect>0} + \underbrace{\frac{\partial E}{\partial c'}\frac{\partial c'}{\partial V_S}}_{indirect\ effect<0}$$

$$= \frac{(1-\tau)}{r}\left(\frac{V_0}{V_S}\right)^{-X}\left\{\frac{X}{V_S}(c-c') - \left[1-\left(\frac{V_S}{V_B'}\right)^{-X}\right]\frac{\partial c'}{\partial V_S}\right\}. \tag{A.1}$$

Now we need to determine the derivative of c' with respect to V_S. For this we apply the implicit function theorem in equation (8), according to which the derivative of c' with respect to V_S is:

$$\frac{\partial c'}{\partial V_S} = -\frac{\partial H/\partial V_S}{\partial H/\partial c'}, \tag{A.2}$$

where

$$H(c', V_S) = c'/r + [(1-\alpha)V_B' - c'/r](V_S/V_B')^{-X} - \beta c/r$$
$$- \beta[(1-\alpha)V_B - c/r](V_S/V_B)^{-X}.$$

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We compute the partial derivatives:

$$\frac{\partial H}{\partial V_S} = \beta \left[(1-\alpha)V_B - c/r\right] X V_S^{-X-1} V_B^X - \left[(1-\alpha)V_B' - c'/r\right] X V_S^{-X-1} V_B'^X$$

$$= X V_S^{-1} \left(\frac{c'}{r} - \frac{\beta c}{r}\right),$$

where we have used equation (8) in the second equality.

$$\frac{\partial H}{\partial c'} = \frac{1}{r} - \frac{(X+1)V_S^{-X}V_B'^X}{r} + (1-\alpha)(X+1)V_S^{-X}V_B'^X \frac{(1-\tau)X}{r(X+1)}$$

$$= \frac{1}{r} \left\{1 + V_S^{-X}V_B'^X \left[(1-\alpha)(1-\tau)X - X - 1\right]\right\}.$$

Replacing the last two results in equation (A.2), we obtain:

$$\frac{\partial c'}{\partial V_S} = \frac{X V_S^{-1}(\beta c - c')}{1 + V_S^{-X}V_B'^X \left[(1-\alpha)(1-\tau)X - X - 1\right]}.$$

Coming back to the derivative of equity with respect to V_S in equation (A.1), we obtain:

$$\frac{\partial E}{\partial V_S} = \frac{(1-\tau)}{r} V_0^{-X} \left[V_S^{X-1}X(c-c') + \frac{X V_S^{-1}(\beta c - c')\left(V_B'^X - V_S^X\right)}{1 + V_S^{-X}V_B'^X[p-1]}\right]$$

$$= \frac{(1-\tau)}{r} V_0^{-X} V_S^{-1} X \left[V_S^X(c-c') + \frac{(\beta c - c')\left(V_B'^X - V_S^X\right)}{1 + V_S^{-X}V_B'^X[p-1]}\right]$$

$$= \frac{(1-\tau)}{r} V_0^{-X} V_S^{-1} X \left[\frac{V_S^X(c-c') + V_B'^X(c-c')[p-1]}{1 + V_S^{-X}V_B'^X[p-1]}\right.$$

$$\left. + \frac{(\beta c - c')\left(V_B'^X - V_S^X\right)}{1 + V_S^{-X}V_B'^X[p-1]}\right]$$

$$= \left[\frac{(1-\tau)}{r} V_0^{-X} V_S^{-1} X\right] \frac{\left(V_S^X - V_B'^X\right)(1-\beta)c + V_B'^X(c-c')p}{1 + V_S^{-X}V_B'^X[p-1]},$$

where $p = (1-\alpha)(1-\tau)X - X$.

Analyzing the sign of this result, we observe that the first factor is positive as the new coupon is inferior to the initial one. The denominator of the fraction represents the derivative of H with respect to c'. As the right hand

side of equation (8) does not depend on c', this is also the derivative of $d(V_S, c', V_B(c'))$ with respect to c'. Furthermore, as we consider coupon values strictly below the coupon that maximizes debt value at $V_S(C_{max}(V_S))$, we know from Leland (1994) that this derivative is positive in this region, so that the denominator in the fraction is positive. Finally, the numerator of the fraction is negative. Therefore we obtain that the derivative of equity with respect to V_S is negative: equity is decreasing in V_S. □

PROOF OF COROLLARY 2. (ii) Replacing $V_S^* = V_B^*$ in equation (8) we obtain equation (10). To show that the ratio c'/c solves an equation that does not depend on c, rewrite the left hand side of equation (10) as follows:

$$\frac{c'^*}{r}\left(1 - \left(\frac{V_B^*}{V_B'^*}\right)^{-X}\right) + (1-\alpha)V_B'^*\left(\frac{V_B^*}{V_B'^*}\right)^{-X} = \beta(1-\alpha)V_B^*.$$

Plugging $V_B^* = Ac^*$ and $V_B'^* = Ac'^*$ with $A = \frac{(1-\tau)}{r+0.5\sigma^2}$ in this equation leads to

$$\frac{1}{r(1-\alpha)A}\frac{c'^*}{c}\left(1 - \left(\frac{c}{c'^*}\right)^{-X}\right) + \frac{c'^*}{c}\left(\frac{c}{c'^*}\right)^{-X} = \beta.$$

(iii) Follows by definition of the new default threshold.

(iv) To derive equation (12), plug the optimal renegotiation threshold into equation (6) and replace $d(V_B^*, c'^*, V_B(c'^*))$ by the expression given in equation (10). □

PROOF OF THE DYNAMICS OF THE RENEGOTIATION SURPLUS. We know that $S_i = E_i^+(V_S) + (\beta - 1)D_i(V_S)$. For notational purposes, let $V_{Bi} = \frac{(1-\tau)c_i}{r+0.5\sigma^2} \equiv Ac_i$ and $\gamma \equiv \frac{c_{i+1}}{c_i}$. Then we can express the surpluses at time i and at time $i+1$ as:

$$S_i = V_{Bi} - \frac{(1-\tau)c_{i+1}}{r}\left[1 - \left(\frac{c_{i+1}}{V_{Bi}}\right)^X m\right] + (\beta - 1)(1-\alpha)V_{Bi}$$

$$= Ac_i - \frac{(1-\tau)\gamma c_i}{r}\left[1 - \left(\frac{\gamma}{A}\right)^X m\right] + (\beta - 1)(1-\alpha)Ac_i.$$

$$S_{i+1} = V_{Bi+1} - \frac{(1-\tau)c_{i+2}}{r}\left[1 - \left(\frac{c_{i+2}}{V_{Bi+1}}\right)^X m\right] + (\beta - 1)(1 - \alpha)V_{Bi+1}$$

$$= Ac_{i+1} - \frac{(1-\tau)\gamma c_{i+1}}{r}\left[1 - \left(\frac{\gamma}{A}\right)^X m\right] + (\beta - 1)(1 - \alpha)Ac_{i+1}$$

$$= A\gamma c_i - \frac{(1-\tau)\gamma^2 c_i}{r}\left[1 - \left(\frac{\gamma}{A}\right)^X m\right] + (\beta - 1)(1 - \alpha)A\gamma c_i$$

$$= \gamma S_i.$$

Therefore, we have $S_{i+1} = \gamma S_i = \gamma(\gamma S_{i-1}) = \cdots = \gamma^i S_1$. $\qquad\square$

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Figures

a) $c < C_{max}(V_t)$,
$V_t > \tilde{V}$

b) $c = C_{max}(V_t)$,
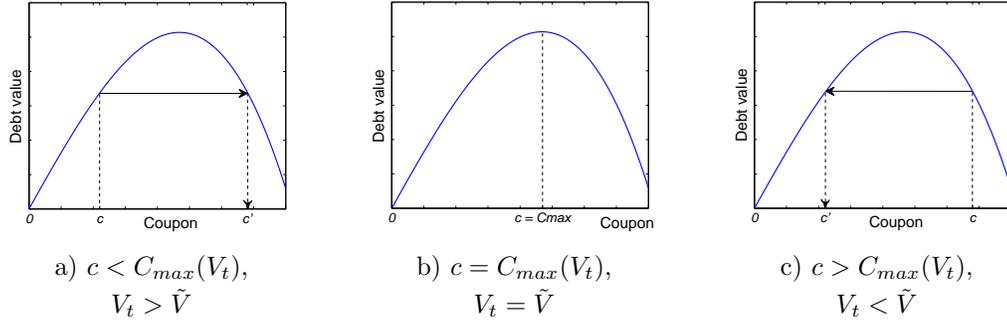$V_t = \tilde{V}$

c) $c > C_{max}(V_t)$,
$V_t < \tilde{V}$

Figure 1: Renegotiation. Debt value as a function of the coupon for three different asset values V_t. Panel a): asset value above the threshold $\tilde{V}$ (equivalent to an initial coupon value c below the coupon which maximizes the debt value C_{max}). Panels b) and c): asset value equal to $\tilde{V}$ (equivalent to c equal to C_{max}) and asset value below $\tilde{V}$ (equivalent to c above C_{max}) respectively. Renegotiation is only possible in panel c), when the initial coupon c is reduced to a new coupon value c'.

a) Maximum coupon reduction
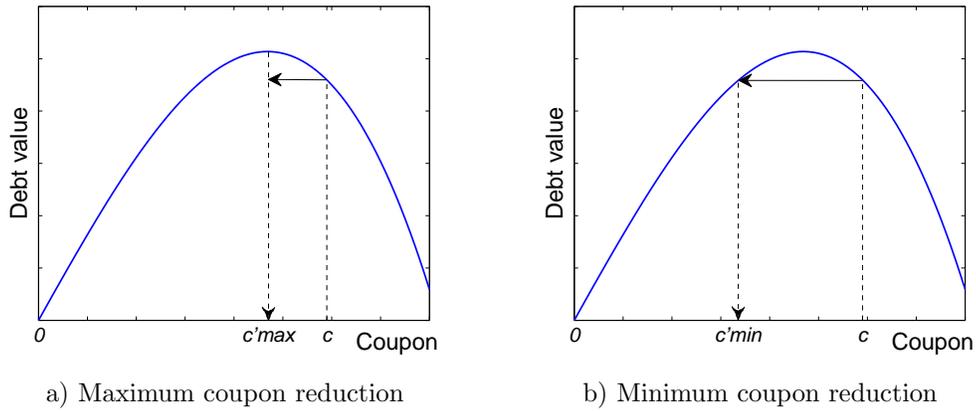
b) Minimum coupon reduction

Figure 2: Coupon Reduction Range. Debt value as a function of the coupon for two different levels of the bargaining power. Panel a): the initial coupon c will be reduced at least till a new coupon c'_{max} (in this case creditors have all bargaining power). Panel b): the initial coupon c will be reduced at most till a new coupon c'_{min} (in this case the equity holder has all bargaining power).
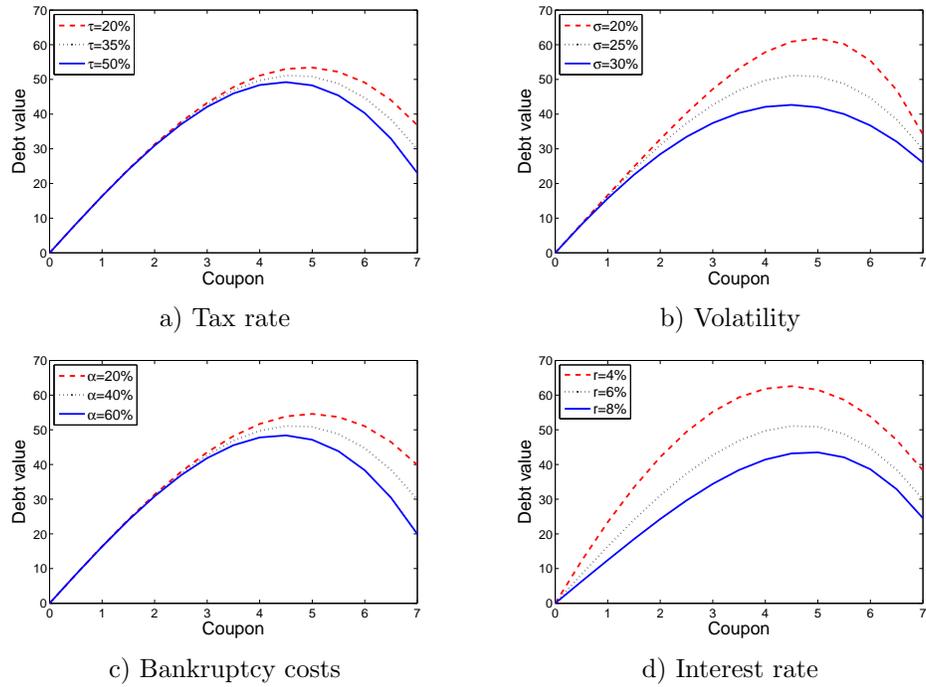
Figure 3: Comparative statics. Comparative statics of debt value as a function of the coupon with respect to four parameters: the tax rate τ (panel a)), volatility σ (panel b)), bankruptcy costs α (panel c)), and interest rate r (panel d)).
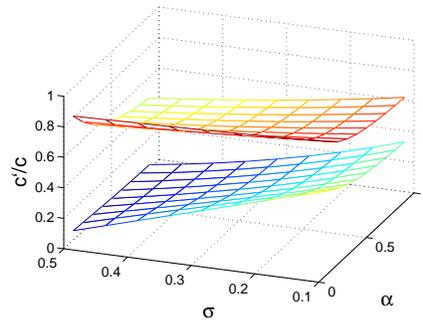

Figure 4: Maximum versus minimum coupon reduction. Relative coupon value c'/c as a function of business (σ) and financial risk (α). The upper surface represents the minimum coupon reduction, where creditors have all bargaining power. The bottom surface represents the maximum coupon reduction, where the equity holder has all bargaining power and the coupon is reduced to the minimum value accepted by creditors.
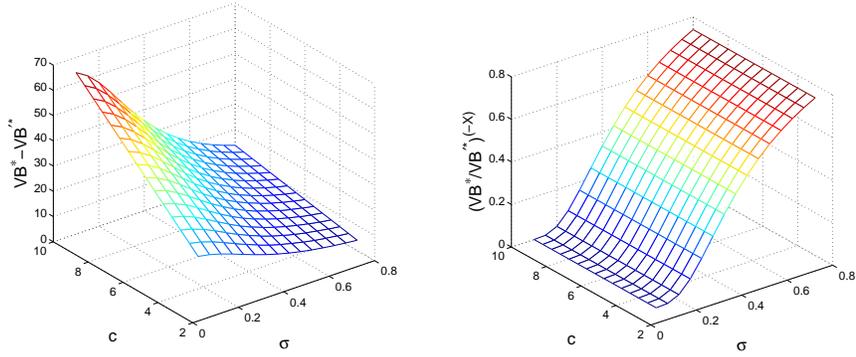
a) Difference in default thresholds   b) Proxy for default probability at V_S

Figure 5: Extra life due to renegotiation. The plots are presented as a function of the initial coupon c and volatility σ, given by two distinct measures. In panel a) the distance between the Leland default threshold V_B^* and the post-renegotiation default threshold $V_B'^*$ is plotted. Panel b) represents the present value (at renegotiation, when the asset value equals V_B^*) of \$1 to be obtained conditional on default post-renegotiation, when the asset value reaches $V_B'^*$.
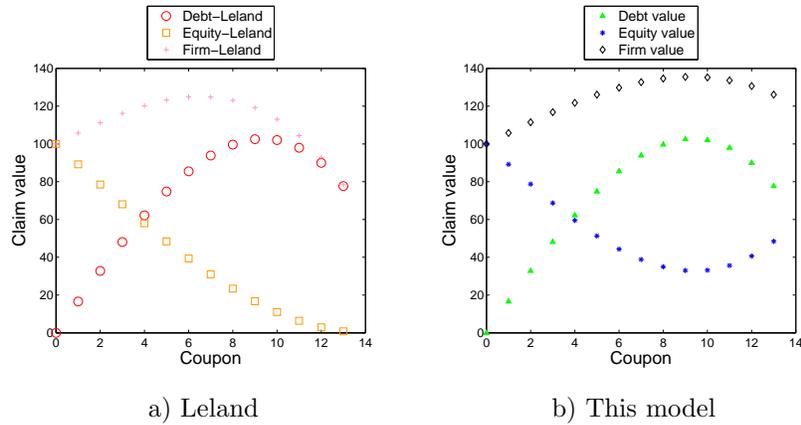
a) Leland   b) This model

Figure 6: Capital structure at inception. Leland's model is presented in panel a), while our model in panel b). Debt, equity and firm values at time 0 are plotted as a function of the coupon for $\beta = 1$, equity holder has all bargaining power.
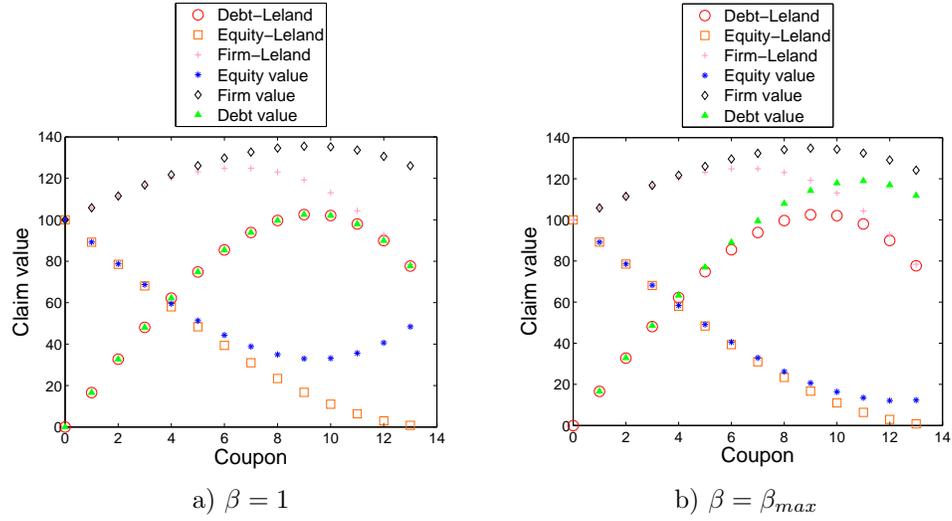
Figure 7: Sharing of the renegotiation surplus. Panel a) overlaps Figure 6 a) and b), that is, claim values given by Leland and claim values given by our model. In panel a) the equity holder has all bargaining power ($\beta = 1$). Panel b) presents the case $\beta = \beta_{max}$, where creditors have all bargaining power.
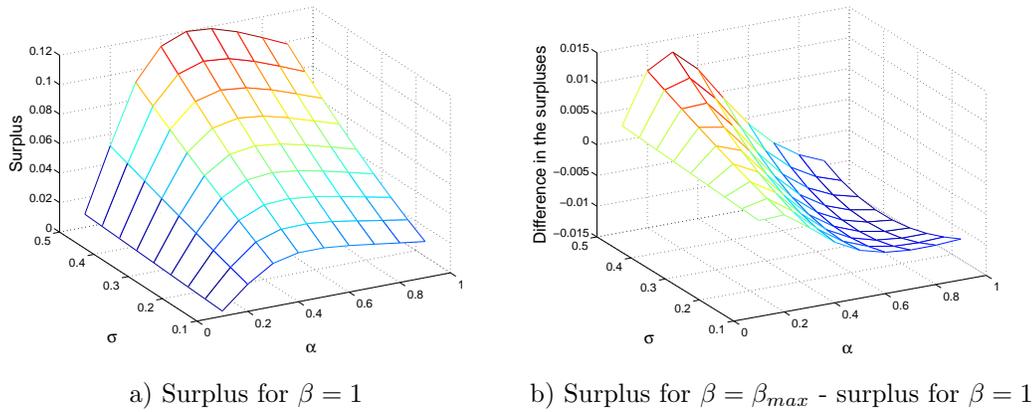

Figure 8: Renegotiation surplus. The plots are presented as a function of business and financial risk (volatility σ and bankruptcy costs α). The surplus for $\beta = 1$ is shown in panel a). The difference in the surpluses for $\beta = \beta_{max}$ and $\beta = 1$ is shown in panel b.

Tables

Table 1: Parameter values for the baseline case

Parameter	Value
V_0	100
r	6%
τ	35%
σ	25%
α	40%
c	7%

Table 2: Numerical results of the baseline case

Results	$c = 5\%$	$c = 7\%$	$c = 9\%$	$C^* = 9.27\%$	$C_{max} = 9.37\%$
$V_S^* = V_B^*$	35.62	49.86	64.11	66.03	66.75
$(V_0/V_S)^{-X}$	0.14	0.26	0.43	0.45	0.46
c'_{min}	1.37	1.91	2.46	2.53	2.56
c'_{max}	3.34	4.67	6.01	6.19	6.26

Table 3: Comparative statics

Variables/Parameters	τ	σ	α	r
$V_S^* = V_B^*$	-	-	0	-
$(V_0/V_S)^{-X}$	-	?	0	-
c'^*/c	-	-	-	+

35

Table 4: Maximum increase in debt value at renegotiation: β_{max}

Cases/Parameters	τ	σ	α	r	β_{max}
Baseline case	0.35	0.25	0.4	0.06	1.71
Variation in τ	0.15	0.25	0.4	0.06	1.39
Variation in σ	0.35	0.2	0.4	0.06	1.81
Variation in α	0.35	0.25	0.2	0.06	1.37
Variation in r	0.35	0.25	0.4	0.04	1.63

Table 5: From junk to investment-grade bond

	Junk bond (no renegotiation)	Investment-grade bond (renegotiation)
Probability of default	1	0.08
Recovery rate	0.26	0.07
Leverage	100%	50.16%
Credit spreads	1740	40